RZAYEV O.G.

BUCKLING AROUND THE PENNY-SHAPED INTERFACE CRACK

Abstract

In the framework of the piece-wise homogeneous body model with use of the Three-Dimensional Linearized Theory of Stability the buckling problem of the circular sandwich plate with two parallel interface penny-shaped cracks is studied. The rotationally symmetrical buckling is considered and it is assumed that the lateral boundary of the plate is clamped and circumferentially compressed inward through this clamp by fixed radial displacement. Corresponding eigen-value and boundary value problems are solved numerically by employing FEM, local buckling of he coating layers around the cracks is investigated, the numerical results illustrating the influence of the problem parameters to this phenomenon are presented.

1. Introduction.

According to [1,2] the beginning of the failure of a protective coating is observed as its partial lifting from the surface of the basic material. It is known that this phenomenon is modelled as a local buckling of the coating part in compression along the coated surface. In this case it is assumed that there are cracks between the coating layer and the basic material and the local buckling of this layer is studied. As a result of this study a critical coating thickness is determined for a certain compressive external force, note that a similar failure mechanism is also applied for a near surface delamination of unidirectional-layered composite materials in compression, as it is well known, the compressive strength of structures made from laminated composite materials may be reduced several times by the presence of the delamination damage, which is modelled as a crack and the stability loss around this crack is studied. Note that in the many cases the necessity arises for employing of the Three-Dimensional Linearized Theory of Stability (TDLTS) [3] for investigation of these problems. Such investigations were made in [4-6] and in many others the review of which are given in [7]. However, the investigations carried out by TDLTS were made in the framework of the following assumption: the material of the composite is taken as a homogeneously anisptropic one with normalized mechanical properties. Consequently, the crack in this material was assumed as a macro-crack. In other words it was assumed that the crack length is significantly greater than the characteristic minimum size of the components of the composite.

It should be noted that in the framework of this assumption we could not distinguish the inter-layer crack from the into-layer crack. Therefore, the approaches developed in the framework of this assumptions cannot be applied for the investigation of the local buckling of the coating layer around the interface crack.

In [8] and in many other investigations listed therein the piece-wise-homogeneous body model is used. However, in these investigations only the internal cracks were considered, but not the problems related to the near-surface delamination. Therefore, the approach [8] cannot be applied for the coating is developed for the delamination problems around the near-surface micro-crack. All investigations are made on the clamped sandwich circular plate. In this case the corresponding eigen-value problem is solved numerically by employing FEM.

2. Formulation of the problem and method of solution.

Consider the circular sandwich plate with the geometry shown in fig. 1. Assume that the materials of the layers of the plate are isotropic, homogeneous and linear elastic.

Assume also that the materials of the upper and lower layers are the same. We suppose that between the middle and the upper layer as well as the lower layer of the plate there is a penny-shaped crack whose location is shown in fig. 1. Attempt to investigate the stability loss (delamination) around these cracks.

We associate with the middle layer of the plate Lagrangian cylindrical coordinate system $Or\theta z$ (fig. 1). Assume that the plate occupies the region $\{0 \le r \le R,$ $0 \le \theta \le 2\pi, -h/2 \le z \le h/2\}$ and the penny-shaped cracks occur in $\{z = \pm(h/2 - h_u),$ $0 \le r \le R\}$. In the framework of the above-stated we suppose that the plate is compressed circumferentially and inward through the clamp by fixed radial displacement in the lateral boundary $r = R$. The values of the normal forces with intensity p, which is shown in fig. 1b, is calculated after solution procedure through the well-known operations.

We will denote the values related to the upper and lower layers by upper indices (1) and (3) respectively, however the values related to the middle layer by (2). Thus, we investigate rotationally symmetrical stability loss of the plate shown in fig. 1. For this purpose we use the second version of a small deformation theory of TDLTS [3]. Within each layer we write the stability loss equations

$$\frac{\partial}{\partial r}T_{rr}^{(k)} + \frac{\partial}{\partial z}T_{rz}^{(k)} + \frac{1}{r}\left\{T_{rr}^{(k)} - \sigma_{\theta\theta}^{(k)} - \sigma_{\theta\theta}^{(k)}\frac{u_r^{(k)}}{r}\right\} = 0,$$

$$\frac{\partial}{\partial r}T_{zr}^{(k)} + \frac{1}{r}T_{zr}^{(k)} + \frac{\partial}{\partial z}T_{zz}^{(k)} = 0, \qquad (1)$$

mechanical relation

$$\sigma_{rr}^{(k)} = \lambda^{(k)}\left(\varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)}\right) + 2\mu^{(k)}\varepsilon_{rr}^{(k)}$$

$$\sigma_{\theta\theta}^{(k)} = \lambda^{(k)}\left(\varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)}\right) + 2\mu^{(k)}\varepsilon_{\theta\theta}^{(k)},$$

$$\sigma_{zz}^{(k)} = \lambda^{(k)}\left(\varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)}\right) + 2\mu^{(k)}\varepsilon_{zz}^{(k)}, \quad \sigma_{rz}^{(k)} = 2\mu^{(k)}\varepsilon_{rz}^{(k)}, \qquad (2)$$

and the geometrical relations

$$\varepsilon_{rr}^{(k)} = \frac{\partial u_r^{(k)}}{\partial r}, \quad \varepsilon_{\theta\theta}^{(k)} = \frac{u_r^{(k)}}{r}, \quad \varepsilon_{rz}^{(k)} = \frac{1}{2}\left(\frac{\partial u_r^{(k)}}{\partial z} + \frac{\partial u_z^{(k)}}{\partial r}\right), \quad \varepsilon_{zz}^{(k)} = \frac{\partial u_z^{(k)}}{\partial z}. \qquad (3)$$

Assume that the following conditions are satisfied

$$T_{rz}^{(1)}\Big|_{z=(h/2-h_u)+0} = T_{zz}^{(1)}\Big|_{z=(h/2-h_u)+0} = 0,$$

$$T_{rz}^{(2)}\Big|_{z=(h/2-h_u)-0} = T_{zz}^{(2)}\Big|_{z=(h/2-h_u)-0} = 0,$$

$$T_{rz}^{(2)}\Big|_{z=-(h/2-h_u)+0} = T_{zz}^{(2)}\Big|_{z=-(h/2-h_u)+0} = 0,$$

$$T_{rz}^{(3)}\Big|_{z=-(h/2-h_u)-0} = T_{zz}^{(3)}\Big|_{z=-(h/2-h_u)-0} = 0 \quad \text{for} \quad r \in [0, R_0], \qquad (4)$$

$$T_{rz}^{(1)}\Big|_{z=(h/2-h_u)} = T_{rz}^{(2)}\Big|_{z=(h/2-h_u)}, \; T_{zz}^{(1)}\Big|_{z=(h/2-h_u)} = T_{zz}^{(3)}\Big|_{z=(h/2-h_u)},$$

$$T_{rz}^{(2)}\Big|_{z=-(h/2-h_u)} = T_{rz}^{(3)}\Big|_{z=-(h/2-h_u)}, \; T_{zz}^{(2)}\Big|_{z=-(h/2-h_u)} = T_{zz}^{(2)}\Big|_{z=-(h/2-h_u)},$$

$$u_r^{(1)}\Big|_{z=(h/2-h_u)} = u_r^{(2)}\Big|_{z=(h/2-h_u)}, \; u_z^{(1)}\Big|_{z=(h/2-h_u)} = u_z^{(2)}\Big|_{z=(h/2-h_u)},$$

$$u_r^{(2)}\Big|_{z=-(h/2-h_u)} = u_r^{(3)}\Big|_{z=-(h/2-h_u)}, \quad u_z^{(2)}\Big|_{z=-(h/2-h_u)} = u_z^{(3)}\Big|_{z=-(h/2-h_u)} \quad \text{for} \quad r \in [R_0, R], \qquad (5)$$

$$T_{rz}^{(1)}\Big|_{z=h/2}=0\,,\ T_{zz}^{(1)}\Big|_{z=h/2}=0\,,\ T_{rz}^{(3)}\Big|_{z=-h/2}=0\,,\ T_{zz}^{(3)}\Big|_{z=-h/2}=0\quad\text{for}\quad 0\le r\le R\,, \tag{6}$$

$$u_r^{(k)}\Big|_{r=R}=0\,,\ u_z^{(k)}\Big|_{r=R}=0\quad\text{for}\quad z\in(-h/2,+h/2). \tag{7}$$

In (1), (4)-(6) the following notation has been introduced

$$T_{rr}^{(k)}=\sigma_{rr}^{(k)}+\sigma_{rr}^{(k)0}\frac{\partial u_r^{(k)}}{\partial r}+\sigma_{rz}^{(k)0}\frac{\partial u_r^{(k)}}{\partial z}\,,$$

$$T_{rz}^{(k)}=\sigma_{rz}^{(k)}+\sigma_{rz}^{(k)0}\frac{\partial u_r^{(k)}}{\partial r}+\sigma_{zz}^{(k)0}\frac{\partial u_r^{(k)}}{\partial z}\,,$$

$$T_{zr}^{(k)}=\sigma_{rz}^{(k)}+\sigma_{rr}^{(k)0}\frac{\partial u_z^{(k)}}{\partial r}+\sigma_{rz}^{(k)0}\frac{\partial u_z^{(k)}}{\partial z}\,,$$

$$T_{zz}^{(k)}=\sigma_{zz}^{(k)}+\sigma_{rz}^{(k)0}\frac{\partial u_z^{(k)}}{\partial r}+\sigma_{zz}^{(k)0}\frac{\partial u_z^{(k)}}{\partial z}\,. \tag{8}$$

The other notation used in (1)-(7) is conventional. Note that the values indicated by upper indices 0 and entering (1)-(8) relate to the precritical state and are determined from the following boundary-value problem

$$\frac{\partial\sigma_{rr}^{(k)0}}{\partial r}+\frac{\partial\sigma_{rz}^{(k)0}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}^{(k)0}-\sigma_{\theta\theta}^{(k)0}\right)=0\,,\quad \frac{\partial\sigma_{rz}^{(k)0}}{\partial r}+\frac{\partial\sigma_{zz}^{(k)0}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(k)0}=0\,,$$

$$\sigma_{rr}^{(k)0}=\lambda^{(k)}\left(\varepsilon_{rr}^{(k)0}+\varepsilon_{\theta\theta}^{(k)0}+\varepsilon_{zz}^{(k)0}\right)+2\mu^{(k)}\varepsilon_{rr}^{(k)0}\,,$$

$$\sigma_{\theta\theta}^{(k)0}=\lambda^{(k)}\left(\varepsilon_{rr}^{(k)0}+\varepsilon_{\theta\theta}^{(k)0}+\varepsilon_{zz}^{(k)0}\right)+2\mu^{(k)}\varepsilon_{\theta\theta}^{(k)0}\,,$$

$$\sigma_{zz}^{(k)0}=\lambda^{(k)}\left(\varepsilon_{rr}^{(k)0}+\varepsilon_{\theta\theta}^{(k)0}+\varepsilon_{zz}^{(k)0}\right)+2\mu^{(k)}\varepsilon_{zz}^{(k)0}\,,$$

$$\sigma_{rz}^{(k)0}=2\mu^{(k)}\varepsilon_{rz}^{(k)0}\,,\ \ \varepsilon_{rr}^{(k)0}=\frac{\partial u_r^{(k)0}}{\partial r}\,,\ \ \varepsilon_{\theta\theta}^{(k)0}=\frac{u_r^{(k)0}}{r}\,,$$

$$\varepsilon_{rz}^{(k)0}=\frac{1}{2}\left(\frac{\partial u_r^{(k)0}}{\partial z}+\frac{\partial u_z^{(k)0}}{\partial r}\right)\,,\ \varepsilon_{zz}^{(k)0}=\frac{\partial u_z^{(k)0}}{\partial z}\,; \tag{9}$$

$$\sigma_{rz}^{(1)0}\Big|_{z=(h/2-h_u)+0}=0\,,\quad \sigma_{zz}^{(1)0}\Big|_{z=(h/2-h_u)+0}=0\,,\ \ \sigma_{rz}^{(2)0}\Big|_{z=(h/2-h_u)-0}=0\,,$$

$$\sigma_{zz}^{(2)0}\Big|_{z=(h/2-h_u)-0}=0\,,\ \sigma_{rz}^{(2)0}\Big|_{z=-(h/2-h_u)+0}=0\,,\ \sigma_{zz}^{(2)0}\Big|_{z=-(h/2-h_u)+0}=0\,,$$

$$\sigma_{rz}^{(3)0}\Big|_{z=-(h/2-h_u)-0}=0\,,\ \sigma_{zz}^{(3)0}\Big|_{z=-(h/2-h_u)-0}=0\quad\text{for}\quad r\in[0,R_0], \tag{10}$$

$$\sigma_{rz}^{(1)0}\Big|_{z=(h/2-h_u)}=\sigma_{rz}^{(2)0}\Big|_{z=(h/2-h_u)}\,,\ \ \sigma_{zz}^{(1)0}\Big|_{z=(h/2-h_u)}=\sigma_{zz}^{(2)0}\Big|_{z=(h/2-h_u)}\,,$$

$$\sigma_{rz}^{(2)0}\Big|_{z=-(h/2-h_u)}=\sigma_{rz}^{(3)0}\Big|_{z=-(h/2-h_u)}\,,\ \ \sigma_{zz}^{(2)0}\Big|_{z=-(h/2-h_u)}=\sigma_{zz}^{(3)0}\Big|_{z=-(h/2-h_u)}\,,$$

$$u_r^{(1)0}\Big|_{z=(h/2-h_u)}=u_r^{(2)0}\Big|_{z=(h/2-h_u)}\,,\ \ u_z^{(1)0}\Big|_{z=(h/2-h_u)}=u_z^{(2)0}\Big|_{z=(h/2-h_u)}\,,$$

$$u_r^{(2)0}\Big|_{z=-(h/2-h_u)}=u_r^{(3)0}\Big|_{z=-(h/2-h_u)}\,,\ \ u_z^{(2)0}\Big|_{z=-(h/2-h_u)}=u_z^{(3)0}\Big|_{z=-(h/2-h_u)}\quad\text{for}\quad r\in[R_0,R], \tag{11}$$

$$\sigma_{rz}^{(1)0}\Big|_{z=h/2}=\sigma_{zz}^{(1)0}\Big|_{z=h/2}=0\,,\ \sigma_{rz}^{(3)0}\Big|_{z=-h/2}=\sigma_{zz}^{(3)0}\Big|_{z=-h/2}=0\quad\text{for}\quad r\in(0,R), \tag{12}$$

$$u_r^{(k)0}\Big|_{r=R}=-U\,,\ \ u_z^{(k)0}\Big|_{r=R}=0\quad\text{for}\quad z\in(-h/2,+h/2). \tag{13}$$

Thus, the investigation of the stability loss problem of the considered plate is reduced to the solution of the boundary-value problem (9)-(13) and eigen-value problem (1)-(8). Each of these problems is solved by employing FEM and in this case it is taken the problem symmetry with respect to $z=0$ plane into account. In other words we consider only the sub-region $\{0 \le r \le R\ ,\ 0 \le z \le h/2\}$ and this sub-region is divided into 120 rectangular Lagrange family quadratic elements, with 533 nodes and 1027 NDOF. Under determination of the precritical state, i.e. under solution of the boundary-value problem (9)-(13) we employ the classical Ritz technique, for numerical investigation we give the values to U (13) and after solution of the problem (9)-(13) determine the values of p (fig. 1) from the relation

$$hp = \int_{-h/2}^{-(h/2-h_u)} \sigma_{rr}^{(3)0}\Big|_{r=0} dz + \int_{-(h/2-h_u)}^{(h/2-h_u)} \sigma_{rr}^{(2)0}\Big|_{r=0} dz + \int_{(h/2-h_u)}^{h/2} \sigma_{rr}^{(1)0}\Big|_{r=0} dz\ . \qquad (14)$$

The analyses of the numerical results show that the stresses $\sigma_{zz}^{(k)0}$, $\sigma_{rz}^{(k)0}$ differ from zero in the very near vicinity of the lateral boundary $r = R$ and can be taken as zero outside this region. Therefore the existence of the crack in the plate does not change the stress distribution from the one without crack in the precritical state. The values of the precritical state correspond to that obtained for the whole plate without the crack. So, we determine the values related to the precritical state and for solution of the eigen-value problem (1)-(3) introduce the following functional

$$\Pi\left(u_r^{(1)}, u_r^{(2)}, u_r^{(3)}, u_z^{(1)}, u_z^{(2)}, u_z^{(3)}\right) = \frac{1}{2}\sum_{k=1}^{3} \iint_{\Omega'^{(k)}} P^{(k)} r\, dr dz\ , \qquad (15)$$

where

$$\Omega'^{(1)} = \{0 \le r \le R, (h/2 - h_u) \le z \le h/2\} - \{0 \le r \le R_0\ ,\ z = (h/2 - h_u) + 0\},$$

$$\Omega'^{(2)} = \{0 \le r \le R, (h/2 - h_u) \le z \le -(h/2 - h_u)\} - \{0 \le r \le R_0\ ,\ z = (h/2 - h_u) - 0\},$$

$$\Omega'^{(3)} = \{0 \le r \le R, -h/2 \le z \le -(h/2 - h_u)\} - \{0 \le r \le R_0\ ,\ z = -(h/2 - h_u) - 0\},$$

$$P^{(k)}(r,z) = T_{rr}^{(k)}\varepsilon_{rr}^{(k)} + \left[\sigma_{\theta\theta}^{(k)} + \sigma_{\theta\theta}^{(k)0}\frac{u_r^{(k)}}{r}\right]\varepsilon_{\theta\theta}^{(k)} + T_{rz}^{(k)}\varepsilon_{rz}^{(k)} + +T_{zr}^{(k)}\varepsilon_{rz}^{(k)} + +T_{zz}^{(k)}\varepsilon_{zz}^{(k)}\ . \qquad (16)$$

From the relation

$$\delta\Pi\left(u_r^{(1)}, u_r^{(2)}, u_r^{(3)}, u_z^{(1)}, u_z^{(2)}, u_z^{(3)}\right) = 0 \qquad (17)$$

we obtain the equations (1) and boundary conditions (4)-(6). In this way we prove that the equations (1), (4)-(6) are Eurel equations of the functional (15), (16). Thus, applying the version of the FEM technique described in [9] to equation (17) we obtain the corresponding homogeneous linear algebraic equation for the nodal values of the displacement. Note that upon dividing the region $\{0 \le r \le R, 0 \le z \le h/2\}$ into finite elements, according to [10] and others, we replace the nodes in the finite elements associated with the crack tip. This replacement is shown in fig. 2. In this way we keep the needed singularity of the stresses and strains at the crack tips.

3. Numerical results and discussions.

We assume that $E^{(1)} = E^{(3)}$, $\nu^{(1)} = \nu^{(3)}$ and $\nu^{(1)} = \nu^{(2)} = 0.3$, where $E^{(k)}$ and $\nu^{(k)}$ $(k = 1,2,3)$ are Young's modulus and Poisson coefficients respectively. Introduce the parameter $\beta = h_u/(2R_0)$ and analyze the influence of the problem parameters on the values of $p_{cr}/E^{(2)}$. Note that this analysis will be made below in the framework of the following two statements which are established as a result of various numerical investigation:

1. In the cases, where $\beta < h/(2R)$ the values of the $p_{cr}/E^{(2)}$ do not depend on the geometrical parameters of the whole plate, these values depend only on the parameter β. Consequently, this situation allows us to conclude that the results obtained for the cases $\beta < h/(2R)$ relate to the local buckling of the coating layer which is on the crack. Moreover, according to these results the interaction between the upper and lower cracks can be neglected completely under determination of the values of the critical force.

2. The trustiness of the PC programs and algorithms composed by authors are established by comparing the corresponding results with those obtained in [6]. Note that in [6] the local buckling problem of the half-space near-surface penny-shaped crack had been investigated with employing double integral equation technique and for obtaining the numerical results the Galerkin method had been used. Moreover, it was assumed that the material of the half-space is a multilayered composite consisting of the two alternating layers which lie in the planes $z = const$. According to the present notation, Young's modulus and Poisson's coefficients for these layer's materials can be denoted as $E^{(1)}$, $\nu^{(1)}$ and $E^{(2)}$, $\nu^{(21)}$ respectively. It should be noted that in [6] it was also assumed that the near-surface penny-shaped crack is a macro-crack and the material of the half-space is modelled as a transversally-isotropic one, whose isotropy axis coincides with the Oz axis. Table 1 shows the values of $p_{cr}/\left((1-c)E^{(1)} + cE^{(2)}\right)$ obtained by applying both present and [6] approaches in the case where $c = 0.3$ (c is a filler concentration), $\beta = 1/16$, $\nu^{(1)} = \nu^{(2)} = 0{,}3$ for various values of $E^{(2)}/E^{(1)}$. Note that the results given in Table 1 and related to the present approach had been obtained in the framework of the 1st statement. The comparison of the corresponding results given in Table 1 shows that the numerical results obtained in the framework of the present approach are very close to those obtained in [6]. This situation guarantees the validity of the algorithms and programs used in the present investigations.

Table 1

$E^{(2)}/E^{(1)}$	Results of [6]	Present results
1	0.0167	0.0178
10	0.0140	0.0154
25	0.0126	0.0129

Now we return to the analysis of the results, which are obtained in the framework of the above-mentioned statements for the local buckling of the coating layer around the penny-shaped interface micro-crack. In fig. 3 the graphs of the relations between $p_{cr}/E^{(2)}$ and $\log_e\left(E^{(1)}/E^{(2)}\right) = \log\left(E^{(1)}/E^{(2)}\right)$ are given for various β. While constructing these graphs, the values of the $E^{(1)}/E^{(2)}$ have been changed as follows: $0.01 \le E^{(1)}/E^{(2)} \le 100$ for $\beta = 0.04,\ 0.008$; $0.01 \le E^{(1)}/E^{(2)} \le 50$ for $\beta = 0.012$; $0.01 \le E^{(1)}/E^{(2)} \le 10$ for $\beta = 0.16, 0.20$ because under $E^{(1)}/E^{(2)} > 50$ (for $\beta = 0.12$) and under $E^{(1)}/E^{(2)} > 10$ (for $0.16, 0.20$) the instability (failure) of the middle layer material arises in an earlier stage of the loading than the local buckling of the coating layer.

The graphs given in fig. 3 show that the dependence between $p_{cr}/E^{(2)}$ and $E^{(1)}/E^{(2)}$ has a non-monotonic character for each of the values of β. Moreover, these graphs indicate that the values of $p_{cr}/E^{(2)}$ increase monotonically with β.

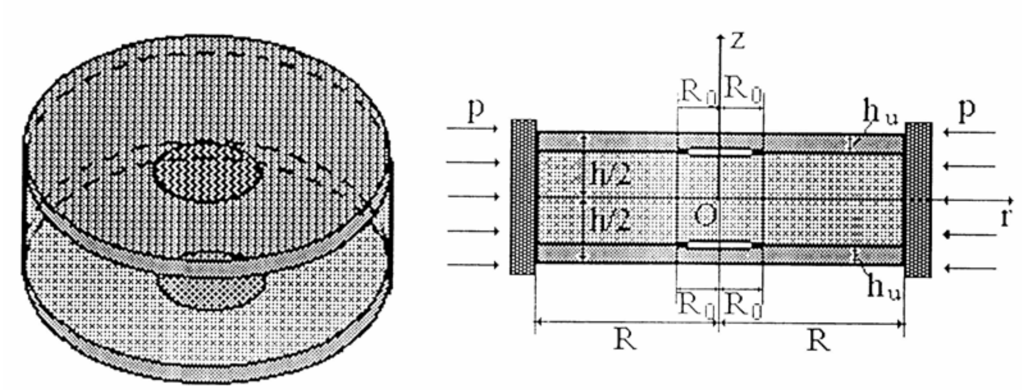

Fig. 1.

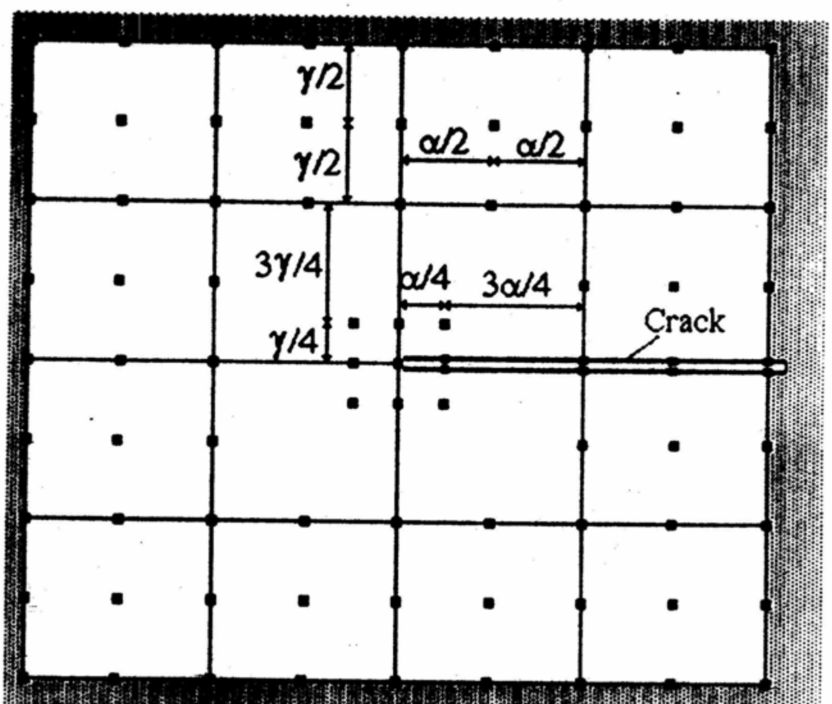

Fig. 2.

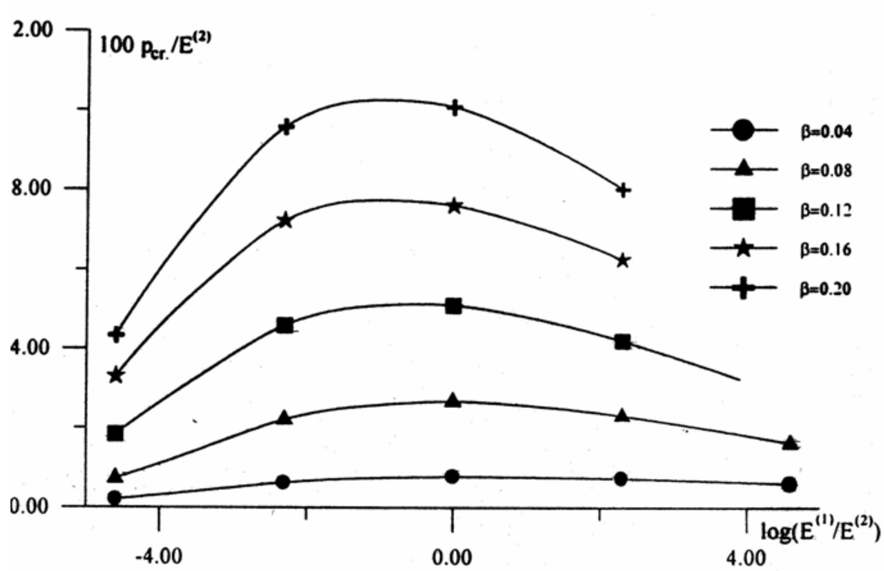

Fig. 3.

References

[1]. Kolesov V.S., Cherkunov V.V. *Critical size of coating defects.* Phys.Chemis. of working of mater., 1987, №1, p.78-83. (in Russian)
[2]. Diao D.F., Sawaki Y. *Fracture mechanism of ceramic coating during wear.* Thin Solid Films, 270, p.362-366.
[3]. Guz A.N. *Fundamentals of the Three-Dimensional Theory of Stability of Deformable Bodies.* Springer-Verlag 556, 1999.
[4]. Nazarenko V.M. *On the fracture of materials under compression along the near surface ceack in plane-deformation state.* Prikl.Mekan.22, 1986, №10, p.72-81. (In Russian)
[5]. Nazarenko V.M. *Plane problem of fracture of materials in compression along near-surface crack.* Prikl.Mekan.22, 1986, № 12, p.96-104. (in Russian)
[6]. Guz A.N., Nazarenko V.M. *Symmetric failure of the half-space with penny-shaped crack n compression.* Theor.Appl.Fract., Mech., 1985, №3, p.233-245.
[7]. Guz A.N. *Mechanics of compressive failure of composite materials.* Kiev: Nauk. dumka., 1990, p.630. (in Russian)
[8]. Guz I.A. *Study of the composite resistance under compression along two parallel structural cracks on the interface.* Int.Appl.Mechan., 30, 1994, №11, p.18-25.
[9]. Akbarov S.D., Guz A.N. *Mechanics of curved composites.* Kluwer Academic Publishers, 2000, 464p.
[10]. Benzley S.E. *Representation of singularities with isoparametric finite elements.* Inst.Jour.Numer., Methods in Engin.8, 1974, p.537-545.

Orudjali G. Rzayev
Institute of Mathematics & Mechanics of NAS Azerbaijan.
9, F.Agayev str., 370141, Baku, Azerbaijan.
Tel.:39-47-20(off.).

[Rzayev O.G.]

Received September 5, 2001; Revised December 17, 2001.
Translated by author.