Vagif D. GADJIEV, Sabira R. AKHADOVA

APPROXIMATE - ANALYTICAL SOLUTION OF STABILITY PROBLEM OF NONHOMOGENEOUS ELASTIC RECTANGULAR PLATE LYING ON NONHOMOGENEOUS BASE

Abstract

In the paper the stability problem of nonuniformly elastic nonhomogeneous plate under the action of two-sided pressure is considered. It is assumed that modulus of elasticity depends on two space coordinates, and the Poisson's coefficient is a constant quality.

Differential equation is partial differential equation with variable coefficients.

Solution is constructed by means of method of small parameter and Burnov-Galerkin method.

As is known [1,2,4] solutino of stability problem of nonhomogeneous plates and shells is based on application of numerical and approximative analytical methods. In the given paper solving of the stated problem will be realized by means of method of small parameter and Bubnov-Galerkin method.

Let's rectangular plate of dimension $(a \times b \times h)$ be under the action of two-sided pressure with loading of intensities $\underline{P}$ and Q.It$'$s assumed that modulus of elasticity is the function of coordinate of the width and lenth of the plate. The coordinate system was choosen in the following form: axes X and Y are on mean plane and Z axis is perpendicular to them

$$E = E_0 f_D (x) \cdot f_2 (z) , \quad v = const. \tag{1}$$

Here E_0 corresponds to homogeneous case, function $f_0 (x)$ itself and its derivatives till the second order inclusive are continuous functinos. On the basis of [4] one can show that in the given case stability equation subject to resistance of the base has the following form:

$$\begin{gathered} (1 + \varepsilon f_1 (x)) \nabla w + 2\varepsilon f_1' (x) \frac{\partial}{\partial x} (\triangle w) + \\ + \varepsilon f_1'' (x) \left(\frac{\partial^2 w}{\partial x^2} + v \frac{\partial^2 w}{\partial y^2} \right) + \tilde{k} (1 + \varepsilon \varphi (x)) w - \\ - \underline{P}^* \left(\frac{\partial^2 w}{\partial x^2} + \beta \frac{\partial^2 w}{\partial y^2} \right) = 0. \end{gathered} \tag{2}$$

Here $\varphi (x)$ is a continuous function and $0 \leq \varepsilon \ll 1$ holds. Here also the following denotations were accepted:

$$\tilde{k} = (D_0 J)^{-1} ; \;\; J = 12 \int_{0,5}^{0,5} f (p) \rho^2 d\rho ; \;\; \underline{P}^* = \underline{P} (D_0 J)^{-1}$$

$$\rho = z \cdot h^{-1};\ \beta = P \cdot Q^{-1},\quad D_0 = \frac{E_0}{12\left(1-v^2\right)h^3}$$

Here ∇ and $\triangle$ are biharmonic and harmonic operators respectively.

We'll seck for solution of (2) in the following form:

$$w = w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + ... + \varepsilon^n w_n$$

$$P^* = P_0^* + \varepsilon P_1^* + \varepsilon^2 P_2^* + ... + \varepsilon^n P_n^*. \tag{3}$$

Substituting (3) into (2) and euating the terms with the same power of parameter ε we'll obtain the following system of equations

$$\nabla w_0 + \tilde{k} w_0 - P_0^* \left(\frac{\partial^2 w_0}{\partial x^2} + \beta \frac{\partial^2 w_0}{\partial y^2}\right) = 0$$

$$\nabla w_1 + \tilde{k} w_1 - P_1^* \left(\frac{\partial^2 w_1}{\partial x^2} + \beta \frac{\partial^2 w_1}{\partial y^2}\right) = -f_1(x)\nabla w_0 - 2f_1'(x)\frac{\partial}{\partial x}\left(\nabla w_0\right) -$$

$$-f_1''(x)\left(\frac{\partial^2 w_0}{\partial x^2} + \beta \frac{\partial^2 w_0}{\partial y^2}\right) - \tilde{k}\varphi(k)\, w_0, \tag{4}$$

....

$$\nabla w_n + \tilde{k} w_n - P_n^* \left(\frac{\partial^2 w_n}{\partial x^2} + \beta \frac{\partial^2 w_n}{\partial y^2}\right) = -f_1(x)\nabla w_{n-1} -$$

$$-2f_1'(x)\frac{\partial}{\partial x}\left(\nabla w_{n-1}\right) - f_1''(x)\left(\frac{\partial^2 w_{n-1}}{\partial x^2} + \beta \frac{\partial^2 w_{n-1}}{\partial y^2}\right) - \tilde{k}\varphi(k)\, w_{n-1}.$$

Note that in many cases when nonhomogeneous plates are produced function $f_1(x)$ is linear one and stability equation is simplified [7] and system (4) takes on the following form: $\left(f_1'' = a^{-1}x\right)$:

$$\nabla w_0 + \tilde{k} w_0 - P_0^* \left(\frac{\partial^2 w_0}{\partial x^2} + \beta \frac{\partial^2 w_0}{\partial y^2}\right) = 0$$

$$\nabla w_1 + \tilde{k} w_1 - P_1^* \left(\frac{\partial^2 w_1}{\partial x^2} + \beta \frac{\partial^2 w_1}{\partial y^2}\right) = -a^{-1}x\nabla w_0 - 2a^{-1}\frac{\partial}{\partial x}\left(\triangle w_0\right) - \tilde{k}\varphi(x)\, w_0$$

....

$$\nabla w_n + \tilde{k} w_n - P_n^* \left(\frac{\partial^2 w_n}{\partial x^2} + \beta \frac{\partial^2 w_n}{\partial y^2}\right) =$$

$$-a^{-1}x\nabla w_{n-1} - 2a^{-1}\frac{\partial}{\partial x}\left(\triangle w_{n-1}\right) - \tilde{k}\varphi(x)\, w_{n-1}. \tag{5}$$

Note that for l_0 the multifold pinning solution at zero approximation can be found in the following form:

$$w_0 = c_{mn}\sin\frac{m\pi}{\alpha}x \cdot \sin\frac{n\pi}{b}y. \tag{6}$$

In this case from the first equation of (5) subject to (6) we'll obtain

$$P_0^* = \frac{\left(\frac{m\pi}{\alpha}\right)^4\left[1 + \left(\frac{n}{m}\frac{a}{b}\right)^2\right]^2 + \tilde{k}}{1+\beta}. \tag{7}$$

Note that if $\beta = 0$ we obtain solution of analogous problem for one-sided pressure. Condition $\tilde{k} = 0$ corresponds to solution of problem for dissimilar on thickness plate disregarding the resistance of base. Formula (7) is solution of problem for dissimilar on thickness plate with regard to Winkler resistance.

System of equations (5) beginning with the first approximation can be written in the following form

$$\nabla w_{i-1} + \tilde{k} w_{i-1} - P_{i-1}^{*}\left(\frac{\partial^2 w_{i-1}}{\partial x^2} + \beta \frac{\partial^2 w_{i-1}}{\partial y^2}\right) = F_{i-1}(x,y)\,,\ i = 2,3,... \qquad (8)$$

where

$$F_{i-1} = -a^{-} x \nabla w_{i-1} - 2a^{-1}\frac{\partial}{\partial x}(\triangle w_{i-1}) - \tilde{k}\varphi(x)\, w_{i-1}, \qquad (9)$$

One can solve equation (8) by the one of analytical methods. In the given case we'll apply the Bubnov-Galerkin method, moreover we'll seek for functions $w_{i-1}(x,y)$ in the following form:

$$w_{i-1} = c_{i-1}\psi_{i-1}(x)\,\eta_{i-1}(y)\,. \qquad (10)$$

Here $\psi_{i-1}(x)$ and $\eta_{i-1}(y)$ satisfy the correspondent boundary conditions.

Taking into account (10) in (8) subject to the above-mentioned method we'll obtain P_1^* in the following form:

$$P_1^* = \left(\sqcap_{i,1} + \tilde{k}\sqcap_{i,2} - \sqcap_{i,3}\right)\sqcap_{i,0}^{-1}\,. \qquad (11)$$

Here the following denotations were accepted

$$\sqcap_{i,0} = c_i \int_0^a\int_0^b \left(\eta_{i-1}(y)\frac{d^2\psi_{i-1}}{dx^2} + \beta\psi_{i-1}(x)\frac{d^2\eta_{i-1}}{dy^2}\right)\psi_{i-1}(x)\,\eta_{i-1}(y)\,dxdy$$

$$\sqcap_{i,1} = c_i \int_0^a\int_0^b \left(\eta_{i-1}(y)\frac{d^4\psi_{i-1}}{dx^4} + 2\frac{d^2\psi_{i-1}}{dx^2}\frac{d^2\eta_{i-1}}{dy^2} + \psi_{i-1}(x)\frac{d^4\eta_{i-1}}{dy^4}\right)\times$$
$$\times\psi_{i-1}(x)\,\eta_{i-1}(y)\,dxdy$$

$$\sqcap_{i,2} = c_i \int_0^a\int_0^b \psi_{i-1}^2(x)\,\eta_{i-1}^2(y)\,\varphi(x)\,dxdy, \qquad (12)$$

$$\sqcap_{i,3} = c_i \int_0^a\int_0^b F_{i-1}(x,y)\,\psi_{i-1}(x)\,\eta_{i-1}(y)\,dxdy.$$

Setting particular values of $\psi_i(x)$, $\eta_i(y)$, $\varphi(x)$ from (10) one can easily find the value of P_i^*.

The simplest case is cylindrical form of buckling at $\beta = 0$.

For this case $\sqcap_{1,0}, \sqcap_{1,1}, \sqcap_{1,2}, \sqcap_{1,3}$ have the following values

$$\sqcap_{1,0} = c_1\int_0^a\left(\psi_1\frac{d^2\psi_1}{dx^2}\right)dx;\quad \sqcap_{1,1} = c_1\int_0^a\left(\psi_1\frac{d^4\psi_1}{dx^4}\right)dx$$

$$\sqcap_{1,2} = c_1 \int_0^a \psi_1^2 (x)\, dx; \ \sqcap_{1,3} = c_1 \int_0^a F_1 (x)\, \psi_1 (x)\, dx \tag{13}$$

Introducing denotation $\bar{P} = P \cdot P_e^{-1}$ we'll obtain the following relation

$$\bar{P} = \left(1 + k \cdot \nu_0^{-1} \cdot \sqcap_{1,2} \cdot \sqcap_{1,0}^{-1} - \sqcap_{1,3} \cdot \sqcap_{1,0}\right) \cdot J^{-1} \tag{14}$$

$$\left(F_1 = -a^{-1} \cdot x \frac{d^4 w_0}{dx^4} - 2a^{-1} \frac{d^3 w_0}{dx^3} - k\varphi (x)\, w_0\right)$$

Here P_e corresponding to the values of Euler loading.

From formula (14) at $J = 1,\ \tilde{k} = 0, \sqcap_{i,3} = 0$ we'll obtain the value $P_e = 1$ i.e. $\bar{\mathrm{P}}$=1

Calculations were carried out for the cases

$a)$ $f_2 = 1 + \varepsilon\rho$; $a) f_2 = 1 + \varepsilon\rho^2$; $b) f_3 = 1 + \varepsilon e^{\rho}$,where $\varepsilon \in [0, 1]$.

Results of the carried out calculations are cited in the form of tables and dependence graphs between typical parameters.

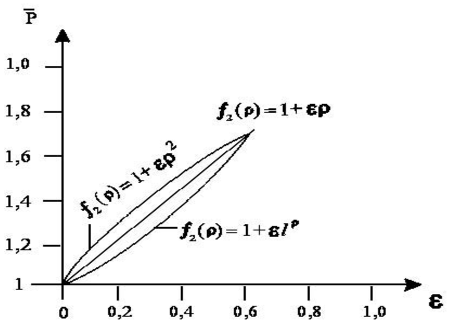

Fig. 1.

Tab. 1.

ε	$\bar{P}$	$\bar{P}$
0,2	1,208	1,118
0,4	1,460	1,365
0,6	1,668	1,649
f_2	$1+\varepsilon\rho^2$	$1+\varepsilon l^{\rho}$

References

[1]. Lomakin V.A. *On application of method of small parameter for estimation of stresses in nonhomogeneous elastic media.* MTT, 1972, p.35-39. (Russian)

[2].ShikulaY.N. *Method of small parameter in corrected stability problem of plates of variable rigidity PM*, v.24, No4, 1983, p.128-131. (Russian)

[3].Volmir A.S.*Stability of deformable systems.* M., 1976. (Russian)

[4].Gadjiyev V.D.,Sofiyev A.G.,Mirzoyev R.D. *On eigen oscillation of nonhomogeneous rectangular plates.* Trudy IMM AN Azerbaijan, v.IV (XII), Baku, 1986, p.103-109. (Russian)

[5]. Boliashvili P.A. *On some problems of stability of rods at variable elasticity modulus.* I.A. "Stroitel'stva i arkhitektura" 36, Gruz, NII TI, Tbilisi, 1985, 8p. (Russian)

[6]. El'sgolts L.E. *Differential equation and calculus of variations.* M., 1965. (Russian)

[7]. Gutas V. *Effect of nonhomegeneous elastic plastic transition in thin rotating disco.* Indian Pure and Apple. math., 1994 25.10, p.1089-1097.

Vagif D. GADJIEV,
Institute of Mathematics & Mechanics of NAS of Azerbaijan.
9, F.Agayev str., 370141, Baku, Azerbaijan.
Tel.:38-65-13(apt.).

Sabira R. AKHADOVA
Gandja State University.
Shakh Ismail Khatai str., 187, 370000, Gandja, Azerbaijan.

Received September 10, 2002; Received November 25, 2002.
Translated by Agayeva R.A