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THE DISPERSED FAILURE OF A LAMINATED HEAVY HALF-PLANE WITH A CIRCULAR APERTURE

Abstract

In this problem dispersion of failure around the aperture contour in infinitely heavy complex plane having an aperture was investigated. Here forces in rectilinear boundary of domain and forces uniformly distributed inside the aperture were given. Solving this problem Suvorov-Akhundov "failure theory of isotropic bodies" was used.

Strength and stability characteristics of buried constructions is its main quality index. At the development of buried constructions the investigation of strength problem is one of the necessary problems. On the other hand, failure process is a main course of deformed solids mechanics.

Realization of buried buildings in rheological complex and heterogeneous rocks and also refinement of computation for the strength require to take into consideration very important factors, which were not considered to present day. One of such factors is failure process in rocks.

The problems of finding of maximal stress on the contour of an aperture in elastic and hereditarily elastic mediums were considered to present day. However, such factors as failure, distribution of failure region or changing the mechanical qualities in failure region were not provided. Namely these factors were considered in this paper.

Assume that for the given medium conditions of plane deformation are valid. Then the domain will be in the form of two infinitely heavy laminated half-planes, which have upright aperture. We assume the medium to be isotropic defective.

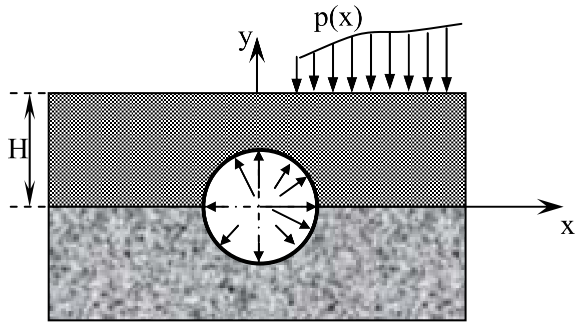

Fig. 1

It is clear from fig.1, that the center of aperture located on infinite half-plane is situated at a depth of H from rectilinear vertical boundary of plane. In this problem the failure process in two laminated mediums was investigated characterized by physical properties G_1, K_1 and G_2, K, and also having regional aperture.

Solving of problem was investigated in each of layers. Here system of equations is formed of physical dependencies and equations of deformation compatibility. The boundary conditions for these equations are defined by forces given on rectilinear domain of a half-plane and uniformly distributed on a contour apertures.

The main defining correlations are

$$3K\varepsilon_0 = \left(1 + N^*\right)\sigma\ ;\ \ 2G_0 Э_{ij} = \left(1 + M^*\right)S_{ij}\ ,$$

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$$\sigma = \sigma_{ii}\ ;\ \varepsilon = \varepsilon_{ii}\ ;\ S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma\delta_{ij}\ ;\ э_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon\delta_{ij}\ .$$

Here $э_{ij}$ and s_{ij} are strain and stress deviators respectively, G_0 and K_0 are instantaneous coefficient of elasticity, M^* and N^* are destruction operators of hereditary type.

Applying defining correlations we can obtain the dependence between strain and stress components

$$\varepsilon_{ij} = \frac{1}{2G_0}\left(1 + M^*\right)\sigma_{ij} + \frac{1}{3}\left\{\left(\frac{1}{3K_0} - \frac{1}{3G_0}\right) + \left(\frac{1}{3K_0}N^* - \frac{1}{2G_0}M^*\right)\right\}\sigma\delta_{ij}\ . \qquad (1)$$

The criterion of failure will be

$$\sigma_u + M^*\sigma_u = \sigma_M\ . \qquad (2)$$

Here σ_M is instantaneous ultimate stress limit, σ_u is stress intensity

$$\sigma_u = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 - \sigma_{11}\sigma_{22} - \sigma_{22}\sigma_{33} - \sigma_{11}\sigma_{33} + 3\sigma_{12}^2}\ . \qquad (3)$$

Since, the problem is a plane one, then equations of motion will be

$$\begin{cases} \dfrac{\partial\sigma_{11}}{\partial x_1} + \dfrac{\partial\sigma_{12}}{\partial x_2} = 0, \\ \dfrac{\partial\sigma_{12}}{\partial x_1} + \dfrac{\partial\sigma_{22}}{\partial x_2} + \gamma = 0. \end{cases} \qquad (4)$$

Here γ is specific gravity of rock.

Equations of deformation compatibility will have the following form

$$\frac{\partial^2\varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2\varepsilon_{22}}{\partial x_1^1} = 2\frac{\partial^2\varepsilon_{12}}{\partial x_1\partial x_2}\ . \qquad (5)$$

The criteria were given by means of stresses, therefore we express the problem trough the stresses.

For that, taking into account (1) and (5), we obtain the following system

$$\begin{cases} \dfrac{1}{2G_0}\left(1 + M^*\right)\left(\dfrac{\partial^2\sigma_{11}}{\partial x_2^2} + \dfrac{\partial^2\sigma_{22}}{\partial x_1^2}\right) + \dfrac{1}{3}\left\{\left(\dfrac{1}{3K_0} + \dfrac{1}{2G_0}\right) + \left(\dfrac{1}{3K_0}N^* + \dfrac{1}{2G_0}M^*\right)\right\}\times \\ \times\left(\dfrac{\partial^2\sigma_{11}}{\partial x_2^2} + \dfrac{\partial^2\sigma_{22}}{\partial x_2^2} + \dfrac{\partial^2\sigma_{33}}{\partial x_2^2} + \dfrac{\partial^2\sigma_{11}}{\partial x_1^2} + \dfrac{\partial^2\sigma_{22}}{\partial x_1^2} + \dfrac{\partial^2\sigma_{33}}{\partial x_1^2}\right) = \dfrac{1}{G_0}\left(1 + M^*\right)\dfrac{\partial^2\sigma_{12}}{\partial x_1\partial x_2}, \\ \dfrac{1}{2G_0}\left(1 + M^*\right)\sigma_{33} + \dfrac{1}{3}\left\{\left(\dfrac{1}{3K_0} - \dfrac{1}{3G_0}\right) + \left(\dfrac{1}{3K_0}N^* - \dfrac{1}{2G_0}M^*\right)\right\}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right) = 0. \end{cases} \qquad (6)$$

This system of equations with equilibrium equations (4) forms by stress component an isolated system of equations.

As was noted, in the system N^* and M^* are integral operators of hereditary type, characterizing failure process.

Since the amount of failure formed by volumetric deformation significantly smaller than the amount of failures formed due to volumetric friction, $N^* = 0$.

Then full system of equations will be

$$\begin{cases} \frac{1}{2G_0}\left(1+M^*\right)\left(\frac{\partial^2\sigma_{11}}{\partial x_2^2}+\frac{\partial^2\sigma_{22}}{\partial x_1^2}\right)+\frac{1}{3}\left\{\left(\frac{1}{3K_0}+\frac{1}{2G_0}\right)+\frac{1}{2G_0}M^*\right\}\times \\ \times\left(\frac{\partial^2\sigma_{11}}{\partial x_2^2}+\frac{\partial^2\sigma_{22}}{\partial x_2^2}+\frac{\partial^2\sigma_{33}}{\partial x_2^2}+\frac{\partial^2\sigma_{11}}{\partial x_1^2}+\frac{\partial^2\sigma_{22}}{\partial x_1^2}+\frac{\partial^2\sigma_{33}}{\partial x_1^2}\right)=\frac{1}{G_0}\left(1+M^*\right)\frac{\partial^2\sigma_{12}}{\partial x_1\partial x_2}, \\ \frac{1}{2G_0}\left(1+M^*\right)\sigma_{33}+\frac{1}{3}\left\{\left(\frac{1}{3K_0}-\frac{1}{3G_0}\right)-\frac{1}{2G_0}M^*\right\}\left(\sigma_{11}+\sigma_{22}+\sigma_{33}\right)=0\,, \\ \frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{12}}{\partial x_2}=0\,, \\ \frac{\partial\sigma_{12}}{\partial x_1}+\frac{\partial\sigma_{22}}{\partial x_2}-\gamma_2=0\,. \end{cases} \tag{7}$$

In system of equations (7) values G_0 and K_0 in the first layer were taken as $G_0 = G_1$, $K_0 = K_1$, for the second layer $G_0 = G_2$, $K_0 = K_2$.

Since forces on the segment $[a,b]$ of rectilinear domain of half-plane are taken uniformly distributed, and out of the segment all forces are taken equal to zero, then here boundary conditions will be as following:

$$\sigma_{xy} = -q\,;\ \sigma_{yy} = -p\,;\quad y = H,\ a \le x \le b\,. \tag{8}$$

Because of uniform distribution of forces inside of circle, the boundary conditions on the contour of aperture will be the following

$$\sigma_{rr} = -q\,;\ \sigma_{r\varphi} = 0\,;\ r = R\,. \tag{9}$$

At the point at infinity stress tends to the natural stress, i.e. on a heavy half-plane, where aperture is absent

$$\sigma ij \to \sigma_{ij}^0\,;\ x^2 + y^2 \to \infty\,. \tag{10}$$

Here σ_{yy}^0 are stresses corresponding to initial natural state, where

$$\begin{aligned} \sigma_{yy}^0 &= -\gamma(H-y), \\ \sigma_{yy}^0 &= -\lambda\gamma(H-y). \end{aligned} \tag{11}$$

Here λ $(0<\lambda<1)$ is the construction coefficient, in elastic state $\lambda = \nu/(1-\nu)$, ν is Poisson coefficient.

In obtained systems of equations we turn to dimensionless quantities:

$$\tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{p}\,;\ \tilde{G}_0 = \frac{G_0}{p}\,;\ \tilde{K}_0 = \frac{K_0}{p}\,;\ \tilde{x}_1 = \frac{x_1}{H}\,;\tilde{x}_2 = \frac{x_2}{H}\,. \tag{12}$$

Analytical solving of (7) is very difficult, therefore here numerical method and finite net method were applied.

For that we pass from infinite half-plane onto finite rectangle. Its incremental dimensions are defined during the numerical computations. Damaging operator is of the form:

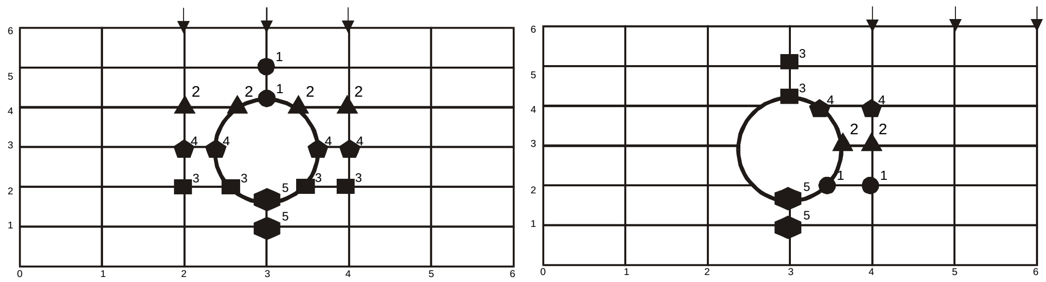
$$M^*\sigma_{ij} = \sum_{k=1}^{n}\delta_k\int_{t_k^-}^{t_k^+}M(t-\tau)\sigma_{ij}(\tau)d\tau + \int_{t_{n+1}^-}^{t}M(t-\tau)\sigma_{ij}(\tau)d\tau\,, \tag{13}$$

$$\sigma_{u_k} - \sigma_{u_{k-1}} > 0\,. \tag{14}$$

Here we take $\delta_1 = \delta_n = \frac{1}{2}$, $\delta_k = 1$, $M(t_n - t_k) = m$, $k = 2,...,n-1$ here computations are carried out for prime steady kernel

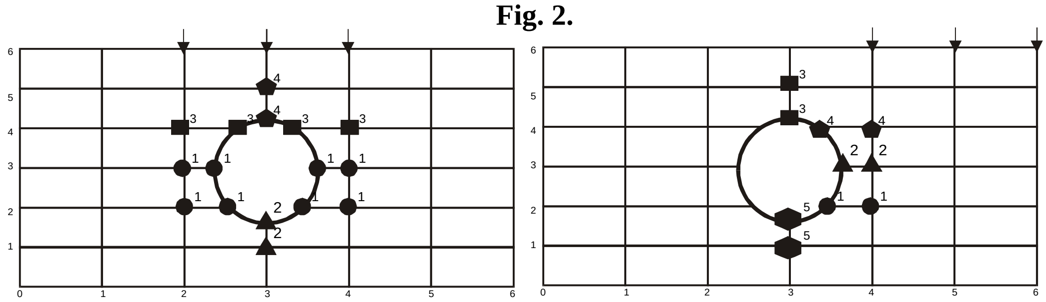
$$\widetilde{G}_0' = \frac{\widetilde{G}_0}{10^2}; \quad \widetilde{K}_0' = \frac{\widetilde{K}_0}{10^2}. \tag{15}$$

In these design parameters stress is defined in all nodes and correctness of failure criteria in all nodes is checked. Thus, as a result of these computations its possible to follow by nodes the increase of failure. The result of one of such type computations is demonstrated on figures 1-2.



a) b)

Fig. 2.



a) b)

Fig. 3.

\The computations were carried out for the two states more concretely for symmetric and non-symmetric weight location in the aperture (2b, 3b).

For the symmetric state the initial failure corresponds to stresses in the known elastic problem. This proves the accuracy of computations.

For the symmetric state with increase of lateral shear ruptured zone expands from the aperture to cheeks, i.e. failure process accelerates.

For the non-symmetric state ruptured zone along the aperture increases non-symmetrical, at this time initial failure depends on the construction coefficient. With increasing of lateral shear coefficient the ruptured zone changes along the domain containing the lower part of the aperture and direction of surface forces.

On the second plot for the state $\lambda = 0$ failure points are shown.

On the third and third plots failure points for the state $\lambda = 1$ are demonstrated.

On this plot the following denotations are accepted

- ● initial failure
- ▲ second failure
- ■ third failure

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