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ON MODELLING PROBLEM OF DELAYED FRACTURE PROCESSES OF CONSTRUCTIONS

Abstract

By using the kinetic equation of damage accumulation in hereditarily elastic bodies constructed by the author in the case of arbitrary time loading, the mathematical conditions of modelling delayed fracture process of given construction with the help of analogous construction from the other or same material are obtained.

Assume that it arises the necessity of prediction of strength states of construction which by its working purpose must be under mechanical stress for significant times by using the estimations of strength states of analogous construction from the same or other material for short interval of time. Such kind prediction may be satisfied by modelling fracture process of the considered construction. We'll hold the kinetic conception; we'll consider the fracture process as a process of gradual accumulation of damages of materials at deformation and durability [1,2,3]. The durability of construction is determined by time, for which the discontinuity of details of construction holds. Discontinuity, occurs in that case, when accumulated damages reach definite quantity.

Under the modelling of fracture process we'll understand the creation in model construction (in future of a model) for a bounded time interval such amount of accumulated damages that may have a full scale construction (in future a nature) in exploitation for a long time. The supposed fracture modelling pursues also the aim to determine the parameters of necessary experiments permitting to estimate the fracture process of given construction for short time interval.

We'll assume that the initial states of nature and model are natural; in this construction the initial stresses and strains are absent and they weren't subjected to the previous deformation. Under such initial data, we'll model the delayed fracture process of constructions made of hereditarily elastic material. We use the obtained formula in [4] for scalar of accumulated damages in hereditarily elastic body under the arbitrary stresses

$$\Pi\left(t,x\right)=H\left(t-t\right)\left\{-\frac{t_1^{1+\lambda}\left(\sigma_*,T\right)}{t_0^{1+\lambda}\left(\sigma_*,T\right)-t_1^{1+\lambda}\left(\sigma_*,T\right)}+\right.$$

$$\left.(1+\lambda)\int\limits_0^t\frac{(t-\tau)^{\lambda}\,d\tau}{t_0^{1+\lambda}\left(\sigma_*\left(\tau\right),T\left(\tau\right)\right)-t_1^{1+\lambda}\left(\sigma_*\left(\tau\right),T\left(\tau\right)\right)}\right\}. \tag{1}$$

Here t is the time, $x=(x_1,x_2,x_3)$ are the coordinates of points of body, $\Pi\left(t,x\right)$ is a scalar characterizing the accumulated damages, $T=T\left(t,x\right)$ is the temperature counted off from some initial temperature, $\sigma_*=\sigma_*\left(t,x\right)$ is some so-called equivalent stress. It's a function of invariant quantities: intensity of the stress $\sigma_+=\left(3s_{ij}s_{ij}/2\right)^{1/2}$ and average stress $\sigma=\sigma_{ij}\delta_{ij}/3$, where $s_{ij}=\sigma_{ij}-\sigma\delta_{ij}$ is deviator of the stress tensor σ_{ij}; δ_{ij} is a Kronecker symbol, $t_1=t_1\left(\sigma_*,T\right)$, $t_0=$

$t_0(\sigma_*, T)$ are experimentally defined functions for each material time to appearance of damages in body and time to failure for different constants $\sigma_* = \sigma_*^{(k)} = const$, $T = T_k = const \quad (k = 1, 2, ...)$ respectively, m is an experimentally defined constant; $H(t)$ is a Heavyside unit function, t is the time to appearance of damage in body under the action of arbitrary $\sigma_* = \sigma_*(t, x)$, $T = T(t, x)$. It's determined from (1) under the condition $\Pi(t, x) = 0$, which in [4] is called damage condition. If we denote the time to failure (durability) under the action of arbitrary $\sigma_* = \sigma_*(t, x)$, $T = T(t, x)$ by t_*, then by definition it is found from (1) under the condition $\Pi(t_*, x) = 1$ (long-term strength) [4]. Assume that, in the formula (1) besides of the material functions and constants, the equivalent stress $\sigma_*(t, x)$ and the temperature field $T(t, x)$ from solution of corresponding problems are also known.

We leave the quantities relative to nature without index, denote the quantities relative to models by using index "M". For relation of modelling process with the help of the formula (1) under the condition $t > t$, $t_M > t_M$, we equate $\Pi_M(t_M, x_\alpha)$ and $\Pi(t, x_\alpha)$ at the point (x_α), where the degree of accumulated damages are maximum:

$$(1+\lambda_M)\int_0^{t_M} \frac{(t_M-\tau)^{\lambda_M}\,d\tau}{t_{0M}^{1+\lambda_M}(\sigma_{*M}(\tau), T_M(\tau)) - t_{1M}^{1+\lambda_M}(\sigma_{*M}(\tau), T_M(\tau))} -$$

$$-(1+\lambda)\int_0^{t} \frac{(t-\tau)^{\lambda}\,d\tau}{t_{0}^{1+\lambda}(\sigma_{*}(\tau), T(\tau)) - t_{1}^{1+\lambda}(\sigma_{*}(\tau), T(\tau))} =$$

$$= \frac{t_1^{1+\lambda_M}(\sigma_{*M}, T_M)}{t_{0M}^{1+\lambda_M}(\sigma_{*M}, T_M) - t_{1M}^{1+\lambda_M}(\sigma_{*M}, T_M)} - \frac{t_1^{1+\lambda}(\sigma_*, T)}{t_0^{1+\lambda}(\sigma_*, T) - t_1^{1+\lambda}(\sigma_*, T)} \tag{2}$$

Since the equation (2) contains the sufficient arbitrariness, it allows to realize the modelling of fracture process of given construction with the help of analogous construction from the other or same material. If the modelling time t_M is given, then the modelling condition (2) σ_{*M} and by the same taken the parameters of exterior load which must be applied to model construction, may be determined. If the parameters of exterior loading from (2) are given we can determine the modelling time t_M.

Note that the modelling condition (2) doesn't impose the constraint on coincidence of geometrical dimensions of models and natures, they must be only geometrical similar. Besides, note that it's necessary to realize the modelling of failure of constructions from metal working at the higher temperature by using the constructions from plastic, only at monotone load, since a lot of metals at elevated temperature unlike plastic, have a property to have residual deformation at off loading. Experimental data processing [4] shows that for most materials it can be approximately accepted: $t_1(\sigma_*, T)/t_0(\sigma_*, T) \approx A = const$, $t_{1M}(\sigma_{*M}, T_M)/t_{0M}(\sigma_{*M}, T_M) \approx A_M = const$. Subject to these approximations, the modelling condition (2) is rewritten in the following form:

$$\left(1-A^{1+\lambda}\right)\left[-A_M^{1+\lambda_M}+(1+\lambda_M)\int_0^{t_M}\frac{(t_M-\tau)^{\lambda_M}\,d\tau}{t_{0M}^{1+\lambda_M}\left(\sigma_{*M}(\tau),T_M(\tau)\right)}\right]=$$

$$=\left(1-A_M^{1+\lambda_M}\right)\left[-A^{1+\lambda}+(1+\lambda)\int_0^{t}\frac{(t-\tau)^{\lambda}\,d\tau}{t_0^{1+\lambda}\left(\sigma_*(\tau),T(\tau)\right)}\right] \qquad (3)$$

Now consider the concrete approximation of material functions t_0 and t_{0M}. We select two variants of approximations

variant 1

$$t_0=t_{s1}\left(\frac{\sigma_*}{\sigma_{s1}}\right)^{-\alpha}\left(\frac{T_{s1}}{T}\right)^{æ};\quad t_{0M}=t_{s1M}\left(\frac{\sigma_{*M}}{\sigma_{s1M}}\right)^{-\alpha_M}\left(\frac{T_{s1M}}{T_M}\right)^{æ_M} \qquad (4)$$

variant 2

$$\begin{gathered}t_0=t_{s2}\exp\left[\beta\left(1-\tfrac{\sigma_*}{\sigma_{s2}}\right)+d\left(1-\tfrac{T}{T_{s2}}\right)\right]\\ t_{0M}=t_{s2M}\exp\left[\beta_M\left(1-\tfrac{\sigma_{*M}}{\sigma_{s2M}}\right)+d_M\left(1-\tfrac{T_M}{T_{s2M}}\right)\right]\end{gathered} \qquad (5)$$

Here $\alpha, æ, \alpha_M, æ_M, \beta, \beta_M, d, d_M$ are constants which for each material are determined from experimental curves of long-term strength: $\sigma_{s1}, \sigma_{s1M}, \sigma_{s2}, \sigma_{s2M}$ are stresses, $T_{s1}, T_{s1M}, T_{s2}, T_{s2M}$ are the temperatures of reduction to pure quantities which are constant and are chosen from variation interval $\sigma_*, \sigma_{*M}, T, T_M$; $t_{s1}, t_{s1M}, t_{s2}, t_{s2M}$ are the times to failure at $\sigma_*=\sigma_{s1}$, $T=T_{s1}$; $\sigma_{*M}=\sigma_{s1M}$, $T_M=T_{s1M}$; $\sigma_*=\sigma_{s2}$, $T=T_{s2}$; $\sigma_{*M}=\sigma_{s2M}$, $T_M=T_{s2M}$ respectively.

Subject to (4) the modelling condition of fracture process (3) is transformed in the following from

$$\int_0^{t_M}(t_M-\tau)^{\lambda_M}\,\sigma_{*M}^{\alpha_M(1+\lambda_M)}(\tau)\,T_M^{æ_M(1+\lambda_M)}(\tau)\,d\tau=$$

$$=Q_1+Q_2\int_0^{t}(t-\tau)^{\lambda}\,\sigma_*^{\alpha(1+\lambda)}(\tau)\,T^{æ(1+\lambda)}(\tau)\,d\tau. \qquad (6)$$

The following notations are accepted in condition (5)

$$Q_1=T_{s1M}^{æ_M(1+\lambda_M)}Q_3;\ Q_2=\frac{T_{s1M}^{æ_M(1+\lambda_M)}}{T_{s1}^{æ(1+\lambda)}}Q_4;$$

$$Q_3=\frac{\left(A_M^{1+\lambda_M}-A^{1+\lambda}\right)t_{s1M}^{1+\lambda_M}\sigma_{s1M}^{\alpha_M(1+\lambda_M)}}{(1+\lambda_M)\left(1-A^{1+\lambda}\right)};$$

$$Q_4=\frac{(1+\lambda)\left(1-A_M^{1+\lambda_M}\right)t_{s1M}^{1+\lambda_M}\sigma_{s1M}^{\alpha_M(1+\lambda_M)}}{(1+\lambda)\left(1-A^{1+\lambda}\right)t_{s1}^{1+\lambda}\sigma_{s1}^{\alpha(1+\lambda)}}.$$

By using (5) in condition (3) the following expressions are obtained

$$\int_0^{t_M}(t_M-\tau)^{\lambda_M}\exp\left[(1+\lambda_M)\left(\beta_M\frac{\sigma_{*M}(\tau)}{\sigma_{s2M}}+d_M\frac{T_M(\tau)}{T_{s2M}}\right)\right]d\tau=$$

$$=D_1+D_2\int_0^{t}(t-\tau)^{\lambda}\exp\left[(1+\lambda)\left(\beta\frac{\sigma_*(\tau)}{\sigma_{s2}}+d\frac{T(\tau)}{T_{s2}}\right)\right]d\tau. \quad (7)$$

Here $D_1=D_3\exp\left[(1+\lambda_M)d_M\right],\ D_2=D_4\exp\left[(1+\lambda_M)d_M-(1+\lambda)d\right]$;

$$D_3=\frac{t_{s2M}^{1+\lambda_M}\left(A_M^{1+\lambda_M}-A^{1+\lambda}\right)\exp\left[(1+\lambda_M)\beta_M\right]}{(1+\lambda_M)\left(1-A^{1+\lambda}\right)};$$

$$D_4=\frac{t_{s2M}^{1+\lambda_M}(1+\lambda)\left(1-A_M^{1+\lambda_M}\right)}{t_{02}^{1+\lambda}(1+\lambda_M)\left(1-A^{1+\lambda}\right)}\exp\left[(1+\lambda_M)\beta_M-(1+\lambda)\beta\right].$$

In the case of isothermal loading ($T=T_{s1},\ T_M=T_{s1M}$ or $T=T_{s2},\ T_M=T_{s2M}$) the conditions (6) and (7) have the form

$$\int_0^{t_M}(t_M-\tau)^{\lambda_M}\sigma_{*M}^{\alpha_M(1+\lambda_M)}(\tau)\,d\tau=W_3+Q_4\int_0^{t}(t-\tau)^{\lambda}\sigma_*^{\alpha(1+\lambda)}(\tau)\,d\tau \quad (8)$$

$$\int_0^{t_M}(t_M-\tau)^{\lambda_M}\exp\left[(1+\lambda_M)\frac{\sigma_{*M}(\tau)}{\sigma_{s2M}}\right]d\tau=$$

$$=D_3+D_4\int_0^{t}(t-\tau)^{\lambda}\exp\left[(1+\lambda)\beta\frac{\sigma_*(\tau)}{\sigma_{s2}}\right]d\tau. \quad (9)$$

If the full-scale and model construction are made from the same material, then their mechanical characterizations coincide $\lambda_M=\lambda,\ \alpha_M=\alpha,\ æ_M=æ,\ \beta_M=\beta,\ d_M=d$. Besides if we take $\sigma_{s1M}=\sigma_{s1},\ T_{s1M}=T_{s1}$ (the first variant) , then we'll have $t_{s1M}=t_{s1}$. Moreover, if we take $\sigma_{s2M}=\sigma_{s2},\ \ T_{s2M}=T_{s2}$ (the second variant), then we obtain $t_{s2M}=t_{s2}$. in addition $Q_1=Q_3=0,\ Q_2=Q_4=1;\ D_1=D_3=0,\ D_2=D_4=1$.

In this case the conditions (6) and (7) are converted to the following relations, respectively

$$\int_0^{t_M}(t_M-\tau)^{\lambda}\sigma_{*M}^{\alpha(1+\lambda)}T_M^{æ(1+\lambda)}(\tau)=\int_0^{t}(t-\tau)^{\lambda}\sigma_*^{\alpha(1+\lambda)}(\tau)\,T^{æ(1+\lambda)}(\tau)\,d\tau \quad (10)$$

$$\int_0^{t_M} (t_M-\tau)^\lambda \exp\left[(1+\lambda)\left(\beta\frac{\sigma_{*M}(\tau)}{\sigma_{s2}}+d\frac{T_M(\tau)}{T_{s2}}\right)\right]d\tau=$$

$$=\int_0^{t} (t-\tau)^\lambda \exp\left[(1+\lambda)\left(\beta\frac{\sigma_{*}(\tau)}{\sigma_{s}}+d\frac{T(\tau)}{T_{s}}\right)\right]d\tau. \qquad (11)$$

In the case of isothermal loading of full-scale and model construction from the same material, we'll have the following modelling conditions which correspond to considered above variants 1 and 2

$$\int_0^{t_M} (t_M-\tau)^\lambda \sigma_{*M}^{\alpha(1+\lambda)}(\tau)\,d\tau=\int_0^{t} (t-\tau)^\lambda \sigma_{*}^{\alpha(1+\lambda)}(\tau)\,d\tau \qquad (12)$$

$$\int_0^{t_M} (t_M-\tau)^\lambda \exp\left[(1+\lambda)\,\beta\frac{\sigma_{*M}(\tau)}{\sigma_{s2}}\right]d\tau=$$

$$=\int_0^{t} (t-\tau)^\lambda \exp\left[(1+\lambda)\,\beta\frac{\sigma_{*}(\tau)}{\sigma_{s2}}\right]d\tau. \qquad (13)$$

It follows to note that the conditions (6) and (7), (8) and (9), (12) and (13) are alternative modelling conditions. For he given modelling time of fracture process they determine the external force which have to act on model construction, in order to create in it such amount of damages that has full-scale construction in time t under the action in it of the given impressed force and vice versa.

Finally note that the choice for application of one of the obtained relations as a modelling condition is dictated basically by experimentally damage curves processing, by long-term strength of materials of model and full-scale construction, their stress state and temperature fields.

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