GYULMAMEDOV M.Kh.

FORCED VIBRATIONS OF A SPHERICAL SHELL UNDER MIXED BOUNDARY CONDITIONS ON ITS FACE

Abstract

In the present paper the forced vibration of spherical shell is considered. The stated problem is exactly solved. The roots of variance equation are asymptotically investigated according to the problem we construct asymptotical formulas for displacements and stains corresponding to these roots.

Consider the forced vibration of a spherical shell under the homogemeous boundary conditions on its face

$$u_r = 0,\ \tau_{r\theta} = 0,\ \tau_{r\varphi} = 0 \ \text{when}\ r = r_k\ \ (k = 1,2)\,, \tag{1}$$

and the following boundary conditions are given on the other part of the boundary

$$\sigma_\theta = Q_\theta^{(s)}(r,\varphi)e^{i\omega t}\,,\ \tau_{r\theta} = Q_{r\theta}^{(s)}(r,\varphi)e^{i\omega t}\,,\ \tau_{r\varphi} = Q_{r\varphi}^{(s)}(r,\varphi)e^{i\omega t}\ \text{when}\ \theta = \theta_s\ \ (s = 1,2). \tag{2}$$

Here the other non-homogeneous boundary condition are possible, too.

Using the results of [2, 4] we obtain the following boundary problems

$$\begin{cases} L_1(u_r,\Phi) = 0, \\ L_2(u_r,\Phi) = 0, \end{cases} \tag{3}$$

$$\begin{cases} [u_r]_{r=r_k} = 0, \\ \left[\dfrac{\partial \Phi}{\partial r} - \dfrac{1}{r}\Phi\right]_{r=r_k} = 0 \quad (k = 1,2), \end{cases} \tag{4}$$

and

$$L_3(F) = 0\,, \tag{5}$$

$$\left[\frac{\partial F}{\partial r} - \frac{1}{r}F\right]_{r=r_k} = 0 \quad (k = 1,2), \tag{6}$$

where

$$L_1(u,\Phi) = \frac{2(1-\nu)}{1-2\nu}\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2}{r^2}u\right) + \frac{1}{r^2}\Delta_0 u + $$
$$+ \frac{1}{1-2\nu}\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{4\nu - 3}{r^2}\right)\Delta_0\Phi + \lambda^2 u\,,$$

$$L_2(u,\Phi) = \frac{1}{1-2\nu}\frac{1}{r}\left(\frac{\partial u}{\partial r} + \frac{4-4\nu}{r}u\right) + \frac{2(1-\nu)}{1-2\nu}\frac{1}{r^2}\Delta_0\Phi + \frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r} + \lambda^2\Phi\,,$$

$$L_3(F) = \frac{\partial^2 F}{\partial r^2} + \frac{2}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\Delta_0 F + \lambda^2 F\,,\ \Delta_0 = \frac{\partial^2}{\partial \theta^2} + ctg\theta\frac{\partial}{\partial \theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}\,.$$

Note that the boundary value problem (5), (6) is investigated in [3]. Therefore here we investigate the boundary value problem (3) and (4). Here r, θ, φ are spherical coordinates, u_r, u_θ, u_φ are components of displacement vector, $\sigma_r, \sigma_\theta, \sigma_\varphi, \tau_{r\theta}$, $\tau_{r\varphi}, \tau_{\theta\varphi}$ are components of strain tensor, G is a Lame coefficient, ω is frequency of vibrations, ν is a Poisson coefficient, ε is a small parameter characterizing the thickness of a spherical shell, λ is a pure frequency parameter, z is a spectral parameter. Using the results of [4], from (3), (4) we obtain the following spectral problem

$$\begin{cases} \left(a'+\frac{2}{r}a\right)'+\frac{1-2\nu}{2(1-\nu)}\left(\lambda^2-\frac{z^2-\frac{1}{4}}{r^2}\right)a-\frac{1}{2(1-\nu)}\frac{1}{r}\left(z^2-\frac{1}{4}\right)\left[b'-\frac{3-4\nu}{r}b\right]=0, \\ \frac{1}{1-2\nu}\left(\frac{1}{r}a'+\frac{4-4\nu}{r^2}a\right)+b''+\frac{2}{r}b'+\left[\lambda^2-\frac{2(1-\nu)}{1-2\nu}\frac{z^2-\frac{1}{4}}{r^2}\right]b=0, \end{cases} \tag{7}$$

$$\begin{cases} [a]_{r=r_k}=0\,, \\ \left[b'-\frac{1}{r}b\right]_{r=r_k}=0 \quad (k=1,2). \end{cases} \tag{8}$$

As is known the solution of a system of equations (7) has the following form:

$$a(r)=\frac{1}{\sqrt{r}}\left\{C_{1z}\left[\alpha J_z'(\alpha r)-\frac{1}{2r}J_z(\alpha z)\right]+C_{2z}\left[\alpha Y_z'(\alpha r)-\frac{1}{2r}Y_z(\alpha r)\right]-\right.$$
$$\left.-\frac{1}{r}\left(z^2-\frac{1}{4}\right)J_z(\lambda r)C_{3z}-\frac{1}{r}\left(z^2-\frac{1}{4}\right)Y_z(\lambda r)C_{4z}\right\},$$
$$b(r)=\frac{1}{\sqrt{r}}\left\{C_{1z}\frac{1}{r}J_z(\alpha z)+C_{2z}\frac{1}{r}Y_z(\alpha r)-C_{3z}\left[\lambda J_z'(\lambda r)+\frac{1}{2r}J_z(\lambda z)\right]-\right.$$
$$\left.-C_{4z}\left[\lambda Y_z'(\lambda r)+\frac{1}{2r}Y_z(\lambda r)\right]\right\}. \tag{9}$$

Here $J_z(x)$, $Y_z(x)$ are Bessel's functions of 1-st and 2-nd kinds, respectively.

Satisfying the homogeneous boundary conditions (8) we obtain variance equation in the form of

$$\Delta_1=\frac{1}{r_1r_2}\left(z^2-\frac{1}{4}\right)\left\{\left(z^2-\frac{1}{4}\right)L_z^{(0,0)}(\lambda)\left[-4\alpha^2r_1r_2L_z^{(1,1)}(\alpha)+6\alpha r_1L_z^{(0,0)}(\alpha)+\right.\right.$$
$$\left.+6\alpha r_2L_z^{(0,1)}(\alpha)-9L_z^{(0,0)}(\alpha)\right]+\left[\varphi_3(z,r_1)L_z^{(0,0)}(\lambda)-2\lambda r_1L_z^{(1,0)}(\lambda)\right]\times$$
$$\times\left[2\alpha^2r_1r_2L_z^{(1,1)}(\alpha)-3\alpha r_1L_z^{(1,0)}(\alpha)-\alpha r_2L_z^{(0,1)}(\alpha)+\frac{3}{2}L_z^{(0,0)}(\alpha)\right]+$$
$$+\left[\varphi_3(z,r_2)L_z^{(0,0)}(\lambda)-2\lambda r_2L_z^{(0,1)}(\lambda)\right]\left[2\alpha^2r_1r_2L_z^{(1,1)}(\alpha)-\alpha r_1L_z^{(1,0)}(\alpha)-\right.$$
$$\left.\left.-3\alpha r_2L_z^{(0,1)}(\alpha)+\frac{3}{2}L_z^{(0,0)}(\alpha)\right]-\frac{32}{\pi^2}\right\}+\frac{1}{4r_1r_2}\left[4\alpha^2r_1r_2L_z^{(1,1)}(\alpha)-2\alpha r_1L_z^{(1,0)}(\alpha)-\right.$$
$$\left.-2\alpha r_2L_z^{(0,1)}(\alpha)+L_z^{(0,0)}(\alpha)\right]\left[-4\lambda^2r_1r_2L_z^{(1,1)}(\lambda)+2\lambda r_2\varphi_3(z,r_1)L_z^{(0,1)}(\lambda)+\right.$$
$$\left.+2\lambda r_1\varphi_3(z,r_2)L_z^{(1,0)}(\lambda)-\varphi_3(z,r_1)\varphi_3(z,r_2)L_z^{(0,0)}(\lambda)\right]=0\,, \tag{10}$$

where $\varphi_3(z,r)=2z^2-\frac{3}{2}-\lambda^2r^2$, $\alpha^2=\frac{1-2\nu}{2(1-\nu)}\lambda^2$,

$$L_z^{(s,l)}(\beta)=J_z^{(s)}(\beta r_1)Y_z^{(l)}(\beta r_2)-J_z^{(l)}(\beta r_2)Y_z^{(s)}(\beta r_1) \quad (s,l=0,1).$$

The transcendental equation (10) has a denumerable set of zeros and the following homogeneous solutions correspond to them

$$u_r = \frac{1}{\sqrt{r}} \sum_{k=1}^{\infty} C_k u_{rk} T_k(\theta, \varphi) e^{i\omega t}, \quad u_\theta = \frac{1}{\sqrt{r}} \sum_{k=1}^{\infty} C_k u_{\theta k} \frac{\partial T_k}{\partial \theta} e^{i\omega t}, \tag{11}$$

$$u_\varphi = \frac{1}{\sqrt{r}} \sum_{k=1}^{\infty} C_k u_{\theta k} \frac{\partial T_k}{\partial \varphi} e^{i\omega t}; \sigma_\theta = \frac{G}{r^2\sqrt{r}} \sum_{k=1}^{\infty} C_k \left[\sigma_{\theta k} T_k(\theta, \varphi) + \sigma_{\varphi k} \frac{\partial^2 T_k}{\partial \theta^2} \right] e^{i\omega t},$$

$$\sigma_\varphi = \frac{G}{r^2\sqrt{r}} \sum_{k=1}^{\infty} C_k \left\{ \sigma_{\theta k} T_k(\theta, \varphi) + \sigma_{\varphi k} \left[\frac{\partial T_k}{\partial \theta} + \frac{1}{\sin^2\theta} \frac{\partial^2 T_k}{\partial \varphi^2} \right] \right\} e^{i\omega t},$$

$$\sigma_r = \frac{G}{r^2\sqrt{r}} \sum_{k=1}^{\infty} C_k \sigma_{rk} T_k(\theta, \varphi) e^{i\omega t}, \; \tau_{r\varphi} = \frac{G}{r^2\sqrt{r}} \sum_{k=1}^{\infty} C_k \tau_{rk} \frac{\partial T_k}{\partial \varphi} \frac{1}{\sin\theta} e^{i\omega t},$$

$$\tau_{r\theta} = \frac{G}{r^2\sqrt{r}} \sum_{k=1}^{\infty} C_k \tau_{rk} \frac{\partial T_k}{\partial \theta} e^{i\omega t}, \; \tau_{\theta\varphi} = -\frac{G}{r^2\sqrt{r}} \sum_{k=1}^{\infty} C_k \sigma_{\varphi k} \frac{\partial T_k}{\partial \varphi} \frac{ctg\theta}{\sin\theta} e^{i\omega t}. \tag{12}$$

Here

$$u_{rk} = \alpha Z'_{z_k}(\alpha r) - \frac{1}{2r} Z_{z_k}(\alpha r) - \frac{1}{r}\left(z_k^2 - \frac{1}{4}\right) Z_{z_k}(\lambda r),$$

$$u_{\theta k} = \frac{1}{r} Z_{Z_k}(\alpha r) - \lambda Z'_{z_k}(\lambda r) - \frac{1}{2r} Z_{z_k}(\lambda r), \sigma_{\varphi k} = 2 Z_{z_k}(\alpha r) - 2\lambda\, r Z'_{z_k}(\lambda r) - Z_{z_k}(\lambda r),$$

$$\sigma_{\theta k} = 2\alpha r Z'_{z_k}(\alpha r) - \left(1 + \frac{\nu}{1-\nu}\lambda^2 r^2\right) Z_{z_k}(\alpha r) - 2\left(z_k^2 - \frac{1}{4}\right) Z_{z_k}(\lambda r),$$

$$\sigma_{rk} = \varphi_2(z_k, r) Z_{z_k}(\alpha r) - 4\alpha r Z'_{z_k}(\alpha r) + \left(z_k^2 - \frac{1}{4}\right)\left(3 Z_{z_k}(\lambda r) - 2\lambda r Z'_{z_k}(\lambda r)\right),$$

$$\tau_{rk} = 2\alpha r Z'_{z_k}(\alpha r) - 3 Z_{z_k}(\alpha r) + 2\lambda r Z'_{z_k}(\lambda r) - \varphi_3(z_k, r) Z_{z_k}(\lambda r),$$

where

$$Z_{z_k}(x) = C_{1z_k} J_{z_k}(x) + C_{2z_k} Y_{z_k}(x), \quad \varphi_2(z, r) = 2z^2 + \frac{3}{2} - \lambda^2 r^2,$$

C_k are arbitrary constants.

Now we pass to the investigation of roots of the equation (10). For this we expand the equation (10) in series by ε

$$D(z, \lambda, \varepsilon) = \frac{4}{\pi^2} \frac{1}{(1-\nu)^2} \varepsilon^2 \Big\langle \Big\{ 4(1-\nu)^2 \lambda^4 z^2 - 2(1-\nu)(1-2\nu)\lambda^6 -$$

$$-(1-\nu)(5-9\nu)\lambda^4 \Big\} + \frac{\varepsilon^2}{3} \Big\{ 16(1-\nu)^2 \lambda^4 z^4 + 2\big[-2(1-\nu)(5-8\nu)\lambda^6 -$$

$$-2(10\nu^2 - 12\nu + 3)\lambda^4 + 8(1-\nu)^2\big] z^2 + 2(8\nu^2 - 10\nu + 3)\lambda^8 + (4\nu^2 + \nu - 1)\lambda^6 -$$

$$(23\nu^2 - 38\nu + 14)\lambda^4 - 4(1-\nu)^2 \Big\} + \frac{\varepsilon^4}{45} \Big\{ 128(1-\nu)^2 \lambda^4 z^6 + 4\big[-32(2-3\nu) \times$$

$$\times (1-\nu)\lambda^6 - 4(13\nu^2 - 10\nu - 1)\lambda^4\big] z^4 + (\ldots) z^2 + \ldots \Big\} + \ldots \Big\rangle. \tag{13}$$

For the equation (13) the following statement is valid.

The equation (13) at finite λ has two groups of zeros, where the first group consists of two zeros $z_k = O(1)$ $(k = 1,2)$, and the second group contains a denumerable set of roots which have the order $o(\varepsilon^{-1})$.

For the first group we seek z_k in the form of the expansion

$$z_k = z_{k_0} + \varepsilon^2 z_{k_2} + \dots . \tag{14}$$

After substitution (14) in (13) we obtain

$$z_{k_0}^2 = \frac{1}{4(1-\nu)}\left[2(1-2\nu)\lambda_0^2 + (5-9\nu)\right],$$

$$z_{k_2} = \frac{1}{24(1-\nu)^2}\frac{1}{z_{k_0}}\left\{16(1-\nu)^2 z_{k_0}^4 + 2\left[-2(1-\nu)(5-8\nu)\lambda_0^2 - 2\left(10\nu^2 - 12\nu + 3\right) + \right.\right.$$
$$\left. + 8(1-\nu)^2\lambda_0^{-4}\right] z_{k_0}^2 + 2\left(8\nu^2 - 10\nu + 3\right)\lambda_0^4 + \left(4\nu^2 + \nu - 1\right)\lambda_0^2 - \left(23\nu^2 - 38\nu + 14\right) -$$
$$\left. - 4(1-\nu)^2\lambda_0^{-4}\right\}. \tag{15}$$

From (15) it's obvious that when $\lambda_0^2 \ge -\dfrac{5-9\nu}{2(1-2\nu)}$ we have two real and when $\lambda_0^2 < -\dfrac{5-9\nu}{2(1-2\nu)}$- two imaginary roots. Some penetrating solutions correspond to these groups of roots.

For constructing the asymptotic of zeros of the second group we'll seek z_n $(n = 3,4,...)$ in the form of

$$z_n = \frac{\delta_n}{\varepsilon} + O(\varepsilon). \tag{16}$$

Substituting (16) in the variance equation (10) and transforming it with the help of asymptotical expansions of the functions $J_z(x), Y_z(x)$ for large z for δ_n we have

$$sh^2 2\delta_n = 0. \tag{17}$$

We studied the asymptotical properties z of zeros of variance equation, assuming that the frequency parameter λ is finite when $\varepsilon \to 0$. Consider the case when λ unboundedly increases when $\varepsilon \to 0$ we'll call such vibrations super high-frequency. For such λ the following statement is valid.

If $\lambda \to \infty$ when $\varepsilon \to 0$, then for equation (13) the following limit cases are possible

$$\text{a) } \lambda\,\varepsilon \to const \text{ when } \varepsilon \to 0\ , \tag{18}$$
$$\text{b) } \lambda\,\varepsilon \to \infty \text{ when } \varepsilon \to 0,$$

under which zeros of (13) unboundedly increase.

Consider case a). In this case ($z \sim \lambda$, $\lambda\varepsilon \to const$, $z\varepsilon \to const$ when $\varepsilon \to 0$) we'll seek z_n and λ in the form of

$$z_n = \frac{\delta_n}{\varepsilon} + O(\varepsilon), \qquad \lambda = \frac{\lambda_0}{\varepsilon}. \tag{19}$$

Substituting (19) in (10) and transforming it with the help of asymptotical expansion of the functions $J_z(x), Y_z(x)$ for large z and x, for δ_n we obtain the following equation

$$\beta_n^4 sh2\beta_n sh2\gamma_n = 0, \tag{20}$$

where

$$\beta_n = \sqrt{\delta_n^2 - \frac{1-2\nu}{2(1-\nu)}\lambda_0^2}\ ,\quad \gamma_n = \sqrt{\delta_n^2 - \lambda_0^2}\ .$$

For the given λ the equation (20) determines the denumerable set z_n.

Finally in case b), denoting $\mu_k\varepsilon$ by x_k, $\lambda\varepsilon$ by y and using asymptotical expansion of Bessel's function, we can represent asymptotics of the variance equation (10) at first member in the form of

$$\beta_k^4 sh2\beta_k sh2\gamma_k = 0\,, \tag{21}$$

where

$$\beta_k = \sqrt{x_k^2 - \frac{1-2\nu}{2(1-\nu)}y^2}\ ,\qquad \gamma_k = \sqrt{x_n^2 - y^2}\ .$$

As is seen the equation (20) is valid in case b), too.

Assuming $r = 1 + \varepsilon\eta, -1 \le \eta \le 1$ and using the formulas (11), (12) we represent asymptotical formulas for displacements and strains corresponding to roots of the equation (17). For the series of roots $ch^2\delta = 0$ we obtain

$$u_r = \pm i\sum_{n=1}^{\infty} C_n\delta_n^2\left[ch\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}\,,\ u_\varphi = \pm i\,\varepsilon\sum_{n=1}^{\infty} C_n\delta_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\varphi}\frac{1}{\sin\theta}e^{i\omega t}\,,$$

$$u_\theta = \pm i\,\varepsilon\sum_{n=1}^{\infty} C_n\delta_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t}\,,\ \sigma_r = \pm\frac{i2G}{\varepsilon}\sum_{n=1}^{\infty}\delta_n^3 C_n\left[sh\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}\,,$$

$$\sigma_\theta = O(\varepsilon),\ \sigma_\varphi = \pm\frac{i2G}{\varepsilon}\sum_{n=1}^{\infty}\delta_n^3 C_n\left[-sh\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}\,,$$

$$\tau_{r\theta} = \pm i\,2G\sum_{n=1}^{\infty}\delta_n^2 C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t}\,,$$

$$\tau_{r\varphi} = \pm i\,2G\sum_{n=1}^{\infty}\delta_n^2 C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\varphi}\frac{1}{\sin\theta}e^{i\omega t}\,,$$

$$\tau_{\theta\varphi} = \pm i\,\varepsilon G\sum_{n=1}^{\infty}\delta_n C_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial}{\partial\varphi}\left[\frac{\partial T_n}{\partial\theta} - ctg\theta\,T_n(\theta,\varphi)\right]\frac{2}{\sin\theta}\,e^{i\omega t}\,. \tag{22}$$

For the series of roots $sh^2\delta = 0$

$$u_r = \pm\sum_{n=1}^{\infty} C_n\delta_n^2\left[sh\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}\,,\ u_\varphi = \pm\varepsilon\sum_{n=1}^{\infty} C_n\delta_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\varphi}\frac{1}{\sin\theta}e^{i\omega t}\,,$$

$$u_\theta = \pm\varepsilon\sum_{n=1}^{\infty} C_n\delta_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t}\,,\sigma_r = \pm\frac{2G}{\varepsilon}\sum_{n=1}^{\infty}\delta_n^3 C_n\left[ch\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}\,,$$

$$\sigma_\theta = O(\varepsilon)\,,\ \sigma_\varphi = \pm\frac{2G}{\varepsilon}\sum_{n=1}^{\infty}\delta_n^3 C_n\left[-ch\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}\,,$$

$$\tau_{r\theta} = \pm 2G\sum_{n=1}^{\infty}\delta_n^2 C_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t}\,,$$

$$\tau_{r\varphi} = \pm 2G\sum_{n=1}^{\infty}\delta_n^2 C_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\varphi}\frac{1}{\sin\theta}e^{i\omega t}\,,$$

$$\tau_{\theta\varphi} = \pm\varepsilon G\sum_{n=1}^{\infty}\delta_n C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial}{\partial\varphi}\left[\frac{\partial T_n}{\partial\theta} - ctg\theta\,T_n(\theta,\varphi)\right]\frac{2}{\sin\theta}e^{i\omega t}\,. \tag{23}$$

are obtained.

Here C_n are arbitrary constants.

The problem on satisfaction of boundary conditions on end-walls of shell with the help of a class of homogeneous solutions isn't considered in the present paper. This question is considered in [5] where with the help of the Hamilton variational principle, the boundary value problem is reduced to a solution of an infinity system of linear algebraic equations.

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Murshud Kh. Gyulmamedov
Institute of Deep Oil and Gas Deposits of NAS Azerbaijan.
33, H.Javid av., 370143, Baku, Azerbaijan.
Tel.:39-57-17(off.).

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