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ON TRANSVERSE IMPACT ON THE FLEXIBLE FILAMENT

Abstract

In the paper the construction of solving the problem on transverse impact with constant velocity by obtuse rigid wedge on elastic filament is given. It's assumed that the velocity of wave of strong break is more than the velocity of elastic wave in the filament.

The stress state of an elastic filament at impact on it by solid in some cases essentially depends on geometry of bombarding body and has various applications in many fields of techniques. Beginning from [2] some problems on impact by wedge on flexible filament [3, 4, 6] are solved. In the present paper the solution of the problem on transverse impact by symmetric wedge having plane fore-part (fig. 1) on flexible elastic filament is given.

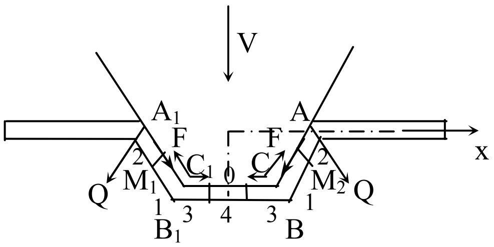

Fig. 1.

§1. Let the transverse impact by rigid symmetric wedge with plane fore-part with the constant velocity V be performed on infinite long flexible linear-elastic rectilinear non-strained filament. It's assumed that after impact a part of the filament A_1B_1BA covers the surface of the bombarding named wedge, and the velocity of the points A and A_1 are more than the velocity of elastic wave of filament. In addition from the right of the point A and from the left of the point A_1 the filament is at rest state $[1, 2]$ since $b = Vctg\gamma > a_0$. Here γ is an angle between the initial position of filament and check of wedge $BA\left(B_1A_1\right)$; $a_0 = \sqrt{E\rho^{-1}}$ is velocity of elastic wave; b is velocity of the break point $A\left(A_1\right)$. In the domain A_1B_1BA in filament four elastic waves whose fronts are the points M_1, C_1, C, M_2, and two waves of strong break A, A_1 [1, 2] (fig. 1). Denote the length BB_1 by $2L$. The behavior of the filament in the domains $A_1M_1B_1C_1O$ and AM_2BCO are the same. The velocity of particles of filament in the domains $OB\left(OB_1\right)$ and $BA\left(B_1A_1\right)$ are directed along the check of wedge respectively. In the domains CO and C_1O the filament is at rest to the time $t = \dfrac{L}{a_0}$ relative to wedge.

Since the impact on flexible linear-elastic filament is performed with constant velocity, then in originating domains the filaments determining the parameters are constant. In fig. 1 B, B_1 are stationary break joints and the motion of filaments relative to these breaks are taken as motion via fixed block [5] and the deformations aren't discontinuous at these points. It's assumed that the friction is absent in covering domain between the filament and wedge. We'll supply the unknown parameters of motion of flexible filament originating in domains 1, 2, 3, 4, 5, 11, 31 with corresponding indices.

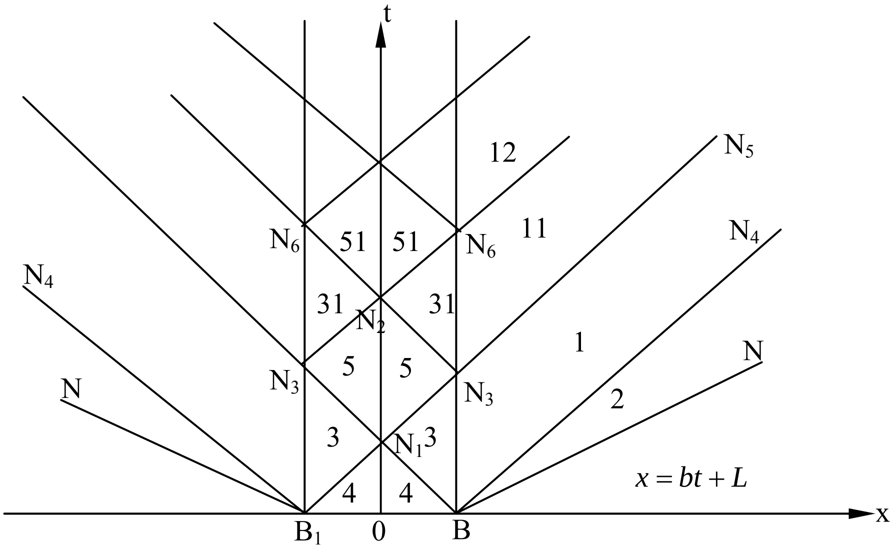

Fig. 2.

The wave picture of motion of filament after impact in the plane (x,t) is shown in fig. 2. The following designations are accepted: ε is deformation; ϑ is velocity of particles of filament; σ is stress; E is Young's modulus; ρ is density; t is time; x is Lagrangian coordinate.

At supersonic regime the conditions at the break point A have the form [2]

$$\vartheta_2 = 0\ ;\ \ \varepsilon_2 = \sec\gamma - 1;\ \ (\sigma_2 = E\varepsilon_2) \tag{1.1}$$

for the first regime when $F = \mu_* Q$;

$$\vartheta_2 = b(\sec\gamma - 1 - \varepsilon_2);\ \ b = V\,ctg\,\gamma,$$

$$\varepsilon_2 = \frac{M^2}{M^2 - tg^2\gamma}\left(tg\gamma_* - tg\frac{\gamma}{2}\right)\sin\gamma\ ;\ \ M = \frac{V}{a_0}\ ; \tag{1.2}$$

$$\sigma_2 = E\varepsilon_2\ ;\ \ \mu_* = tg\gamma_*\ ,$$

for the second regime when $F = \mu_* Q,\ \gamma < 2\gamma_*$;

$$\vartheta_2 = V\cos\gamma(tg\,\gamma - tg\,\gamma_*);$$

$$\varepsilon_2 = \left(tg\gamma_* - tg\frac{\gamma}{2}\right)\sin\gamma\ ; \tag{1.3}$$

$$\sigma_2 = 0\ ,$$

for the third regime when $F = \mu_* Q,\ \gamma > 2\gamma_*$.

Here F, Q are tangent and normal to the surface of body component of the point force, $\mu_* = tg\,\gamma_*$ is a coefficient of coulomb friction at breakpoint.

From (1.3) it follows that $(\vartheta_2 > 0,\ \varepsilon_2 < 0, \sigma_2 = 0)$ for this regime the wrinkling of filament occurs. At first we investigate the behavior of filament when on wave of strong break there are the first and second regimes of motion.

§2. On the fronts C, M_2 (fig. 1) or on the fronts 4-3 (BN_1), 1-2 (BN_4) we have correspondingly

$$\vartheta_3 - \vartheta_4 = -a_0(\varepsilon_4 - \varepsilon_3), \qquad (2.1)$$

$$\vartheta_1 - \vartheta_2 = a_0(\varepsilon_2 - \varepsilon_1). \qquad (2.2)$$

At the point B or on BN_3 (fig. 2) the kinematic condition

$$\vartheta_3 = \vartheta_1 \cos\gamma \qquad (2.3)$$

and the condition

$$\varepsilon_1 = \varepsilon_3 \qquad (2.4)$$

exist.

Since

$$\vartheta_4 = 0\,;\ \varepsilon_4 = 0\,;\ \vartheta_3 = a_0\varepsilon_3\,,\ \left(t < \frac{L}{a_0}\right), \qquad (2.5)$$

then from (2.1)-(2.5) we determine ϑ_1, ϑ_3, ε_1, ε_3 in the form of

$$\begin{aligned}&\vartheta_1 = (1+\cos\gamma)^{-1}(\vartheta_2 + a_0\varepsilon_2)\,;\quad \vartheta_3 = \vartheta_1\cos\gamma\,;\\ &\varepsilon_1 = \varepsilon_3 = (1+\sec\gamma)^{-1}\left(\varepsilon_2 + \vartheta_2 a_0^{-1}\right);\ \ \sigma_1 = \sigma_3 = E\varepsilon_1\,.\end{aligned} \qquad (2.6)$$

Note that in the case of the transverse impact by obtuse wedge $(2L = 0)$ the parameters of motion of filament in domain 1 $(0 \le x < a_0 t)$ have the form [2]

$$\vartheta_1^0 = 0;\ \ \varepsilon_1^0 = \varepsilon_2 + \vartheta_2 a_0^{-1}\,;\ \ \sigma_1^0 = E\,\varepsilon_1^0\,, \qquad (2.7)$$

where ϑ_1^0, ε_1^0, σ_1^0 mean parameters of filament at impact by obtuse wedge.

Here ε_2, ϑ_2 are expressed by the formulas (1.1) or (1.2).

It follows from (2.6), (2.7) that the deformation of filament on fronts 1-2 is appreciable less than at transverse impact by obtuse wedge.

At the time $t = \dfrac{L}{a_0}$ the elastic waves C and C_1 (fig. 1) meet in point O and two reflective elastic waves 3-5 (fig.2) arise.

It's obvious that if two identical waves move to meet each other, then at the meeting moment (at the point O (fig. 1, fig. 2)) the velocity of particles is zero, and here the stress (deformation) is doubled. This section of filament (the point O) is stationary relative to wedge and we'll consider it as embedded end of filament. Thus in domains 5 (fig. 2)

$$\begin{aligned}&\vartheta_5 = 0\,;\ \varepsilon_5 = 2\varepsilon_3 = 2(1+\sec\gamma)^{-1}\left(\varepsilon_2 + \vartheta_2 a_0^{-1}\right);\\ &\sigma_5 = Ee_5 = E2\varepsilon_3\,.\end{aligned} \qquad (2.8)$$

The solutions (2.6), (2.8), (1.1), (1.2) hold when $\dfrac{L}{a_0} \le t < \dfrac{2L}{a_0}$. At the time $t = \dfrac{2L}{a_0}$ after impact the reflected elastic waves 3-5 (fig. 2) interact with stationary breaks B and B_1. Since the behaviour of filament with respect to the point O (the axis $0t$) (fig. 2) is the same we'll consider a problem at the right hand of filament from the point O. When $t = \dfrac{2L}{a_0}$ from the left of the point B the reflective elastic wave is propagated with front 5-31 (N_3N_2), and from right of the point B by BA (fig. 1) the elastic wave is propagated with front 11-1 (N_3N_5). Thus at the period $\frac{2L}{a_0} \le t < \frac{3L}{a_0}$ after impact on the covering domain OBA (fig. 1) on the filament five domains (domain 5, domain 31, domain 11, domain 1 and domain 2) exist.

The unknown parameters ε, ϑ of filament in domains 31 and 11 are determined from the relations

$$\begin{aligned} \vartheta_5 - \vartheta_{31} &= -a_0(\varepsilon_{31} - \varepsilon_5); \\ \vartheta_{11} - \vartheta_1 &= a_0(\varepsilon_1 - \varepsilon_{11}). \end{aligned} \tag{2.9}$$

The kinematic relation at the point B will be

$$\vartheta_{31} = \vartheta_{11} \cos\gamma . \tag{2.10}$$

Condition (2.4) in this case will be of the form:

$$\varepsilon_{11} = \varepsilon_{31} . \tag{2.11}$$

Allowing for (2.6), (2.8), from (2.9) we determine the parameters ε_{11}, ϑ_{11}, ϑ_{31}, ε_{31} in the form of

$$\begin{aligned} &\varepsilon_{11} = \varepsilon_{31} = \frac{3+\cos\gamma}{1+\cos\gamma}\varepsilon_1 ; \ \sigma_{11} = \sigma_{31} = E\varepsilon_{31} ; \\ &\vartheta_{11} = a_0\varepsilon_1 tg^2\frac{\gamma}{2}\sec\gamma ; \quad \vartheta_{31} = \vartheta_{11}\cos\gamma . \end{aligned} \tag{2.12}$$

Here ε_1 is expressed by the formula (2.6).

When $t = \frac{3L}{a_0}$ fronts of elastic waves 5-31 (N_3N_2) (fig. 2) meet at the point O and are reflected from this point O with fronts 31-51 (N_2N_6), consequently for the period $\frac{3L}{a_0} \le t < \frac{4L}{a_0}$ new domain 51 (fig. 2) arise. Since point O is always stationary with respect to bombarding body, then the velocity of particles in domain 51 will be equal to zero.

From the condition on fronts of elastic waves 51-31 $(N_2 - N_6)$

$$\vartheta_{51} - \vartheta_{31} = a_0(\varepsilon_{31} - \vartheta_{51}), \tag{2.13}$$

subject to $\vartheta_{51} = 0$ we determine the deformation ε_{51} in the form of

$$\varepsilon_{51} = \varepsilon_{31} + \vartheta_{31}a_0^{-1} . \tag{2.14}$$

Allowing for (2.12) in (2.14) we obtain

$$\varepsilon_{51} = \frac{4\varepsilon_1}{1+\cos\gamma} ; \quad \sigma_{51} = E\varepsilon_{51} ; \quad \vartheta_{51} = 0 . \tag{2.15}$$

From (2.8), (2.12), (2.15) it follows that the following relation holds

$$\sigma_5 < \sigma_{31} < \sigma_{51} . \tag{2.16}$$

Further we can construct a solution of the problem taking into account the multiple reflections of elastic waves from the points O and B.

§3. Now we investigate a problem when on wave of strong break (at the breakpoint A) (fig. 3) we have the condition (1.3), i.e. when at the point A the wrinkling f filament with respect to the point O is symmetric, then we can consider stress state at right hand of filament from the point O. In fig. 4 the wave scheme of motion of filament in the plane (x,t) is represented. As result of impact at the period $0<t<\frac{L}{a}$ in filament (at the right hand) four domains 1, 2, 3, 4 (fig. 3, fig. 4) arise. In domain 4 the filament is

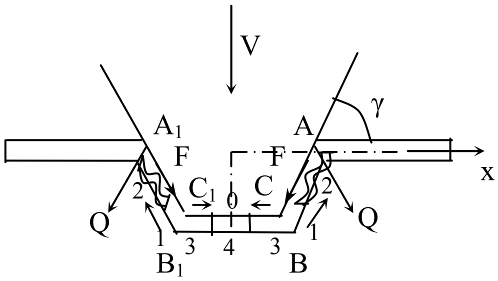

Fig. 3.

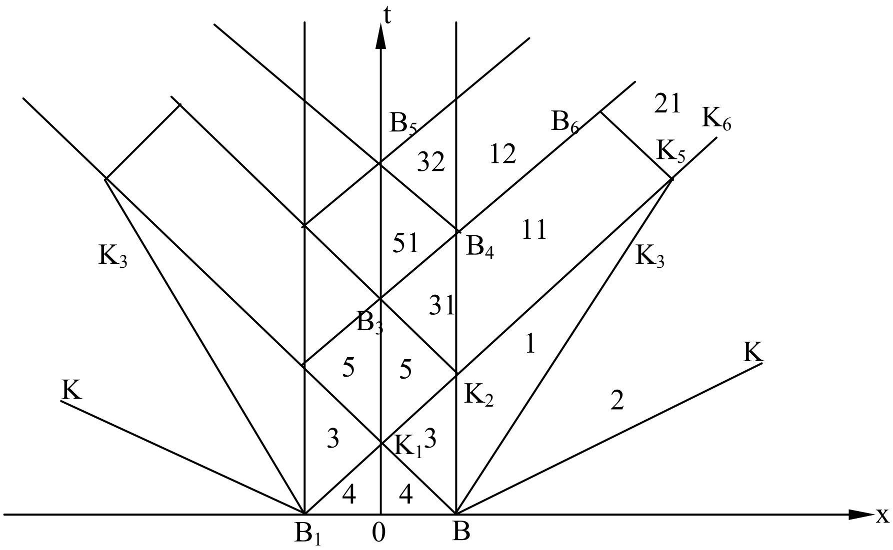

Fig. 4.

at rest. Here the line BK is front of wave of strong break (trajectory of the break point A), BK_3 is a boundary of division of domain with wrinkling (domain 2) and of domain where filament is again stretched (domain 1), BK_1 is front of elastic wave. The straightening front BK_3 is subject to determination in the course of solving the problem, it is propagated with the unknown constant velocity ω $(\omega<a_0)$. In domain 2 the solution of problem is expressed by the formula (1.3). on the expanding front BK_3 1-2 the following conditions are satisfied

$$\sigma_1-\sigma_2=\rho\omega(\vartheta_2-\vartheta_1); \quad (\sigma_2=0); \tag{3.1}$$

$$\vartheta_1-\vartheta_2=\omega(\varepsilon_1-\varepsilon_2). \tag{3.2}$$

Just as we have the conditions (2.3), (2.4) at the point B and the conditions (2.1), (2.5). from the system (2.1), (2.3), (2.4), (2.5), (3.1), (3.2) we determine the unknown parameters ω, ε_1, ε_3, ϑ_3 , ϑ_1 in the form of

$$\omega = a_1 + \sqrt{a_1^2 - a_2}\ ; \quad \varepsilon_1 = \varepsilon_3 = -\varepsilon_2 \frac{\omega^2}{a_0^2 - \omega^2}\ ;$$
$$\vartheta_1 = \vartheta_2 + \frac{a_0^2 \omega}{a_0^2 - \omega^2}\varepsilon_2\ ; \quad \vartheta_3 = \vartheta_1 \cos\gamma\ , \tag{3.3}$$

where

$$a_1 = -\frac{a_0^2 \varepsilon_2}{2(a_0 \varepsilon_2 \sec\gamma - \vartheta_2)}\ ; \qquad a_2 = \frac{\vartheta_2 a_0^2}{a_0 \varepsilon_2 \sec\gamma - \vartheta_2}\ . \tag{3.4}$$

Note that if impact is performed by an obtuse wedge then straightening velocity will be in the form of [2]

$$\omega_0 = \bar{a}_1 + \sqrt{\bar{a}_1^2 - \bar{a}_2}\ ; \ \bar{a}_1 = \frac{\varepsilon_2 a_0^2}{2\vartheta_2}\ ; \ \bar{a}_2 = a_0^2\ . \tag{3.5}$$

From (3.4), (3.5) it follows that

$$a_1 < \bar{a}_1\ ; \ a_2 < \bar{a}_2 \tag{3.6}$$

and consequently the velocity ω is less than the velocity ω_0 $(\omega < \omega_0)$. Here ε_2 $(\varepsilon_2 < 0)$ are determined by the formulas (1.3).

When $t = \frac{L}{a_0}$ the elastic waves B_1K_1 and BK_1 are reflected from the stationary point O and new-domain 5 arises (K_1K_2 is reflected elastic wave) (fig. 4) and the solution is domain 5 will be

$$\vartheta_5 = 0\ ; \ \varepsilon_5 = 2\varepsilon_3 = -\frac{2\varepsilon_2 \omega^2}{a_0^2 - \omega^2}\ ; \ (\varepsilon_2 < 0);$$
$$\sigma_5 = 2\rho a_0^2 \varepsilon_3\ . \tag{3.7}$$

At the period $t = \frac{2L}{a_0}$ the fronts of reflected elastic wave K_1K_2 interact with the stationary break B and from the left of the point B the reflective elastic wave with the front K_2B_3 is propagated, and from the right the elastic wave with the front K_2K_4 (fig. 4) is propagated.

In the domain OBA (fig. 3) in the filament for the period $\frac{2L}{a_0} \le t < \frac{3L}{a_0}$ five domains arise and they are denoted by 2, 1, 11, 31, 5. The solutions in domains 2, 1, 5 are known, we have to determine solutions in domains 11 and 31.

The solutions of problem in domains 11 and 31 are determined from conditions on the fronts K_1B_3, K_2K_4 and have the form (2.9). consequently, the parameters in these domains are determined by the formulas (2.12), but here ε_1, ϑ_1 are expressed by the formulas (3.3), (3.4). When $t = \frac{3L}{a_0}$ the front of the elastic wave K_2B_3 is reflected from the point O with front B_3B_4 and new domain 51 (fig. 4) arises. The unknown parameters ε_{51}, ϑ_{51} in domain 51 are determined by the formulas (2.15), but only ε_1 is expressed by the relations (3.3), (3.4). since the velocity of elastic wave a_0 is more than the velocity of straightening wave ω, then after some period the front of elastic wave K_2K_4 overtakes the straightening wave BK_3 and at the point K_5 interaction of these waves occurs. The interaction time is determined from the relation

$$\omega t + L = a_0(t - t_0);\ t_0 = \frac{2L}{a_0} \tag{3.8}$$

and

$$t = t_1 = \frac{3L}{a_0 - \omega}. \tag{3.9}$$

Note that for the given L, the period $t = t_1$ and consequently wave picture of motion depends on the relation $\frac{\omega}{a_0}$. Let $t_1 > \frac{4L}{a_0}$, i.e. $\frac{1}{4} < \frac{\omega}{a_0} < 1$, then for $t = \frac{4L}{a_0}$ the front of elastic wave B_3B_4 interacts with stationary break B and from the left of the point B the elastic wave in the form of B_4B_5 is reflected, from the right the elastic wave with the front B_4B_6 (fig, 4) is propagated. After the elastic wave B_3B_4 is reflected from the point B $\left(t > \frac{4L}{a_0}\right)$, the front of elastic wave K_2K_4 meet at the point K_5 with the front of expanding wave BK_3. The front of elastic wave K_2K_4 is reflected in the form of K_5B_6, at that time the elastic wave K_2K_4 passing through front BK_5 and propagating in wrinkling domain doesn't influence to behaviour of filament at the break point A, however it influences to the expanding front K_5K_6 (fig. 4), whose velocity is again remains unknown, and we must determine it in the course of solving the problem.

Thus new domains 32, 12, 21 (fig. 4) arise.

For determination of the unknown parameters ε_{32}, ϑ_{32}, ε_{12}, ϑ_{12} at the point B and on the fronts B_4B_5, B_4B_6 we have the following relations

$$\begin{gathered}\vartheta_{32} = \vartheta_{12}\cos\gamma;\ \varepsilon_{32} = \varepsilon_{12};\\ \vartheta_{51} - \vartheta_{32} = -a_0(\varepsilon_{32} - \varepsilon_{51});\\ \vartheta_{12} - \vartheta_{11} = a_0(\varepsilon_{11} - \varepsilon_{12}).\end{gathered} \tag{3.10}$$

Form (3.10) with regard $\vartheta_{51} = 0$ we obtain

$$\begin{gathered}\varepsilon_{32} = \varepsilon_{12} = \frac{1}{1+\cos\gamma}\left[\left(\frac{\vartheta_{11}}{a_0} + \varepsilon_{11}\right)\cos\gamma + \varepsilon_{51}\right];\\ \vartheta_{12} = \frac{1}{1+\cos\gamma}(\vartheta_{11} + a_0\varepsilon_{11} - a_0\varepsilon_{51});\ \vartheta_{32} = \vartheta_{12}\cos\gamma.\end{gathered} \tag{3.11}$$

Here ϑ_{11}, ε_{11}, ε_{51} are expressed by the formulas (2.12), (2.18), but ε_1 participating in these formulas is determined from (3.3), (3.4). Now we determine ϑ_{21}, ε_{21} in domain 21 and the velocity of expanding wave ω_*. For this we have the common conditions on the front K_5K_6 in the form of

$$\begin{gathered}\sigma_{21} - \sigma_2 = \rho\omega_*(\vartheta_2 - \vartheta_{21});\\ \vartheta_{21} - \vartheta_2 = \omega_*(\varepsilon_2 - \varepsilon_{21})\end{gathered} \tag{3.12}$$

and the continuity condition of displacement on the front in the form of

$$\vartheta_{11} - \vartheta_{21} = -a_0(\varepsilon_{21} - \varepsilon_{11}). \tag{3.13}$$

From (3.12), (3.13) we determine ω_*, ε_{21}, ϑ_{21} in the form of

$$\omega_* = a_0\left(\sqrt{\frac{\varepsilon_2^2 a_0^2}{4} + b_1 b_2} + \frac{\varepsilon_2 a_0}{2}\right) b_1^{-1}\ ;$$

$$\varepsilon_{21} = \frac{1}{\omega_* + a_0}\left(\vartheta_2 + \omega_*\varepsilon_2 + a_0\varepsilon_{11} - \vartheta_{11}\right); \qquad (3.14)$$

$$\vartheta_{21} = \frac{1}{\omega_* + a_0}\left[(\vartheta_{11} - a\varepsilon_{11})\omega_* + \vartheta_2 a_0 + \omega_* a_0 \varepsilon_2\right],$$

where

$$b_1 = \vartheta_2 - \vartheta_{11} + a_0(\varepsilon_{11} - \varepsilon_2);\ \ b_2 = \vartheta_2 - \vartheta_{11} + a_0\varepsilon_{11}\,. \qquad (3.15)$$

Let $\mu_* = 0{,}2126;\ \gamma = 36^\circ$; $a_0^{-1} ctg\gamma = 1{,}2$; $M = 0{,}8727$, then from (3.3) and (3.14) we correspondingly obtain

$$\omega\, a_0^{-1} = 0{,}887\ ;\ \ \omega_* a_0^{-1} = 0{,}9098\,. \qquad (3.16)$$

From (3.16) it follows that interaction of elastic wave with front of the straightening wave influences to increase of velocity of straightening wave. The solution of problem with regard to multiple reflections is not difficult.

Note that for the determined combinations of parameters of problem at transverse impact by obtuse wedge the filament is broken at the point of impact $x = 0$ at the moment $t = 0$ under the limiting condition [2]

$$\sigma = \sigma_{np}\ ;\ \ \left(\sigma = 2\rho\, a_0^2 \varepsilon_1^0\right), \qquad (3.17)$$

where σ_{np} is ultimate strength (failure stress) of filament.

But for the same original parameters of the problem at impact by wedge with plane fore-part when $t = 0$ in filament the failure stress doesn't arise. Consequently, the filament may be broken after some period and this case in the present paper isn't considered.

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Received November 7, 2000; Revised September 10, 2001.
Translated by Mirzoyeva K.S.