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TIME MODELLING OF DESTRUCTION PROCESS OF HEREDITARILY ELASTIC PLATE WITH CIRCULAR HOLE AT UNIAXIAL TENSION

Abstract

Analytical formula for destruction time of points of hereditarily elastic plate with circular hole is obtained. The solution of G.Kirsch for stress components is used. The relation determining the stress in model plate from polymeric material on given modeling time is obtained under the influence of that for modeling time in model plate the damage is accumulated as equal number of accumulated damaging in full-score plate from steel fusion of the action time natural stress.

The plate with width $2b$, thickness h and with circular hole of the radius a in the center is subjected to uniaxial tension of the force $P = 2bh\sigma$. We use square Cartesian coordinate system xy. We dispose the coordinate origin in center of hole, we direct the axis along the tension.

1. Stress state. We consider the case $b > 5a$. Here the stress distribution in the case of elastic plate is given by the G.Kirsch known solution [1]. In the most risky section $x = 0$ when $y \ge a$ according to the G.Kirsch solution

$$\sigma_{xx} = \frac{\sigma}{2}\left(2 + \frac{a^2}{y^2} + 3\frac{a^4}{y^4}\right), \quad \sigma_{yy} = \frac{3}{2}\sigma\frac{a^2}{y^2}\left(1 - \frac{a^2}{y^2}\right), \quad \sigma_{xy} = 0 \qquad (1.1)$$

holds.

The solution (1.1) for stress components remains unchangeable and in the case of linear hereditarily elastic plate determining equations that have common form of Volterra equation [2]. The stress intensity in the case of the considered problem is expressed by the formula

$$\sigma_+ = \left(\sigma_{xx}^2 - \sigma_{xx}\sigma_{yy} + \sigma_{yy}^2\right)^{1/2}.$$

In addition subject to (11) we obtain

$$\sigma_+\Big|_{\substack{x=0\\ y\ge a}} = F\left(\frac{a}{y}\right)\sigma(t), \qquad (1.2)$$

where t is time

$$F\left(\frac{a}{y}\right) = \left(1 - \frac{1}{2}\frac{a^2}{y^2} + \frac{25}{4}\frac{a^4}{y^4} - \frac{9}{2}\frac{a^6}{y^6} + \frac{27}{4}\frac{a^8}{y^8}\right)^{\frac{1}{2}}. \qquad (1.3)$$

when $y = a$ we have $F(1) = 3$.

2.Destruction. In [3] the kinetic equation of damaging accumulation in hereditarily elastic body that allows for the existence of incubation time antecedent damaging of material of body is constructed. In particular form it's written in the form of

$$\Pi(t,x) = H(t - t')\left(-\frac{t_1^{1+\lambda}(\sigma_*,T)}{t_0^{1+\lambda}(\sigma_*,T) - t_1^{1+\lambda}(\sigma_*,T)} + (1+\lambda)\int_0^t \frac{(t-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*,T) - t_1^{1+\lambda}(\sigma_*,T)}\right). \qquad (2.1)$$

Here t is time, $x = (x_1, x_2, x_3)$ are coordinates of points of body; $\sigma_* = \sigma_*(t,x)$ is some equivalent stress for which we can accept, for example, stress intensity; $T = T(t,x)$ is temperature counted off from initial temperature; $t_0 = t_0(\sigma_*(t,x)T(t,x))$ is experimentally determined durability for different constants $\sigma_* = const$, $T = const$; $t_1 = t\ (\sigma_*(t,x)T(t,x))$ is introduced by L.Kh. Talybly material function-time antecedent damaging of points of body for different constants $\sigma_* = const$, $T = const$; λ is an experimentally determinated material constant, $H(t)$ is unique Heavyside function; $t' = t'(x)$ is time antecedent damaging of points of body for arbitrary $\sigma_* = \sigma_*(t,x)$, $T = T(t,x)$. It's determined from the conditions $\Pi(t') = 0$ called the damaging condition the durability $t_* = t_*(x)$ for arbitrary $\sigma_* = \sigma_*(t,x)$, $T = T(t,x)$ is determined from the condition $\Pi(t_*) = 1$. There are materials for which the condition $t_1(\sigma_*,T)/t_0(\sigma_*,T) = A = const$ is satisfied. In addition the kinematics equation (2.1) is transformed to the relation

$$\Pi(t,x) = H(t-t')\left[\frac{1}{1-A^{1+\lambda}}\left(-A^{1+\lambda} + (1+\lambda)\int_0^t \frac{(t-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*(\tau,x),T(\tau,x))}\right)\right]. \qquad (2.2)$$

By experimental data [4] the constant A changes in dependence on material and loading in the interval $0{,}4 \le A \le 0{,}7$. In general case $0 \le A < 1$.

The damaging condition $\Pi(t') = 0$ by using (2.2) has the form

$$(1+\lambda)\int_0^{t'} \frac{(t'-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*,T)} = A^{1+\lambda}. \qquad (2.3)$$

The durability condition $\Pi(t_*) = 1$ subject to (2.2) is written in the form of

$$(1+\lambda)\int_0^{t_*} \frac{(t_*-\tau)^\lambda d\tau}{t_0^{1+\lambda}(\sigma_*,T)} = 1. \qquad (2.4)$$

The condition (2.4) coincides with the V.V. Moskvitin durability condition obtained by him in the case $t_1(\sigma_*,T) = 0$ [2].

We can represent the function $t_0(\sigma_*,T)$ in the form of

$$t_0(\sigma_*,T) = t_{os}\exp\left[\beta_0\left(1-\frac{\sigma_*}{\sigma_s}\right) + d_0\left(1-\frac{T}{T_s}\right)\right]\ (\sigma_* \ne 0 \text{ or } T \ne 0). \qquad (2.5)$$

Here σ_s and T_s are stress and reduction temperature that independent of time and are chosen from size of changing σ_* and T; t_{0s} is time to destruction of material when $\sigma_* = \sigma_s$ and $T = T_s$; β_0 and d_0 are material constants.

In fig. 1 and fig. 2 the approximated lines of experimental data between the quantities σ_* and $\lg t_0$ are represented, at tension of model from polymeric material-high density polyethylene (HDPE) when the temperature $T = T_s = 323K$ (fig. 1- A.Ya. Goldman's experiments [4]) and at tension, torsion and tension with torsion of models cut from disk whose material is fusion ЭИ437Б at the temperature $T = T_s = 973K$ (fig. 2- V.P. Sdobirev's experiments [5]). These data are well conformed with the relation (2.5). In addition for HDPE at $T = T_s = 323K$ $\beta_0 = 6{,}67$, for fusion ЭИ437Б at $T = T_s = 973K$, $\beta_0 = 8{,}05$.

Now we accept $\sigma_* = \sigma_+$ and assume $\sigma(t) = \sigma_0 \ln\left(1+\frac{t}{t_a}\right)$, where $t \in [0,t_a]$; $t' \le t_a \le t_*; \sigma_0 = const$. In addition in the case of using (2.5) when $T = T_s$ and (1.2) we transform the integral

$$J=\int_0^{t_a}\frac{(t_a-\tau)^{\lambda}d\tau}{t_0^{1+\lambda}(\sigma_*(\tau))}=\frac{c(\lambda,\beta_0)t_a^{1+\lambda}}{(1+\lambda)t_{0s}^{1+\lambda}(\sigma_s)\exp\left[(1+\lambda)\beta_0\left(2-F\left(\frac{a}{y}\right)\sigma_0/\sigma_s\right)\right]}, \qquad (2.6)$$

where

$$c(\lambda,\beta_0)=(1+\lambda)\int_0^1(1-z)^{\lambda}(1+z)^{(1+\lambda)\beta_0}dz. \qquad (2.7)$$

Allowing for (2.6) when $t_a=t'$ in the damaging condition (2.3) for the time $t'(y)$ antecedent damaging of points of plate when $x=0$, $y\geq a$ we obtain

$$t'=\frac{At_{0s}(\sigma_s)\exp[\beta_0(2-F(a/y)\sigma_0/\sigma_s)]}{c^{1/1+\lambda}(\lambda,\beta_0)}. \qquad (2.8)$$

The using of (2.6) when $t_a=t_*$ in the durability condition (2.4) determines the values of the destruction time t_* of points of plate that lie on the line $x=0$, $y\geq a$

$$t_*=\frac{t_{0s}(\sigma_s)\exp[\beta_0(2-F(a/y)\sigma_0/\sigma_s)]}{c^{1/(1+\lambda)}(\lambda,\beta_0)}. \qquad (2.9)$$

Graphs dependencies $\ln\frac{t'}{t_{0s}}\sim\frac{y}{a}$ (dot lines) and $\ln\frac{t_*}{t_{0s}}\sim\frac{y}{a}$ (full lines) constructed in accordance with formulas (2.8) and (2.9) subject to (1.3) are represented on fig. 3. In addition curves 1 correspond to the value $\sigma_0/\sigma_s=1/3$, curves $2-\sigma_0/\sigma_s=1/6$. Besides, the constants that approximately are fusion data ЭИ437Б when $T=T_s=923K: A=0{,}3$; $\beta_0=8{,}05$; $\lambda=-0{,}3$ are used at these data the graphs of the dependencies $\ln\frac{t'}{t_{0s}}\sim\frac{\sigma_0}{\sigma_s}$ (dot lines) and $\ln\frac{t_*}{t_{0s}}\sim\frac{\sigma_0}{\sigma_s}$ (full line) for the dangerous point $x=0$, $y=a$ are introduced in fig. 4. For example, from these graphics it follows that the point $(0,a)$ of hole when $\sigma_0/\sigma_s=1/3$ is damaged at the time $t'=18{,}18t_{0s}$, fall at the time $t_*=60{,}34t_{0s}$. Between these times damaging accumulation access. If we take $\sigma_0=250\,\text{MPa}$, then $\sigma_s=750\,\text{MPa}$. The value σ_s corresponds to this value of $T=T_s=933K$ at the temperature $t_{0s}=89{,}13$ hour (fig. 2). In addition at the point $(0,a)$ of plate the damaging time and destruction time will be $t'=1620{,}4$ hour, $t_*=5378{,}1$ hour respectively. Besides as we see from fig. 3 by removing from the point $(0,a)$ (when $y>3a$) the quantities t' and t_* tend to a constant quantity that naturally, since in this case the stress intensity decreases and according to the conditions of problem tends to the quantity of σ.

3. Modeling. Consider the modeling variant at the destruction process time of the considered plate that is at uniaxial tension after the laps of significant time big using experiences led at bounded time intervals, on analogous plate from other or same material. Under the modeling we'll understand origination in model construction for bounded time interval, such quantity of damaging accumulation.

The quantities belonging to full-scale plate we keep without indices, to model plate denote by using the index "M". In [6] by using the kinetic equation (2.2) allowing for (2.5) the modeling condition that is written in the form of

$$\int_0^{t_M} (t_M-\tau)^{\lambda_M} \exp\left[(1+\lambda_M)\beta_{0M}\frac{\sigma_*^M}{\sigma_s^M}\right]d\tau =$$

$$= D_1 + D_2\int_0^t (t-\tau)^{\lambda} \exp\left[(1+\lambda)\beta_0\frac{\sigma_*}{\sigma_s}\right]d\tau . \tag{3.1}$$

Here

$$D_1 = \frac{t_{0sM}^{1+\lambda_M}\left(A_M^{1+\lambda_M} - A^{1+\lambda}\right)\exp\left[(1+\lambda_M)\beta_{0M}\right]}{(1+\lambda_M)\left(1-A^{1+\lambda}\right)}, \tag{3.2}$$

$$D_2 = \frac{t_{0sM}^{1+\lambda_M}(1+\lambda)\left(1-A_M^{1+\lambda_M}\right)}{t_{0s}^{1+\lambda}(1+\lambda_M)\left(1-A^{1+\lambda}\right)}\exp\left[(1+\lambda_M)\beta_{0M} - (1+\lambda)\beta_0\right] \tag{3.3}$$

is obtained.

From the condition (3.1) allowing for (3.2), (3.3) and also (1.2), (2.6) at the points $x=0, y\geq a$ and $x_M=0,\ y_M\geq a_M$ we obtain

$$\frac{t_{aM}}{t_{0sM}} = \frac{\exp\left[\beta_{0M}\left(2-F(a_M/y_M)\sigma_0^M\right)\right]}{c^{1/1+\lambda}(\lambda_M,\beta_{0M})}\left\{A_M^{1+\lambda_M} + \frac{1-A_M^{1+\lambda_M}}{1-A^{1+\lambda}}\left[-A^{1+\lambda} + \right.\right.$$

$$\left.\left. + \left(\frac{t_a}{t_{0s}}\right)^{1+\lambda}\frac{c(\lambda,\beta_0)}{\exp\left[(1+\lambda)\beta_0\left(2-F(a/y)\sigma_0/\sigma_s\right)\right]}\right]\right\}^{1/(1+\lambda_M)} \tag{3.4}$$

or

$$\frac{\sigma_0^M}{\sigma_s^M} = -\frac{1}{\beta_{0M}F(a_M/y_M)}\ln\frac{t_{aM}}{t_{0sM}} + \frac{1}{\beta_{0M}F(a_M/y_M)}\left\{ 2\beta_{0M} - \right.$$

$$-\frac{1}{1+\lambda_M}\ln c(\lambda_M,\beta_{0M}) + \frac{1}{1+\lambda_M}\ln\left[A_M^{1+\lambda_M} + \frac{1-A_M^{1+\lambda_M}}{1-A^{1+\lambda_M}}\left(-A^{1+\lambda} + \right.\right.$$

$$\left.\left.\left. + \left(\frac{t_a}{t_{0s}}\right)^{1+\lambda}\frac{c(\lambda,\beta_0)}{\exp\left[(1+\lambda)\beta_0\left(2-F(a/y)\sigma_0/\sigma_s\right)\right]}\right)\right]\right\}. \tag{3.5}$$

The relation (3.4) determines the modeling time t_{aM} during of which at the points $x_M=0$, $y_M\geq a_M$ of model plate for given σ_0^M it's accumulated damaging in the equal number of quantity of accumulated damagings at the points $x=0, y\geq a$, a full-scale plate for the time t_a on action σ_0. The analogous equaling of accumulated damagings holds on fulfillment the relation (3.5) that determines the stress σ_0^M at the points $x_M=0$, $y_M\geq a_M$ for the given modeling time t_{aM}. For construction of graphics we transform the relation (3.5) in the form of

$$\frac{\sigma_0^M}{\sigma_0} = -\frac{1}{50\beta_{0M}F(a_M/y_M)}\ln\frac{t_{aM}}{t_a} + \frac{1}{50\beta_{0M}F(a_M/y_M)}\left\{\ln\frac{t_{0sM}\left(\dfrac{\sigma_0}{50}\right)}{t_{0s}} - \right.$$

$$-\ln\frac{t_a}{t_{0s}}+2\beta_{0M}-\frac{1}{1+\lambda_M}\ln c(\lambda_M,\beta_{0M})+\frac{1}{1+\lambda_M}\ln\left[\quad A_M^{1+\lambda_M}+\right.$$

$$\left.\left.+\frac{1+A_M^{1+\lambda_M}}{1-A^{1+\lambda}}\left(-A+\left(\frac{t_a}{t_{0s}}\right)^{1+\lambda}\frac{c(\lambda,\beta_0)}{\exp[(1+\lambda)\beta_0(2-F(a/y)\sigma_0/\sigma_y)]}\right)\right]\right\}. \qquad (3.6)$$

Assume that full-score plate consists of fusion ЭИ437Б, modeling- from polymeric material HDPE. By uniaxial tension the full-score plate has the temperature $937K$, a modeling- $323K$. In addition, the experimental and some accounting data fusion ЭИ437Б using V.P. Sdobirev's [5] experiments are led above. Analogous data for HDPE- polymeric material by using A.Ya. Goldman's [4] experiments are the following $\beta_{0M}=6{,}67$; $\lambda_M=-0{,}6$; $A_M=0{,}45$. In addition $c(\lambda_M,\beta_{0M})=c(-0{,}6;6{,}67)=2{,}42$. Assume $\sigma_s=3P_0$, $x=0$; $y/a=1$; $x_M=0$, $y_M/a_M=1$; $t_a=t_*$; $t_{aM}=t_{*M}$. By using these and led above data and also data graphics of durability (fig. 1,2) from (3.6) ewe obtain the relation

$$10^3\frac{\sigma_0}{\sigma_s}=-\ln\frac{t_{*M}}{t_0}+5{,}77 \text{ when } \begin{cases}x=0\ , \quad y=a\ ,\\ x_M=0\ , \quad y_M=a_M\ .\end{cases} \qquad (3.7)$$

The graphic of dependence (3.7) is represented in fig. 5. From this dependence it follows, in particular if we take at the point $(0,a)$ of the full-score plate from fusion ЭИ437Б the stress 750 MPa, then by equal destruction times $t_{*M}=t_*$ at the analogous point $(0,a_M)$ of model plate it follows to originate the stress $4{,}33\text{ MPa}$.

Now assume that the materials of full-score and modeling plates are the same. In this case $A_M=A,\ \lambda_M=\lambda,\ \beta_{0M}=\beta_0$ and from (3.6) when $x=0,\ y=a,\sigma_0/\sigma_s=1/3$ it follows

$$\frac{\sigma_0^M}{\sigma_0}=-0{,}125\ln\frac{t_{*M}}{t_*}+1 \quad \text{when} \quad x=0,\ y=a\ . \qquad (3.8)$$

The graphic of dependence $\frac{\sigma_0^M}{\sigma_0}\sim\ln\frac{t_{*M}}{t_*}$ originated in correspondence with the formula (3.8) is represented in fig. 6. It's known that by equal destruction times of model and full-score plate from the same material it must by $\sigma_0^M=\sigma_0$. This obvious conclusion follows also from the obtained relation (3.8).

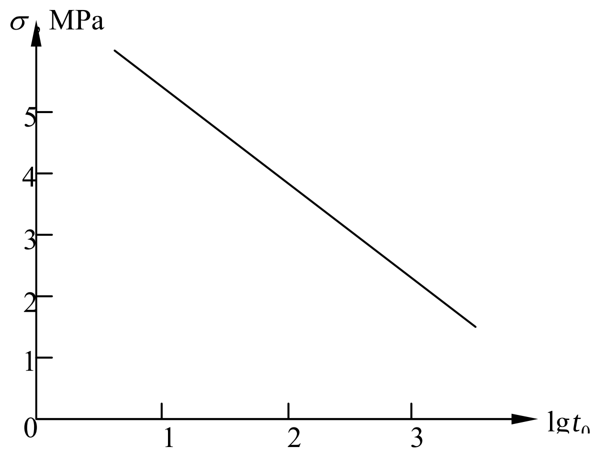

Fig.1. The curve of durability of models from polyethylene of higher density at the temperature $T = 323K$ **(A.Ya. Goldman's experiments [4]).**

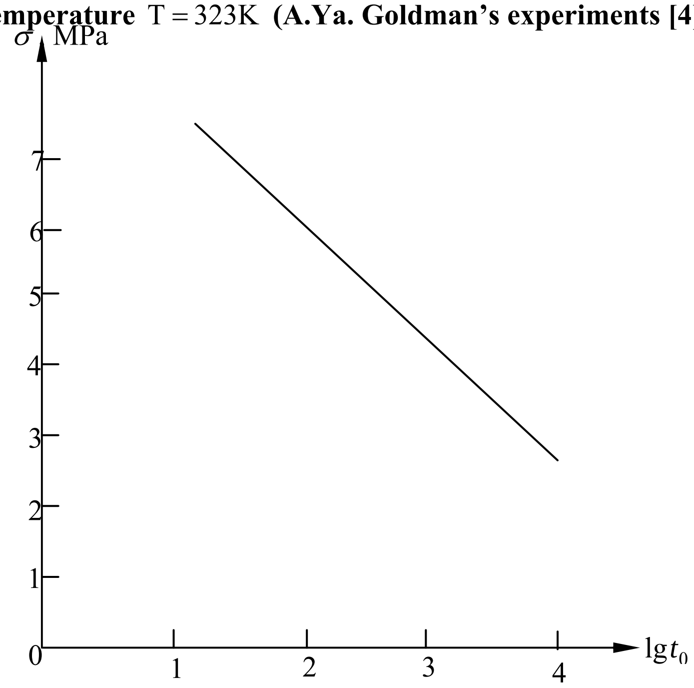

Fig. 2. The curve of durability of models cut from fusion disk ЭИ437Б when $T = 973K$ **(V.P. Sdobirev's experiments [5]).**

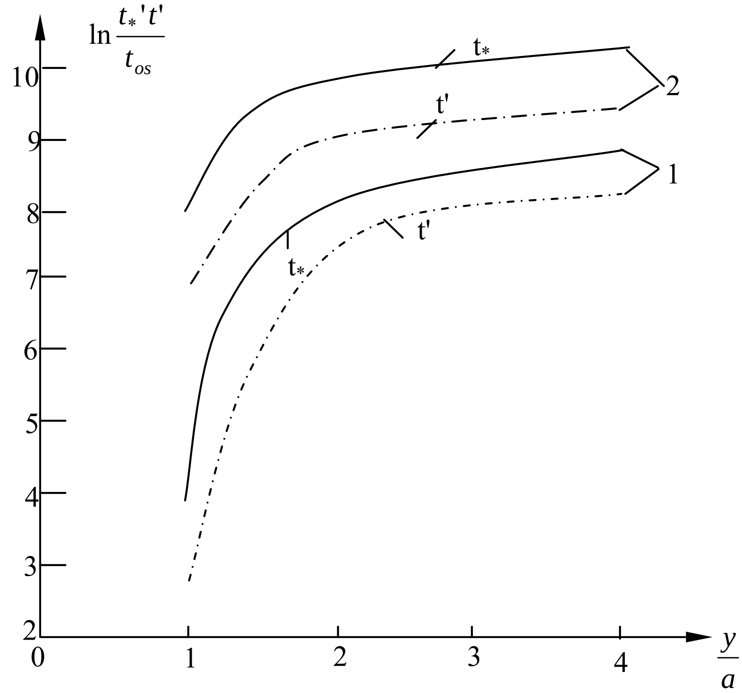

Fig. 3. The curves of damaging (dot line) and durability (full line) of the points $x = 0,\ y \geq a$ of plate with circular hole fusion ЭИ437Б when $\mathrm{T} = 973\mathrm{K}$ $\left(1 - \frac{\sigma_0}{\sigma_s} = \frac{1}{3}; 2 - \frac{1}{6}\right)$.

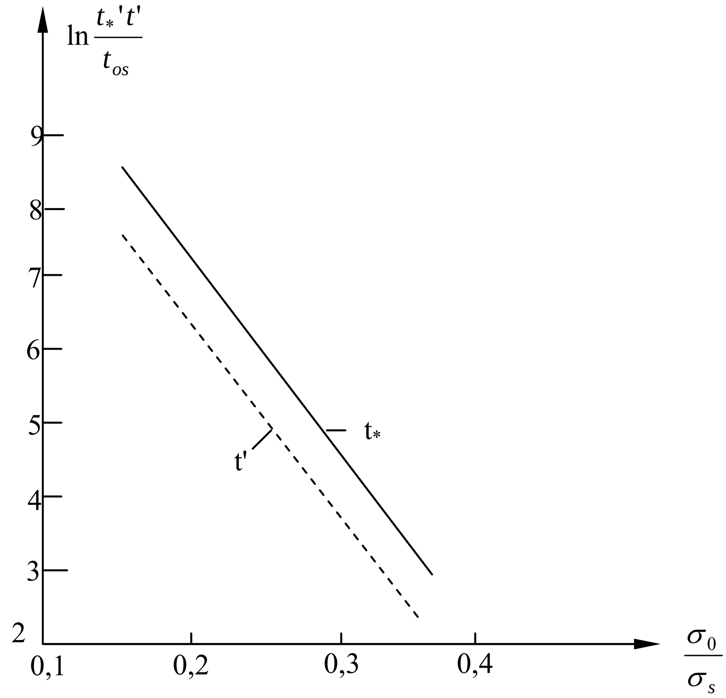

Fig. 4. The curves of damaging (dot line) and durability (full line) of the point $(0, a)$ of plate with circular hole fusion ЭИ437Б in dependence on stress.

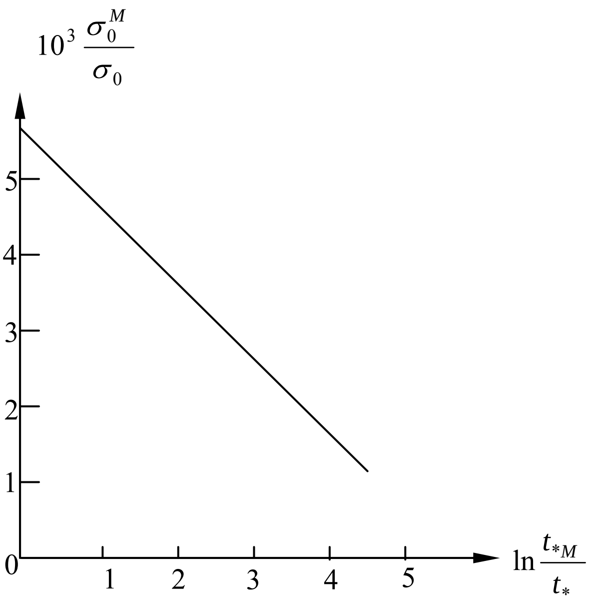

Fig. 5. The curve of modeling of destruction of the points $(0,a)$ of full-score (fusion ЭИ437Б), $(0,a_M)$ model (HDPK) plate with circular hole.

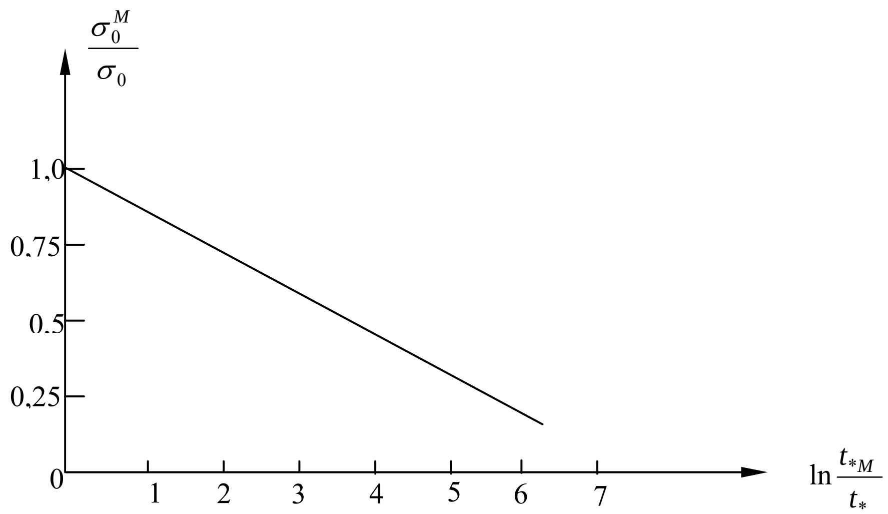

Fig. 6. The curve of modeling of destruction of the points $(0,a)$ of full-score and $(0,a_M)$ model plates from the same material with circular hole.

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