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# DETERMINATION OF A CRITICAL LOADING AT LONGITUDINAL VIBRATION OF A PILE IN THE RESISTING MEDIUM

**Abstract**

*The critical loading of a pile under the longitudinal oscillation in a resisting medium has been obtained by numerical method. The influence of oscillation frequencies and strengthening to the critical value have been studied. The characteristically curves have been constructed.*

The given paper is devoted to the determination of a critical loading of vibration of a pile in resisting medium. The geometrical non-linear vibrations of the mentioned construction are investigated. It is evident [1] that there exist such values of frequencies under which the permutation infinitly increases, i.e. the linear theory isn't applicable. Therefore for these cases it is necessary to take into account the geometrical non-linearity.

The influence of external medium on vibration of a pile is determined within Winkler's linear model [1].

Let's write the equation of longitudinal non-linear vibrations of a pile in resisting medium. It is accepted that there is a full linking with soil on all lateral surface of a pile and at deformation of a pile the coming of a soil doesn't occur. In Cartesian coordinate system we have [2]:

$$\frac{\partial}{\partial x}\left[\sigma\left(1+\frac{\partial u}{\partial x}\right)\right]-\tau_c=\rho\frac{\partial^2 u}{\partial t^2}, \qquad (1)$$

where $u$ is a longitudinal permutation of a pile points, $\rho$ is a density of its material, $t$ is time, $\tau_c$ is the value of soil stress on a lateral surface of pile and by analogous with the Winkler model is accepted in the following form:

$$\tau_c=k_c\cdot u. \qquad (2)$$

Allowing for $\sigma=E\frac{\partial u}{\partial x}\left(1+\frac{1}{2}\frac{\partial u}{\partial x}\right)$, subject to (2) from (1) we'll get:

$$c^2\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}\left(1+\frac{1}{2}\frac{\partial u}{\partial x}\right)\left(1+\frac{\partial u}{\partial x}\right)\right]-\gamma^2 u=\frac{\partial^2 u}{\partial t^2}, \qquad (3)$$

where $c^2=E/\rho$, $\gamma^2=k_c/\rho$, $E$ is an elasticity piles modulus of material.

The boundary conditions we'll take in the form:

$$\text{when } x=0 \quad u=0,$$

$$\text{when } x=L \quad \frac{1}{E}\sigma\left(1+\frac{\partial u}{\partial x}\right)\equiv\frac{\partial u}{dx}\left(1+\frac{1}{2}\frac{\partial u}{\partial x}\right)\left(1+\frac{\partial u}{\partial x}\right)=\tau_0\sin\omega_0 t, \qquad (4)$$

where $\omega_0$ is a frequency of forcing loading, $\tau_0$ is an amplitude, $L$ is a pile length.

So, the equation (3) at the boundary conditions (4) allows us to investigate the longitudinal vibration of pile points in resisting medium. Let's note that in a common case to find the analytical solution of the equation (3) is impossible. Therefore there appears the necessity to apply the approximate methods. One of the effective methods is variational one. In our case we can the Reysner variation principle. This functional in considered case has the form [2]:

$$J=\int_{t_0}^{t_1}\int_V\left\{\sigma\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{dx}\right)^2\right]-\frac{1}{2E}\sigma^2-\frac{1}{2}\rho\left(\frac{\partial u}{d\,t}\right)^2+\frac{1}{2}k_c u^2\right\}dV\,dt+$$

$$+\int_{t_0}^{t_1}\int_F u\sigma\left(1+\frac{\partial u}{\partial x}\right)dF\,dt\Bigg|_{x=0}-E\int_{t_0}^{t_1}\int_F u\,\tau_0\sin\omega_0 t dFdt\Bigg|_{x=L}\,, \tag{5}$$

where the variation values are $\sigma$, $u$; $V$ is a volume, $F$ is an area of cross-section, $(t_0,t_1)$ is an interval time on which the process is investigated. The stationary value of the functional (5) we'll search by Rietz method [2]. Starting from the expected physical behavior of pivot and the solutions of linear problem, let's take the approximate functions in the following form:

$$\sigma=\sin\omega_0 t\left(\sigma_0\cos\frac{\pi x}{2L}+\tau_0\right);\ u=u_0\sin\omega_0 t\sin\frac{\pi x}{2L}\,. \tag{6}$$

Then we'll take that $t_0=0$; $t_1=\frac{\pi}{\omega_0}$. Since the desired values depend only on $x$ then the investigation by $F$ isn't produced. Allowing for the above mentioned we have:

$$J=F\frac{1}{\omega_0}\frac{2L}{\pi}\left\{\sigma_0\left[u_0\frac{\pi^3}{16L}+\frac{\pi^2}{9L^2}u_0^2\right]+\tau_0\left[u_0\frac{\pi^2}{4L}+\frac{\pi^3}{24}u_0^2\right]-\right.$$

$$\left.-\frac{\pi}{4E}\left(\frac{\pi}{4}\sigma_0^2+2\sigma_0\tau_0+\frac{\pi}{2}E\tau_0^2\right)-\frac{\pi^2}{16}\rho\omega_0^2u_0^2+\frac{\pi^2}{16}k_c u_0^2-\frac{\pi^2}{4L}Eu_0\tau_0\right\}. \tag{7}$$

The stationary state conditions of the obtained function (7) gives a system of the equations for the equations for the determination $\sigma_0$ and $u_0$:

$$\frac{\partial J}{\partial\sigma_0}=u_0\frac{\pi^3}{16L}+\frac{4}{9}u_0^2\left(\frac{\pi}{2L}\right)^2-\frac{\pi^2}{8E}\sigma_0-\frac{\pi}{2}\tau_0=0\,,$$

$$\frac{\partial J}{\partial u_0}=\sigma_0\left[\frac{\pi^3}{16L}+\frac{8}{9}\left(\frac{\pi}{2L}\right)^2u_0\right]+\tau_0u_0\left(\frac{\pi}{2L}\right)^2\cdot\frac{\pi}{3}-$$

$$-\frac{\pi^2}{8}\rho\omega_0^2u_0+\frac{\pi^2}{8}k_c u_0=0\,. \tag{8}$$

In case of smallness of permutations from the obtained system (8) follows:

$$u_0\frac{\pi^3}{16L}-\frac{\sigma}{E}\frac{\pi^2}{8}-\frac{\pi}{2}\tau_0=0\,,$$

$$\sigma_0\cdot\frac{\pi^3}{16L}-\rho\omega_0^2u_0\frac{\pi^2}{8}+k_c u_0\frac{\pi^2}{8}=0\,.$$

From here it is easy to get that

$$u_0=\tau_0\frac{2}{L}\left(\frac{\pi^2}{4L^2}-\frac{\rho}{E}\omega_0^2+\frac{k_c}{E}\right)^{-1}=\tau_0\frac{2}{L}\left(\frac{\pi^2}{4L^2}-\frac{1}{c^2}\omega_0^2+k_0^2\right)^{-1},$$

where $k_0=\frac{k_c}{E}$.

The received dependence is rectilinear, moreover the singularity appears when $\omega_0 = \sqrt{\frac{\pi^2}{4}\frac{c^2}{L^2} + \gamma^2}$ .

Let's consider the general case, i.e. the system (8). From this system it follows that

$$P = z\left(1 - za^2\right)^{-1}\left[\frac{64}{81}z^2 + \frac{2\pi}{3}z + \frac{\pi^2}{8}\left(1 - \frac{\omega_0^2}{\omega_{0c}^2} + \frac{4}{\pi^2}k_0^2 L^2\right)\right], \qquad (9)$$

where the following notations are introduced

$$P = \tau_0\ ;\ \ z = \frac{u_0}{L}\ ;\ \ a^2 = \frac{1}{9\pi}\left(32 - 3\pi^2\right);\ \ \omega_{0c}^2 = \frac{\pi^2}{4}\frac{E}{\rho L^2}\ .$$

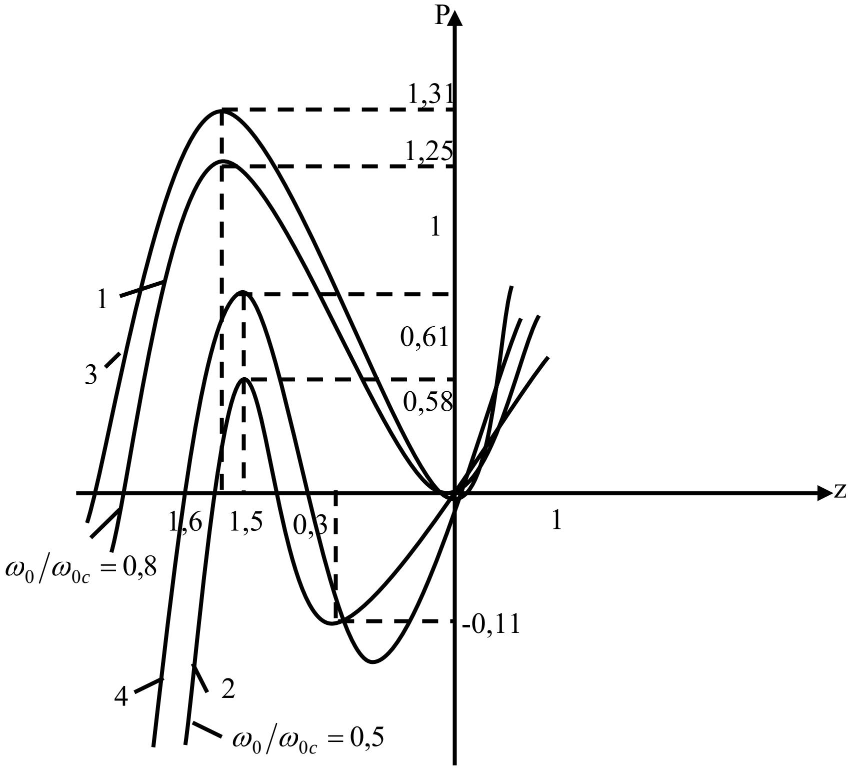


**Fig. 1. The dependence of the loading $P$ from $z$**
**(for curves 1 and 2 $k_c = 0$ ; for curves 3 and 4 $k_c = 2{,}4 \cdot 10^4\ \mathrm{kg/cm^4}$ )**

Let's find the value of critical loading. It is determined from the condition $\frac{dP}{dz} = 0$. As to (9) we have:

$$\frac{dP}{dz} = \left[\frac{64}{81}z^2 + \frac{4\pi}{3}z + \frac{\pi^2}{8}\left(1 - \frac{\omega_0^2}{\omega_{0c}^2} + \frac{4}{\pi^2}k_0^2 L^2\right)\right]\left(1 - za^2\right) +$$

$$+a^2\left[\frac{64}{81}z^3+\frac{2\pi}{3}z^2+\frac{\pi^2}{8}\left(1-\frac{\omega_0^2}{\omega_{0c}^2}+\frac{4}{\pi^2}k_0^2L^2\right)\right]=0\,.$$

After the transformation we'll get:

$$\frac{128}{81}a^2z^3-\frac{2\pi^2}{9}z^2-\frac{4\pi}{3}z-\frac{\pi^2}{8}\left(1-\frac{\omega_0^2}{\omega_{0c}^2}+\frac{4}{\pi^2}k_0^2L^2\right)=0\,. \qquad (10)$$

Let's note that the equation (10) when $\omega_0<\omega_{0c}\sqrt{1+\frac{4}{\pi^2}k_0^2L^2}$ have three real roots: $z_1\,z_2$ and $z_3>a^{-2}$.

The problem is realized numerically. For the input data was accepted:

$$L=5\,\text{m},\ \ E=2\cdot10^{11}\,\text{Pa}\ ,\ \ \rho=7800\ \text{kg}/\text{m}^3\ .$$

We find the critical values of loading determining the roots of the equation (10 and substituting in (9). The results of calculation are represented in fig. 1. At it given the dependence of the critical loading $P$ from $z$ for the different $\omega_0/\omega_{0c}$ and $k_c$.

The analysis of investigations shows that:

1) at non-linear vibration there exists the critical loading. It depends on the frequency of action and from the property of soil;
2) at increasing the frequency of action subject to the influence of a soil the critical force decreases and vanishes for $\omega_0/\omega_{0c}\geq0{,}8$. It is show that when $\omega_0/\omega_{0c}\leq0{,}8$ the linear theory isn't acceptable.
3) The calculation of soil influence at given frequencies leads to the increasing of critical loading in comparison with analogical critical loading without a soil.

**References**

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