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ASYMPTOTIC ESTIMATIONS OF EIGENFREQUENCIES OF VIBRATIONS OF STIFENFED CONSTRUCTIONS, FILLED WITH LIQUID

Abstract

At the given paper the eigen frequencies of oscillations of cylindrical shell reinforced by regular system of longitudinal ribs in ideal compressible fluid is investigated. The frequency equations are constructed and realized numerically.

The given paper is devoted to the investigation of eigenfrequencies of oscillations of cylindrical shell, reinforced by regular system of longitudinal ribs, filled with ideal compressible liquid. Under the regular system of longitudinal ribs for shell we understand such a system, in which rigidity of all ribs, their mutual distances are equal. At the given paper the shell is modeleted as structurally-orthotrophic and according to [1], its motion is described by the following system of equations:

$$\left[\left(1+\gamma_c^1\right)\frac{\partial^2}{\partial\xi^2}+\frac{1-\upsilon}{2}\frac{\partial^2}{\partial\theta^2}\right]u+\frac{1+\upsilon}{2}\frac{\partial^2\nu}{\partial\xi\partial\theta}-$$

$$-\left(\upsilon\frac{\partial}{\partial\xi}+\delta_c^1\frac{\partial^3}{\partial\xi^3}\right)w-\rho_1\frac{\partial^2u}{\partial t_1^2}=0,$$

$$\frac{1+\upsilon}{2}\frac{\partial^2\nu}{\partial\xi\partial\theta}+\left[\frac{1-\upsilon}{2}\left(1+4a^2\right)\frac{\partial^2}{\partial\xi^2}+\left(1+a^2\right)\frac{\partial^2}{\partial\theta^2}\right]\nu+$$

$$+\left[-\frac{\partial}{\partial\theta}+(2-\upsilon)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+a^2\frac{\partial^3}{\partial\theta^3}\right]w-\frac{\partial^2u}{\partial t_1^2}=0,$$

$$-\left(\upsilon\frac{\partial}{\partial\xi}+\delta_c^1\frac{\partial^3}{\partial\xi^3}\right)u+\left[-\frac{\partial}{\partial\theta}+(2-\upsilon)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+a^2\frac{\partial^3}{\partial\theta^3}\right]\nu+$$

$$+\left[1+a^2\Delta\Delta+\eta_c^1\frac{\partial^4}{\partial\xi^4}\right]w++\rho_1\frac{\partial^2w}{\partial t_1^2}=\frac{R^2\left(1-\upsilon^2\right)}{Eh}q_z \tag{1}$$

Here u,ν,w-are components of vector of a shell moving, E,υ-is a Young's modulus and Poisson's coefficient of shell material respectively, h,R-is a thickness and radius of shell respectively, $\rho_1=1+\bar{\rho}_c\bar{\gamma}_c^1$, $\bar{\gamma}_c^1$-is a relation of all ribs to the shell weight, $\xi=\frac{x}{R}$, $\theta=\frac{y}{R}$, $t_1=\omega_0t$, $\omega_0=\sqrt{\frac{E}{(1-\upsilon^2)\rho_0R^2}}$, ρ_0-is a density of shell material, $\bar{\rho}_c=\frac{\rho_c}{\rho_0}$, ρ_c-is a density of ribs material,

$$\gamma_c^1=\frac{E_c}{E}\left(1-\upsilon^2\right)\bar{\gamma}_c^1,\quad \eta_c^1=\gamma_c^1\left[\frac{a}{12}\psi_1\bar{\gamma}_c^1h^2+\left(\frac{h_c}{R}\right)^2\right],\ h^2=\frac{h}{R},$$

$a_1 = \frac{2\pi R}{kh}$-is a distance between ribs, referred to thickness of shell, $\delta_c^1 = \frac{h_c}{R}\gamma_c^1$, $a^2 = \frac{h^2}{12R^2}$ q_z-is a pressure from the direction of liquid on the shell.

The linearized wave equation, describing distribution of small disturbances in ideal compressible liquid has the following form [2]:

$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \tag{2}$$

where Φ-is a potential of liquid, c-is a speed of propagation of sound in liquid.

The equations of motion of shell (1) and liquid (2) are implemented by the contact conditions. On contact surface shell-liquid, continuity of radial speeds and pressures

$$\nu_r = \frac{\partial w}{\partial t}, \quad q_z = -P \tag{3}$$

is observed.

The acoustic pressure P and radial speed ν_r in liquid are determined in the following form [2]:

$$P = -\rho_0\frac{\partial\Phi}{\partial t}, \quad \nu_r = \frac{\partial\Phi}{\partial r}. \tag{4}$$

Here ρ_0- is a density of liquid.

Implementing by contact conditions (3) equations of motion of shell and liquid, we come to a contact problem on eigen vibrations stiffened to a shell, filled with liquid. In other words, the problem on eigen vibrations of stiffened cylindrical shell filled with liquid, is reduced to joint integration of equations of shell theory and liquid at fulfilment of the indicated conditions on a surface of their contact:

Moving of shell we'll seek in the following form:

$$u = u_0 e^{\chi\xi}\cos n\theta\cos\omega_1 t_1,$$

$$\nu = \nu_0 e^{\chi\xi}\sin n\theta\cos\omega_1 t_1 \tag{5}$$

$$w = w_0 e^{\chi\xi}\cos n\theta\cos\omega_1 t_1.$$

Here u_0, ν_0, w_0-are unknown constants.

The potential of speeds Φ in liquid we accept in the form:

$$\Phi = Ae^{\chi\xi}K_n(\gamma r)\cos n\theta\sin\omega_1 t_1, \tag{6}$$

where $\gamma^2 = \chi^2 - \frac{\omega_1^2}{c^2}$, K_n- are Bessel functions of the second genus of the n-th order, A-is a constant.

Using formula (4) and contact conditions (3) we can express the constant A through w_0. After the substitution (5) into (1) the problem is reduced to homogeneous system of linear algebraic equations of the third order, whose nontrivial solution is possible only in a case, when ω_1-is a root of its determinant. The equation relative to ω_1 is transcendent, since it is contained in the argument of Bessel function. We can represent this equation in the following form:

$$a_1(\omega_1)\,\omega_1^6 + a_2(\omega_1)\,\omega_1^4 + a_3(\omega_1)\,\omega_1^2 + a_4(\omega_1) = 0. \tag{7}$$

In case, when $\omega_1^2 >> c^2$ the equation (7) becomes an algebraic equation of the sixth order. The coefficients $a_i(\omega_1)$ $(i = 1, 2, ..., 5)$ have inconvenient form, therefore here we don't inscribe them. Note, only that they depend on above reduced geometrical and physical properties, describing materials of ribs and shell. The equation (7) is realized numerically. At calculations it is accepted

$$E = E_c = 6.67 \cdot 10^9 \frac{H}{m^2},\ \rho_0 = \rho_c = 0.26 \cdot 10^4 \frac{Hc^2}{m^4},\quad \upsilon = 0.3,$$

$$R = 160\ mm, h = 0.45\ mm,\ \psi_1 = 10,\quad \varphi_1 = 0.47,\ \chi = 2,\ \frac{h_c}{R} = 0.04$$

The results of investigation of influencing of number of ribs on frequencies of eigen vibrations of cylindrical shell, reinforced by longitudinal ribs and filled with liquid are shown in fig.1. The analysis shows, that with increasing ribs quantity and their thickness frequencies of eigen vibrations of considered constructions increase.

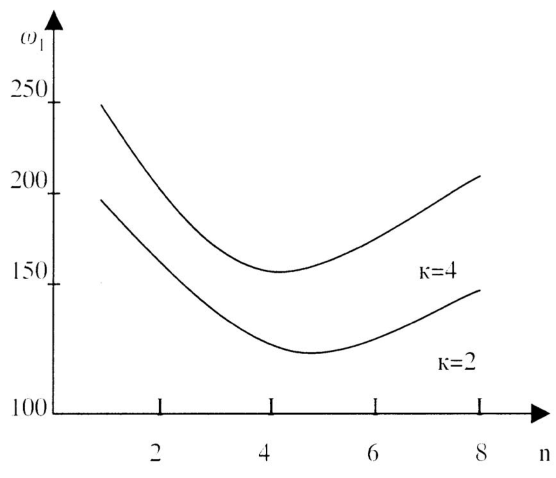

Fig.1 Dependence of eigenfrequency on n.

References

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