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ON EXISTENCE OF ABSORBING SET ON INITIAL BOUNDARY VALUE PROBLEM FOR A NON-LINEAR DEGENERATE EQUATION

Abstract

The paper is devoted to the investigation of properties of solution of initial-boundary value problem for a non-linear degenerate equation. In particular the existence of absorbing set for the considered problem is proved.

In the paper the qualitative properties of a nonlinear parabolic equation with generation

$$\frac{\partial u}{\partial t}-\frac{\partial}{\partial x}\left(\left(|u|^{p_0}+\left|\frac{\partial u}{\partial x}\right|^{p_1}\right)\frac{\partial u}{\partial x}\right)+d\,|u|^{\gamma}\,u+c\,(x)=0 \qquad (*)$$

are investigated.

Some results on smoothness of solution of similar type equations are contained in [1], [2].

We also note that the solvability problems for similar equations under the sufficiently general conditions on non-linearity are considered in [3], [4].

In the present paper the results on smoothness of initial-boundary value problem for the equation (*), obtained in [5] (in the case $d\equiv 0$) are generalized by means of which the existence of absorbing set of the investigated problem is shown. The problem with free boundary was also investigated in the mentioned paper [5]. We also note [6] where the some maximum principle type results were obtained for similar equation.

§1. Investigation of initial-boundary value problem.

Consider the following problem:

$$\frac{\partial u}{\partial t}-D\left(\left(|u|^{p_0}+|Du|^{p_1}\right)Du\right)+d\,|u|^{\gamma}\,u+c\,(x)=0, \qquad (1.1)$$

$$u\,(x,0)=u_0\,(x)\,,\qquad x\in\Omega\equiv(a,b) \qquad (1.2)$$

$$u|_{\Gamma}=0,\qquad \Gamma=\partial\Omega\times(0,T)\,, \qquad (1.3)$$

where $Q\equiv(0,T)\times\Omega;\ p_0\geq 2,\ p_1\geq 2,\ \gamma\geq 0,$ are some real numbers; $D\equiv\frac{\partial}{\partial x}$; $u_0\,(x)\,,\ c\,(x)$ are some functions.

Introduce the following intersection of spaces of the functions $u:Q\rightarrow R:$

$$P_1\,(Q)\equiv L_{\nu}\left(0,T;S_{1,p_0+p_*,2}\,(\Omega)\cap L_{\infty}\left(0,T;\mathring{W}^1_{k+2}\,(\Omega)\right)\cap L_{\mu}\left(0,T;\mathring{S}_{1,p_*,p_1+2}\,(\Omega)\right)\cap\right.$$

$$\cap L_m\left(0,T;S_{1,\gamma+p_*,2}\,(\Omega)\right)\cap W^1_2\,(Q)\cap L_{p_0+2}\left(0,T;\mathring{S}_{2,p_0,2}\,(\Omega)\right)\cap$$

$$\cap L_{p_1+2}\left(0,T;S^1_{1,p_1,2}\,(\Omega)\right)\cap\left\{u|\,u\,(x,0)=u_0\,(x)\right\},$$

where

$$\overset{\circ}{S}_{1,\alpha,\beta}(\Omega)=\left\{u(x)|\int\limits_{\Omega}|u|^{\alpha}|Du|^{\beta}dx<+\infty,\ u|_{\partial\Omega}=0\right\},$$

$$\overset{\circ}{S}_{2,\alpha,\beta}(\Omega)=\left\{u(x)|\int\limits_{\Omega}|u|^{\alpha}\left|D^{2}u\right|^{\beta}dx<+\infty,\ u|_{\partial\Omega}=0\right\},$$

$$\underset{\circ}{S}{}^{1}_{1,\alpha,\beta}(\Omega)=\left\{u(x)|\int\limits_{\Omega}|Du|^{\alpha}\left|D^{2}u\right|^{\beta}dx<+\infty,\ u|_{\partial\Omega}=0\right\},$$

$$\overset{\circ}{S}{}^{1}_{1,\alpha,\beta}(\Omega)=\left\{u(x)|\int\limits_{\Omega}|Du|^{\alpha}\left|D^{2}u\right|^{\beta}dx<+\infty,\ u|_{\partial\Omega}=Du|_{\partial\Omega}=0\right\},$$

($S_{\alpha,\beta,\gamma}$ consider [3] as regards spaces).

Definition. *The solution of problem (1.1)-(1.3) will call the function $u(x,t)\in P_1(Q)$, which satisfies the equation (1.1) in the sense of the space $L_2(Q)$, i.e. for any $\vartheta(x,t)\in L_2(Q)$ the following equality holds*

$$\int\limits_{Q}\frac{\partial u}{\partial t}\cdot\vartheta dxdt-\int\limits_{Q}D\left(\left(|u|^{p_0}+|Du|^{p_1}\right)Du\right)\vartheta dxdt+d\int\limits_{Q}|u|^{\gamma}u\vartheta dxdt+$$

$$+\int\limits_{Q}c(x)\vartheta dxdt=0.$$

Theorem 1. *Let $u_0(x)\in\overset{\circ}{W}{}^{1}_{r}(\Omega)$, $c(x)\in C^{1}[a,b]$, $r=(4p_0-2)(k+2)$. Then for any $p_0\geq 2$, $p_1\geq 2$ the solution of the problem (1.1)-(1.3) is contained in*

$$P_2(Q)\equiv P_1(Q)\cap W^{1}_{\infty}(0,T;L_2(\Omega))\cap\left\{u|\,|Du|^{\frac{p_1}{2}}Du_t\in L_2(Q)\right\}\cap$$

$$\cap\left\{u|\,|u|^{\frac{p_0}{2}}Du_t\in L_2(Q)\right\}.$$

As it was said above the solvability problems of the given problem are investigated in [3], [4]. Theorem 1 confirms that the solution of the problem is contained in $P_2(Q)$ under the mentioned conditions. In other words the problem has more smooth solution at some additional conditions.

The proof of this theorem maybe led by the scheme of the proof of general theorem from [3], [4]. The basic diffuculty, in addition, is in construction of corresponding operator generating the coercive pair with operator generated by the problem (1.1)-(1.3).

The construction of the mentioned operator is sufficiently explicitly stated in [5]. For the brevity of account we omit the proof of theorem 1 we note the corollary from theorem 1 whose proof is led analogously to the proof of the corresponding fact from [5].

Corollary. *At fulfillment of the conditions of theorem 1 for solution of the problem (1.1)-(1.3) the following inclusions are hold*

$$u\in C^{0}\left(0,T;C^{\alpha}\left(\bar{\Omega}\right)\right),\quad 0<\alpha<1,$$

$$Du \in C^0\left(0,T;C^\beta\left(\bar{\Omega}\right)\right), \quad 0<\beta<1.$$

§2. On existence of absorbing set.

At first we prove the following theorem.

Theorem 2. *Let the conditions of theorem 1 be fulfilled and $u(x,t)$ be a solution of the problem (1.1)-(1.3). Then for any $\chi \geq p_1$ the following inequality holds*

$$\|u\|_{L_{\chi+2}(\Omega)} \leq \left(\frac{\delta}{C_1}\right)^{\frac{1}{\gamma+1}} + \frac{1}{(C_1\gamma t)^{\frac{1}{\gamma}}}, \quad \forall t>0,$$

where δ, C_1 are some positive constants.

Proof. Multiply the equation by $|u|^\chi u$ and integrate by Ω. We have

$$\int\limits_\Omega u_t \cdot |u|^\chi u dx - \int\limits_\Omega D\left(|u|^{p_0} Du + |Du|^{p_1} Du\right)|u|^\chi u dx + d\int\limits_\Omega |u|^\chi u\,|u|^\gamma u dx +$$

$$+\int\limits_\Omega c(x)\,|u|^\chi u dx = 0.$$

Hence it follows that

$$\frac{\partial}{\partial t}\int\limits_\Omega \frac{|u|^{\chi+2}}{\chi+2}dx + (\chi+1)\int\limits_\Omega |u|^{p_0+\chi}|Du|^2 dx + (\chi+1)\int\limits_\Omega |Du|^{p_1+2}|u|^\chi dx +$$

$$+d\int\limits_\Omega |u|^{\gamma+\chi+2}dx \leq l\int\limits_\Omega |u|^{\chi+1}dx,$$

where $l = \max|c(x)|$.

Applying the Hōlder inequality to the right hand side of the last inequality we obtain

$$\frac{\partial}{\partial t}\int\limits_\Omega \frac{|u|^{\chi+2}}{\chi+2}dx + (\chi+1)\int\limits_\Omega |u|^{p_0+\chi}|Du|^2 dx + (\chi+1)\int\limits_\Omega |Du|^{p_1+2}|u|^\chi dx +$$

$$+d\int\limits_\Omega |u|^{\gamma+\chi+2}dx \leq \left(\int\limits_\Omega |u|^{\chi+2}dx\right)^{\frac{\chi+1}{\chi+2}} \cdot l\cdot|\Omega|^{\frac{1}{\chi+2}},$$

Thus

$$\frac{\partial}{\partial t}\int\limits_\Omega \frac{|u|^{\chi+2}}{\chi+2}dx + d\int\limits_\Omega |u|^{\gamma+\chi+2}dx \leq \left(\int\limits_\Omega |u|^{\chi+2}dx\right)^{\frac{\chi+1}{\chi+2}} \cdot l\cdot|\Omega|^{\frac{1}{\chi+2}}.$$

Divide the last inequality to integral in the right hand side. Hence it follows that

$$\frac{\frac{1}{\chi+2}\frac{\partial}{\partial t}\int\limits_\Omega |u|^{\chi+2}dx}{\left(\int\limits_\Omega |u|^{\chi+2}dx\right)^{\frac{\chi+1}{\chi+2}}} + d\frac{\int\limits_\Omega |u|^{\gamma+\chi+2}dx}{\left(\int\limits_\Omega |u|^{\chi+2}dx\right)^{\frac{\chi+1}{\chi+2}}} \leq l\cdot|\Omega|^{\frac{1}{\chi+2}}.$$

Hence it follows that

$$\frac{\partial}{\partial t}\|u\|_{L_{\chi+2}(\Omega)}+d\,|\Omega|^{\frac{1}{\chi+\gamma+2}-\frac{1}{\chi+2}}\left(\int\limits_{\Omega}|u|^{\chi+2}\,dx\right)^{\frac{\gamma+1}{\chi+2}}\leq l\cdot|\Omega|^{\frac{1}{\chi+2}}.$$

or

$$\frac{\partial}{\partial t}\|u\|_{L_{\chi+2}(\Omega)}+C_1\,\|u\|^{\gamma+1}_{L_{\chi+2}(\Omega)}\leq\delta.$$

Consequently,

$$\|u\|_{L_{\chi+2}(\Omega)}\leq\left(\frac{\delta}{C_1}\right)^{\frac{1}{\gamma+1}}+\frac{1}{(C_1\gamma t)^{\frac{1}{\gamma}}}.$$

The last implication follows from the following lemma.

Lemma 1. ([7]) *Let $y\,(t)$ satisfy the inequality*

$$y'\,(t)+C_1y^{\gamma+1}\,(t)\leq\delta.$$

Then

$$y\,(t)\leq\left(\frac{\delta}{C_1}\right)^{\frac{1}{\gamma+1}}+\frac{1}{(C_1\gamma t)^{\frac{1}{\gamma}}}\quad\forall t>0.$$

Thus theorem 2 is proved.

Using lemma 1 we can also prove the following fact.

Theorem 3. *Let the conditions of theorem 1 be satisfied. Besides, assume that $d\geq p_0^2+\frac{2^{p_1-1}(p_1+\chi+1)^{2p_1}}{p_1^{p_1}}$, where $\chi=(4p_0-2)\,(k+2)-2,\ p_1\geq 3,\ \gamma\geq\max\{2p_0-2,p_1\}$. Then the following inequality holds*

$$\|Du\|_{L_{\chi+2}(\Omega)}\leq\left(\frac{\alpha}{\beta}\right)^{\frac{1}{p_1+1}}+\frac{1}{(\beta p_1 t)^{\frac{1}{p_1}}}\qquad\forall t>0,$$

where $\alpha,\ \beta$ are some positive constants.

Proof. Multiply the equation (1.1) by $|Du|^{\chi}\,D^2u$ and integrate by Ω

$$-\int\limits_{\Omega}u_t\,|Du|^{\chi}\,D^2udx+\int\limits_{\Omega}|u|^{p_0}\,D^2u\,|Du|^{\chi}\,D^2udx+(p_1+1)\int\limits_{\Omega}|Du|^{p_1+\chi}\times$$

$$\times\left|D^2u\right|^2dx+d\int\limits_{\Omega}|u|^{\gamma}\,|Du|^{\chi+2}\,dx+\int\limits_{\Omega}C\,(x)\,|Du|^{\chi}\,D^2udx+$$

$$+p_0\int\limits_{\Omega}|u|^{p_0-2}\,u\,|Du|^{\chi+2}\,dx=0. \qquad (1.4)$$

In order to estimate the last integral in the left hand side of (1.4) we use the following lemma.

Lemma 2. *For any $a,\ b,\ c\geq 0$ the inequality*

$$a^{p_0-1}b^{\chi+2}c\leq\frac{b^{p_1+\chi}c^2}{p_0}+\frac{a^{p_0}b^{\chi}c^2}{p_0}+p_0a^{\gamma}b^{\chi+2}+p_0b^{\chi+1}, \qquad (1.5)$$

where $\chi > 0,\ p_0 \geq 2,\ p_1 \geq 3,\ \gamma \geq 2p_0 - 2$, holds.

Proof of lemma 2. At first note that if only one of the numbers a, b or c is equal to 0, then the inequality (1.5) is obvious. Consider the case $a \neq 0,\ b \neq 0,\ c \neq 0$ and assume that (1.5) is incorrect.

Then the equivalent inequality

$$1 \leq \frac{b^{p_1-2}c}{p_0 a^{p_0-1}} + \frac{ac}{b^2 p_0} + \frac{p_0 a^{\gamma-p_0+1}}{c} + \frac{p_0}{a^{p_0-1}bc}, \tag{1.6}$$

is also incorrect. But if the last inequality is incorrect, then each of addends in the right hand side of the last inequality is less than unit. Then multiplying the fourth and first addends from (1.6) we obtain

$$\frac{b^{p_1-3}}{a^{2p_0-2}} < 1. \tag{1.7}$$

At the same time multiplying the fourth and second addends we obtain that

$$\frac{1}{a^{p_0-2}b^3} < 1. \tag{1.8}$$

Since $p_0 \geq 2$ and $p_1 \geq 3$, then it follows from (1.7) and (1.8) that $a \geq 1$, since otherwise the alternative inequalities $b^{p_1-3} < 1$ and $b^{-3} < 1$ are satisfied. Thus, $a \geq 1$.

Since by virtue of assumption the third addend from (1.1.6) is also less than unit, then allowing for $a \geq 1$ we obtain

$$1 \leq a^{\gamma-p_0+1} \leq \frac{c}{p_0}.$$

Hence it follows that

$$1 \leq a^{\gamma-p_0+2} \leq \frac{ac}{p_0}.$$

But it follows from the second addend from (1.1.6) that $\frac{ac}{p_0} \leq b^2$. Thus $b^2 \geq 1$. Moreover, multiplying the third and first addends from (1.1.6) we obtain that

$$b^{p_1-2}a^{\gamma-2p_0+2} < 1.$$

But It's impossible, since we showed that a and $b \geq 1$. The lemma is proved.

We return to the proof of theorem 3.

As it was said above we try to estimate the integral from (1.4):

$$p_0 \int\limits_\Omega |u|^{p_0-1} |Du|^{\chi+2}\, dx \leq \cdot \int\limits_\Omega |Du|^{p_1+\chi} \left|D^2u\right| dx + \int\limits_\Omega |u|^{p_0} |Du|^{\chi} \left|D^2u\right|^2 dx +$$

$$+p_0^2 \int\limits_\Omega |u|^{\gamma} |Du|^{\chi+2}\, dx + p_0^2 \int\limits_\Omega |Du|^{\chi+1}\, dx.$$

Substituting the last inequality in (1.4) we obtain:

$$\frac{1}{\chi+2}\frac{\partial}{\partial t}\int\limits_\Omega |Du|^{\chi+2}\, dx + p_1 \int\limits_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx \leq \int\limits_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx +$$

$$+\int_\Omega |u|^{p_0} |Du|^\chi \left|D^2u\right|^2 dx + p_0^2\int_\Omega |u|^\gamma |Du|^{\chi+2} + p_0^2\int_\Omega |Du|^{\chi+1} dx+$$

$$+\int_\Omega \frac{|c'(x)| |Du|^{\chi+1}}{\chi+2} dx - \int_\Omega |u|^{p_0} |Du|^\chi \left|D^2u\right|^2 dx - \int_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx-$$

$$-d\int_\Omega |u|^\gamma |Du|^{\chi+2} dx.$$

Now prove that

$$\frac{1}{2}\int_\Omega |Du|^{p_1+\chi+2} dx \le p_1\int_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx + \frac{1}{2\varepsilon^{p_1}}\int_\Omega |u|^\gamma |Du|^{\chi+2} dx+$$

$$+\frac{1}{2\varepsilon^{p_1+1}}\int_\Omega |Du|^{\chi+1} dx,$$

where

$$\varepsilon = \frac{p_1}{2(p_1+\chi+1)^2}. \tag{1.10}$$

For the further reasonings we use the following lemma.

Lemma 3. *Let* a and b *be arbitrary non-negative numbers,* $\varepsilon > 0,\ \chi \ge 0,\ \gamma \ge p_1 \ge 3$. *Then the inequality*

$$a^{p_1+\chi}b^2 \le \varepsilon^2 a^{p_1+\chi+2} + \frac{b^\gamma a^{\chi+2}}{\varepsilon^{p_1-2}} + \frac{a^{\chi+1}}{\varepsilon^{p_0-1}} \tag{1.11}$$

holds.

Proof of lemma 3. The inequality (1.11) is equivalent to the following

$$1 \le \frac{a^2\varepsilon^2}{b^2} + \frac{b^{\gamma-2}}{a^{p_1-2}\varepsilon^{p_1-2}} + \frac{1}{a^{p_1-1}b^2\varepsilon^{p_1-1}} \tag{1.12}$$

(it's assumed that $a \ne 0,\ b \ne 0$, since in case only one of the numbers is equal to zero, the fulfillment of the inequality (1.11) is obvious).

Consider the various variants

I) $\varepsilon a \le 1,\ b \le 1$; II) $\varepsilon a \ge 1,\ b \le 1$; III) $\varepsilon a \le 1,\ b \ge 1$; IV) $\varepsilon a \ge 1,\ b \ge 1$.

In case of alternatives I), II), III) the inequality (1.12) is fulfilled by virtue of that in case of I) $\frac{1}{(a\varepsilon)^{p_1-1}b^2} > 1$; in case of II) $\frac{(a\varepsilon)^2}{b^2} \ge 1$; in case of III) $\frac{b^{\gamma-2}}{(a\varepsilon)^{p_1-2}} \ge 1$.

Consider case IV). If assume that (1.12) is incorrect, then $(a\varepsilon)^2 \le b^2$. Hence it follows that $a\varepsilon \le b$. Since we assumed that (1.12) is incorrect then $\frac{b^{\gamma-2}}{(a\varepsilon)^{p_1-2}} < 1$. But this is impossible, since

$$\frac{b^2}{(a\varepsilon)^{p_1-2}} = \left(\frac{b}{a\varepsilon}\right)^{p_1-2} \cdot b^{\gamma-p_1} \ge 1.$$

Consequently, the initial assumption is incorrect. Lemma 3 is proved.

We again return to the proof of the theorem.
By virtue of the formula of integration by parts we have

$$\int_\Omega |Du|^{p_1+\chi+2}\,dx = \left| -\int_\Omega |Du|^{p_1+\chi} D^2u \cdot u \cdot (p_1+\chi+1)\,dx \right| \le$$

$$\le \int_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx \cdot (p_1+\chi+1)\,\tilde{\varepsilon} + \frac{1}{\tilde{\varepsilon}}(p_1+\chi+1) \int_\Omega |Du|^{p_1+\chi} |u|^2\,dx. \quad (1.13)$$

Applying lemma 3 to the last integral in the right hand side of (1.13) we obtain

$$\int_\Omega |Du|^{p_1+\chi} |u|^2\,dx \le \varepsilon^2 \int_\Omega |Du|^{p_1+\chi+2}\,dx + \frac{1}{\varepsilon^{p_1-2}} \int_\Omega |u|^\gamma |Du|^{\chi+2}\,dx +$$

$$+ \frac{1}{\varepsilon^{p_1-1}} \int_\Omega |Du|^{\chi+1}\,dx. \quad (1.14)$$

We choose $\tilde{\varepsilon} = \frac{p_1}{p_1+\chi+1}$ (1.13) and substitute (1.14) in (1.13)

$$\int_\Omega |Du|^{p_1+\chi+2}\,dx \le p_1 \int_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx + \frac{(p_1+\chi+1)^2}{p_1} \int_\Omega |Du|^{p_1+\chi} |u|^2\,dx \le$$

$$\le p_1 \int_\Omega |Du|^{p_1+\chi} \left|D^2u\right|^2 dx + \varepsilon^2 \frac{(p_1+\chi+1)^2}{p_1} \int_\Omega |Du|^{p_1+\chi+2}\,dx + \frac{(p_1+\chi+1)^2}{p_1 \varepsilon^{p_1-2}} \times$$

$$\times \int_\Omega |u|^\gamma |Du|^{\chi+2}\,dx + \frac{(p_1+\chi+1)^2}{\varepsilon^{p_1-1} p_1} \int_\Omega |Du|^{\chi+1}\,dx.$$

If we choose ε from the equality $\frac{\varepsilon^2 (p_1+\chi+1)^2}{p_1} = \frac{1}{2}$, we obtain the inequality (1.10). Then it follows from (1.9) and (1.10) that

$$\frac{1}{\chi+2} \frac{\partial}{\partial t} \int_\Omega |Du|^{\chi+2}\,dx + \frac{1}{2} \int_\Omega |Du|^{p_1+\chi+2}\,dx \le$$

$$\le \left(p_0^2 + \frac{\max |c'(x)|}{\chi+2} + \frac{2^{p_1} (p_1+\chi+1)^{2(p_1+1)}}{p_1^{p_1+1}} \right) \int_\Omega |Du|^{\chi+1}\,dx -$$

$$- \left(d - p_0^2 - \frac{2^{p_1-1} (p_1+\chi+1)^{2p_1}}{p_1^{p_1}} \right) \int_\Omega |u|^\gamma |Du|^{\chi+2}\,dx \le$$

$$\le \left(p_0^2 + \frac{\max |c'(x)|}{\chi+2} + \frac{2^{p_1} (p_1+\chi+1)^{2(p_1+1)}}{p_1^{p_1+1}} \right) \int_\Omega |Du|^{\chi+1}\,dx.$$

Further if we act in exactly the same way as at proving theorem 2 we obtain that

$$\left(\int_\Omega |Du|^{\chi+2}\,dx \right)^{\frac{1}{\chi+2}} \le \left(\frac{\alpha}{\beta} \right)^{\frac{1}{p_1+1}} + \frac{1}{(\beta p_1 t)^{\frac{1}{p_1}}} \qquad \forall t > 0.$$

Theorem 3 is proved.

Since we assumed that $\chi + 2 = (4p_0 - 2)(k+2)$, then

$$\|u\|_{\overset{\circ}{W}{}^{(1)}_{(4p_0-2)(k+2)}(\Omega)} \leq c \cdot \left(\left(\frac{\alpha}{\beta} \right)^{\frac{1}{p_1+1}} + \left(\frac{\delta}{c_1} \right)^{\frac{1}{\gamma+1}} + \frac{1}{(\beta p_1 t)^{\frac{1}{p_1}}} + \frac{1}{(c_1 \gamma t)^{\frac{1}{\gamma}}} \right),$$

where c is a constant depending on $u(x,t)$.

The validity of the following theorem follows from theorem (1)-(3).

Theorem 4. *The set*

$$B_0 = \left\{ u(t) \in \overset{\circ}{W}{}^{(1)}_{(4p_0-2)(k+2)}(\Omega), \|u(t)\|_{\overset{\circ}{W}{}^{(1)}_{(4p_0-2)(k+2)}(\Omega)} \leq \right.$$

$$\left. \leq c \left(\left(\frac{\alpha}{\beta} \right)^{\frac{1}{p_1+1}} + \left(\frac{\delta}{c_1} \right)^{\frac{1}{p_1+1}} + \varepsilon \right) \right\}$$

is an absorbing set ([2]) for solution of the problem (1.1)-(1.3) under the conditions of the theorem (1)-(3) and for any $\varepsilon > 0$.

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