JAFAROV A.G.

DELAYED FRACTURE OF THICK TUBE CONTACTING WITH THE PLIABLE MEDIUM

Abstract

In the paper on the base of kinetic theory of Yu,N. Rabotnov damaging by using L.M. Kachanov methodic the problem on scattering destruction situating under the motion of internal pressure of thick pipe is solved, externally contacting with elastic medium modeled by Winkler foundation. The problem is solved in two directions: in interval of elasticity theory and the creeping theory. The formulas for initial incubation period are got. The equations of motion front destruction are constructed and solved. The given estimations indicate on decelerating process of destruction influence of external contact elastic medium.

The scattering fracture of a thick pipe subject to damaging of pipes material on the base of kinetic damaging equation in creeping conditions was investigated in [1], and on the base of model of hereditarily damaging medium in conditions of elastic deformations in [2]. In these investigations external surface of pipe was considered as free of loadings. On practice the pipes contact with surrounding medium and influence of this contact on working resources of the pipe may be significant [7].

At the given paper the problem on scattering destruction of a thick pipe is solved both in case of elastic deformations and in creeping conditions when the pipe is situated in pliable deformable medium. The contact action is modelled by Winkler's foundation [7], i.e. we take that

$$\sigma_r(b) = -k\,u(b), \tag{1}$$

where σ_r is a radial stress in a pipe, u is a radial removal of pipe points, b is an external radius of a pipe, k is a coefficient of a bed external to the medium of the pipe. The pipe is situated under the pressure of internal equally distributed pressure q, i.e.

$$\sigma_r(a) = -q\,. \tag{2}$$

We'll keep of simple kinetic theory of damaging [4], according in which the parameter of damaging doesn't enter in deformation relation and therefore the damages and deformations are determined in elastic problem, or problem on creeping of a pipe.

We'll take the kinetic equation of damaging in the form:

$$\frac{d\omega}{d\,l} = D\left(\frac{\sigma_{экв}}{1-\omega}\right)^n, \tag{3}$$

where $\omega\ (0 < \omega < 1)$ is Yu.N. Robotnov damaging parameter, equal to zero in initial non-damaging state and to unit in the moment of fracture, $\sigma_{экв}$ is equivalent stress, in the capacity of which in conditions of difficult stress state we can take either the greatest stretching stress or intensity of stresses: D and n are empirical constants.

Dividing the equation (3) by the variable ω and time t and integrating by ω from zero to unit we'll get the destruction condition of the form [1]:

$$(n+1)D\int_0^t \sigma^n_{экв}\,d\tau = 0\,. \tag{4}$$

Let's consider first of all the case of elastic deformations. The radial σ, stress y ring σ_θ stress and radial u removal are represented in the following form [6]:

$$\begin{cases} \sigma_r = A + \dfrac{B}{r^2}, \\ \sigma_\theta = A - \dfrac{B}{r^2}, \\ u = \dfrac{1-\nu^2}{E}\left(\dfrac{1-2\nu}{1-\nu}A_r - \dfrac{1}{1-\nu}\dfrac{B}{r}\right), \end{cases} \tag{5}$$

where E is a Young modulus, ν is a Poisson coefficient, r is current radius, and A and B are unknown constants, determined from the boundary conditions (1), (2) for which we get:

$$\begin{cases} B = -q\left(\dfrac{1{,}5k}{bE} + \dfrac{b^2 - a^2}{a^2 b^2}\right)^{-1}, \\ A = \dfrac{1}{b}\left(\dfrac{1{,}5k}{E} - \dfrac{1}{b}\right)B. \end{cases} \tag{6}$$

The prevalent from the stresses will be hoop stress for which we have:

$$\sigma_\theta = q\frac{\left(1 - \dfrac{1{,}5kb}{E}\right) + \left(\dfrac{b}{r}\right)^2}{\dfrac{1{,}5kb}{E} + \left(\dfrac{b}{a}\right)^2 - 1}, \tag{7}$$

which for $k = 0$ (absence of external pressure $\sigma_r(b) = 0$) passes to the known one in paper [6].

Let's introduce the notations:

$$\begin{cases} k_1 = \dfrac{1{,}5kb}{E};\ a_1 = \dfrac{a}{b};\ r_1 = \dfrac{r}{b}, \\ \tilde{\sigma}_\theta = \dfrac{\sigma_\theta}{q};\ \tilde{D} = Dq^n, \\ \tilde{t} = \tilde{D}t. \end{cases} \tag{8}$$

Then from (7) we have

$$\tilde{\sigma}_\theta = \frac{1 - k_1 + r_1^{-2}}{1 + k_1 + a_1^{-2}}. \tag{9}$$

The fracture condition will have the form:

$$(n+1)\int_0^{\tilde{t}} \tilde{\sigma}_\theta^n d\tilde{\tau} = 1. \tag{10}$$

The hoop stress attains its maximal value on an external surface of a pipe, where first of all will occur the fracture whose time (incubation period) will be determined from [10] as:

$$\tilde{t}_0 = \frac{1}{(n+1)\tilde{\sigma}_\theta^n(a_1)} \tag{11}$$

or

$$\tilde{t}_0 = \frac{1}{(n+1)}\left(\frac{a_1^{-2}+k_1-1}{a_1^{-2}+k_1+1}\right)^n \tag{12}$$

The influence of surrounding pliable medium determines the relation of incubation periods involving this medium (12) and at its absence

$$\frac{\tilde{t}_0}{\tilde{t}_0(k_1=0)} = \left(\frac{a_1^{-2}+k_1-1}{a_1^{-2}-k_1+1}\cdot\frac{a_1^{-2}+1}{a_1^{-2}-1}\right)^n . \tag{13}$$

As for $a_1=0{,}5,\, k_1=0{,}25$ and $n=1$ from (13) we have: $\tilde{t}_0/\tilde{t}_0(k_1=0)=1{,}14$, i.e. the calculation of pliable external medium increases the incubation time on 14%.

The motion equation failure of front we'll get by the following scheme given in [1]. For this in (9) we suppose $a_1=\beta(\tau);\, r_1=\beta(t)$. Then

$$\tilde{\sigma}_\theta(\tilde{t},\tilde{\tau}) = \frac{1-k_1+\beta^{-2}(\tilde{t})}{-1+k_1+\beta^{-2}(\tilde{\tau})} . \tag{14}$$

We'll get the equation for redial coordinate $\beta(t)$ failure from (10) subject to (14)

$$(n+1)\int_0^{\tilde{t}}\left(\frac{1-k_1+\beta^{-2}(\tilde{t})}{-1+k_1+\beta^{-2}(\tilde{\tau})}\right)^n d\tilde{\tau} = 1 . \tag{15}$$

Let's consider the case $n=1$, then denoting $\beta^{-2}(\tilde{t})=x(\tilde{t})$ we'll get:

$$2\int_0^{\tilde{t}}\frac{1-k_1+x(\tilde{t})}{-1+k_1+x(\tilde{\tau})}d\tilde{\tau} = 1 . \tag{16}$$

Differentiating by time the received one we'll get the following simple differential equation:

$$\begin{cases}\dfrac{dx}{d\tilde{t}} = 2\dfrac{(1-k_1+x)^2}{1-k_1-x}, \\ x\big|_{\tilde{t}=\tilde{t}_0} = x_0\,;\ \left(x_0=\beta_0^{-2}=a_1^{-2}\right).\end{cases} \tag{17}$$

Its solution is elementary and has the form:

$$2(1-k_1)\frac{x-x_0}{(1-k_1+x_0)(1-k_1+x)} - \ln\frac{1-k_1+x}{1-k_1+x_0} = 2(\tilde{t}-\tilde{t}_0) . \tag{18}$$

We'll estimate the influence of external pliable medium by the comparison of duration of time $\Delta\tilde{t}$ that is necessary to pass the same way $\Delta\beta$. Let $\beta_0=a_1 0{,}5;\, k_1=0{,}25$; $\Delta\beta = 0{,}1$. Then we'll get $\tilde{t}_0/\tilde{t}_0(k_1=0)=1{,}27$, i.e. calculation of pliable surrounding medium decreases the velocity of motion of failure front (increasing period of duration of equal distances).

Now let's consider the problem on scattering fracture of a thick pipe in creeping conditions subject to external contact of a pipe with pliable medium. According to [3] the general representation for stresses and the radial removal u has the form:

$$\begin{cases} \sigma_r = -q - \dfrac{1}{\sqrt{3}} \displaystyle\int_{\vartheta_a}^{\vartheta} \dfrac{s(\vartheta)}{\vartheta} d\vartheta \ ; \ \vartheta = \dfrac{c}{r^2}, \\ \sigma_\theta = -q - \dfrac{1}{\sqrt{3}} \displaystyle\int_{\vartheta_a}^{\vartheta} \dfrac{s(\vartheta)}{\vartheta} d\vartheta + \dfrac{2}{\sqrt{3}} s(\vartheta), \\ \sigma_z = -q - \dfrac{1}{\sqrt{3}} \displaystyle\int_{\vartheta_a}^{\vartheta} \dfrac{s(\vartheta)}{\vartheta} d\vartheta + \dfrac{1}{\sqrt{3}} s(\vartheta) = \dfrac{c}{a^2}, \end{cases} \tag{19}$$

$$u = \frac{\sqrt{3}}{2} \frac{c}{r}. \tag{20}$$

Here q is an internal pressure, c is the unknown constant, $s(\vartheta)$ for power law if creeping has the form:

$$s(\vartheta) = D_* \vartheta^{1/m}. \tag{21}$$

Allowing for (21) in the expression for radial pressure (19), and in the last from (20) in boundary condition (1) we'll get algebraic equation of the m-th degree with respect to the unknown $c_* = c_0^{1/m} = \left(\dfrac{c}{b^2}\right)^{1/m}$:

$$k_0 c_*^m + m\left(\beta^{-2/m} - 1\right) c_* - q_0 = 0, \tag{22}$$

where the following notations for dimensionless values

$$\frac{a}{b} = \beta \ ; \ \frac{c}{b^2} = c_0 \ ; \ \frac{bk}{2D_*} = k_0 \ ; \ \frac{\sqrt{3}q}{D_*} = q_0 \ ; \ c_* = c_0^{1/m} \tag{23}$$

are accepted.

In classical case in the absence of external pressure [3] we have:

$$c_* = \frac{q_0}{m\left(\beta^{-2/m} - 1\right)}. \tag{24}$$

For the investigation of influence of external contacting pliable medium we'll be restricted in the case $m = 3$ that holds [3] for some sorts of steel. Then from (22) we'll get:

$$c_* = \frac{q_0^{1/3}}{2^{1/3} k_0^{1/2}} \left\{ \left(k_0^{1/2} + \left(k_0 + \frac{4}{q_0^2} \left(\beta^{-2/3} - 1\right)^3 \right)^{1/2} \right)^{1/3} + \right.$$
$$\left. + \left(k_0^{1/2} - \left(k_0 + \frac{4}{q^2} \left(\beta^{-2/3} - 1\right)^3 \right)^{1/2} \right)^{1/3} \right\}. \tag{25}$$

It is easy to see that the traditionally opening the indefiniteness of the form $0/0$ at tending of k_0 to zero we'll get the formula (24).

Now at the known value $c = b^2 c_*^m$ the formula (19) gives the formulas for the distribution of pressures at creeping in thick pipe, externally contacting with the elastic medium.

Let's pass to the investigation of fracture process on the base of destruction criterion (4). As equivalent pressure let's accept the intensity of pressures:

$$\sigma_{экв} = \sigma_0 = \frac{1}{2}\left((\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_\theta - \sigma_z)^2\right)^{1/2}. \quad (26)$$

Allowing for (19) in (26) we'll get

$$\sigma_{экв} = \sigma_0 = s(\vartheta) = D_* \vartheta^{1/n} = D_* \left(\frac{c}{r^2}\right)^{1/2} = \frac{c_*}{x^{2/3}}, \quad (27)$$

where

$$x = \frac{r}{b}; \ \frac{a}{b} = \beta_0 \le x < 1. \quad (28)$$

Let's be restricted in later on the case $n = 1$ then the fraction condition (4) will take the form:

$$\frac{1}{2B} = \int_0^t \frac{c_*}{x^{2/3} d\tau}, \quad (29)$$

where $B = D \cdot D_*$.

Since the equivalent pressure receives its greatest value on internal contour there first of all will happen the failure, time of which is determined according to (29) as

$$\frac{1}{2B} = \int_0^{t_0} \frac{c_*\big|_{\beta=\beta_0; x=\beta_0}}{\beta_0^{2/3}}. \quad (30)$$

Whence

$$t_0 = \frac{\beta_0^{2/3}}{2Bc_*\big|_{\beta=\beta_0; x=\beta_0}}. \quad (31)$$

When $k = 0$ we'll get

$$t_0' = \frac{3\left(1 - \beta_0^{3/2}\right)}{2Bq_0}. \quad (32)$$

For $k \ne 0$ we have:

$$t_0 = \frac{\beta_0^{2/3} k_0^{1/3}}{2^{2/3} B q_0^{1/3}} + \left\{\left(k_0^{1/2} + \left(k_0 + \frac{4}{q_0}\left(\beta^{-2/3} - 1\right)^3\right)^{1/2}\right)^{1/3} + \right.$$

$$\left. + \left(k_0^{1/2} - \left(k_0 + \frac{4}{q_0}\left(\beta^{-2/3} - 1\right)^3\right)^{1/2}\right)^{1/3}\right\}^{-1}. \quad (33)$$

For finding the motion equation of front fracture let's put in (27)

$$\frac{a}{b} = \beta = \beta(\tau); \ \frac{r}{b} = x = \beta(t). \quad (34)$$

Then

$$\sigma_{экв}(t,\tau) = \left(\frac{q_0}{2\beta^2(t)}\right)^{1/3} \frac{1}{k_0^{1/2}} \left\{\left(k_0^{1/2} + \left(k_0 + \frac{4}{q_0^2}\left(\beta^{-2/3}(\tau) - 1\right)^3\right)^{1/2}\right)^{1/3} + \right.$$

$$\left. + \left(k_0^{1/2} + \left(k_0 + \frac{4}{q_0^2}\left(\beta^{-2/3}(\tau) - 1\right)^3\right)^{1/2}\right)^{1/3}\right\}. \quad (35)$$

Then the criterion (29) will take the form:

$$\frac{1}{2B}=\int_0^t \sigma_{экв}(t,\tau)d\tau . \qquad (36)$$

For $k=0$ we will get from it:

$$\frac{1}{\alpha}y(t)=\int_0^t \frac{y(\tau)}{1-y(\tau)}d\tau ;\quad \alpha=\frac{2Bq_0}{3};\quad y(t)=\beta^{2/3}(t). \qquad (37)$$

Differentiating the received one by time we'll get:

$$\begin{cases} \dfrac{dy}{dt}=\alpha\dfrac{y}{1-y}, \\ \left| y_{t=t_0'}=y_0 ;\ \left(y_0=\beta_0^{2/3}\right). \right. \end{cases} \qquad (38)$$

Whence we have obvious formula for radial coordinate of fracture front:

$$\frac{2}{3}\ln\frac{\beta(t)}{\beta_0}-\left(\beta^{2/3}(t)-\beta_0^{2/3}\right)=\alpha\left(t-t_0'\right). \qquad (39)$$

When $k\neq 0$ from (36) substituting (35) in (36) and then differentiating the obtained one by time, we'll get the following Cauchy problem for the function $y=\beta^{2/3}(t)$:

$$\begin{cases} \dfrac{dy}{d\tilde{t}}=\left(\left(k_0^{1/2}+\left(k_0+\dfrac{4(1-y)^3}{q_0y^3}\right)^{1/2}\right)^{1/3}+\left(k_0^{1/2}-\left(k_0+\dfrac{4(1-y)^3}{q_0y^3}\right)^{1/2}\right)^{1/3}\right), \\ y|_{\tilde{t}=\gamma t_0}=y_0 ;\ y_0=\beta_0^{2/3} , \end{cases} \qquad (40)$$

where

$$\tilde{t}=\gamma t ;\quad \gamma=\frac{2^{2/3}B\,q_0^{1/3}}{k_0^{1/2}} . \qquad (41)$$

Let's estimate influence of pressure of external elastic medium, $k\neq 0$ by the duration of time interval by failure front to pass the same distance $\Delta\beta=\beta_1-\beta_0$. We'll take $\beta_0=0{,}5$;$\beta_1=0{,}6$; $q_0=4$; $k_0=0{,}25$. Then from (39) we'll get:

$$k=0 ;\ B\Delta t'=0{,}015 ; \qquad k=0{,}25 ;\ B\Delta t=0{,}33 .$$

As before the dates evidence on delay process of fracture influence of external pliable medium which in creeping conditions becomes more applicable, consisting in our case almost in twenty fold increasing the duration of time for the motion of front fracture.

References

[1]. Kachanov L.M. *The base of destruction mechanics.* M., 1974, 312p. (in Russian)
[2]. Boyl J., Spens J. *The analysis of pressures in constructions at creeping.* "Mir", 1968, 360p. (in Russian)
[3]. Rabotnov Yu.N. *The creeping of structure elements.* M., 1966, p.752. (in Russian)
[4]. Edited by Shestirikov S.A. *The regularity of creeping and long strength.* Spravichnik. M., "Mashinovedeniye", 1983, 101p. (in Russian)
[5]. SuvorovaYu.V., Akhundov M.B. *The long destruction of isotrop medium in conditions of difficult pressure state.* M., "Masinovedeniye", 1986, №4, p.40-46. (in Russian)
[6]. Rabotnov Yu.N. *The mechanics of differentiating solid.* M., "Nauka", 1988, 712p. (in Russian)
[7]. Eyubov Ya.A., Ali-zade A.N., Akhundov M.B. *The Winkler method calculation of soil foundation.* Baku, 1999. (in Russian)

Allahverdi G. Jafarov
Azerbaijan State Pedagogical University.
34, U.Hajibeyov str., 370000, Baku, Azerbaijan.
Tel.: 73-75-71(apt).

Received September 3, 2001; Revised November 1, 2001.
Translated by Mamedova V.I.