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MATHEMATICAL MODEL OF DEFORMATION OF SETTLED LOESS GROUNDS WITH ACCOUNT OF HUMIDITY

Abstract

In this paper functional dependencies of physical and mathematical and strength characteristics of ground on the extend of humidification of ground are built on the basis of given tests data for settled loess ground. With accordance of the theory of plastic flow the mathematical model of deformation of settled grounds with account of humidity is worked out. Here the basic model of ground is Grigorian's equation of state and parameters of the model are the function on the extent of water-saturation of ground. The ways of definition of the constants and parameters of the model are given in the paper.

Loess and loess-like grounds possessing settled characteristics are wide spread all around the world. Settled characteristics present great in value and non-uniform in character deformations accompanied by the increase of the volume of soil and significant change of physical and mathematical parameters of ground. Here the principle parameter of loess soils is its humidity, that is water-saturation and porosity. Water-saturation of loess settled grounds is characterized by water filling in grounds pores.

The influence of water-saturation of ground on the process of settlement was studied by many scientists [1-5]. As results of experimental studies show [1-5] depending on the degree of water-saturation of soil the values of physical and mechanical characteristics of soil are sufficiently changing. In [6] according to the test results the model of volume deformation of settled ground with account of humidity was worked out. Here a mechanical parameter – the modulus of volume compression is taken as a function on the degree ground humidity and the appearance of settlements in connection with decrease of compression modulus with increasing humidification of grounds is shown. But strength characteristics of loess grounds also influence the settlement and that is an experimentally stated fact.

With test results [1,2] it was revealed that strength parameters of settled grounds are one-valued functions of humidity. To state these functional dependencies in [1-3] an experimental studies of the number of characteristic settled grounds in Azerbaijan was laboratory tested on cutting device and stabilometer. Results of these tests for three types of settled grounds are given in [1,2] and partially in fig.1. Coefficient of function angle μ (fig. 1a) is taken as equal $\mu = tg\varphi$ (φ - is an angle of inner friction). Depending on humidity the change of Poisson ratio (fig. 1b) was calculated using the relation $\nu = \xi/(1+\xi)$ here ξ is a coefficient of side pressure. Here we should note that analogous dependencies in fig.1 were experimentally obtained in [5]. As it is seen from fig.1 the increase of the value of Poisson ratio is observed with increase of humidity of settled grounds, and coefficient of friction angle decrease. In [1,2] these changes are approximated by linear functions on humidity, which well describe the value of strength parameters of ground from natural moisture content to complete water-saturation and settlement and bearing capacity of loess grounds are defined. According to fig.1 and [1-5] the values of modulus of volume compression (the regularity of changes depending on humidity is shown in fig.2), cohesion forces and coefficient of the angle of inner friction from dry (dried) state till complete water-saturation are decreasing approximately by exponential law with increase if water-saturation of ground. So, functional dependencies of these parameters on the extent of water-saturation of ground may be build in the form

$$K(I_W) = K_{sat}\exp(\alpha_K(1 - I_W)); \quad I_W = W / W_{sat}; \tag{1}$$

$$c(I_W) = c_{sat}\exp(\beta(1 - I_W)), \quad \mu(I_W) = \mu_{sat}\exp(\gamma(1 - I_W)), \tag{2}$$

here $K_{sat}, c_{sat}, \mu_{sat}$ are modulus of volume compression, cohesion force and coefficient of angle of inner friction of completely saturated ground, respectively, α_K, β, γ - empirical dimensionless coefficients characterizing the degree of change of corresponding mechanical characteristics of settled ground, W - current humidity of ground, W_{sat} - humidity corresponding to complete filling ground pores by water.

Now we will state the dependence of modulus of shear on humidity. For that in built dependencies of modulus of compression [5] (fig.2) and Poisson ratio (fig.1b) on humidity according to results of tests we define the changes of shear modulus using known relation [7]

$$G = 3K(1 - 2v)/(2(v + 1)).$$

The regularity of shear modulus change depending on humidity is given in fig.3. It is seen that for shear modulus we could build the dependency in the form

$$G(I_W) = G_{sat}\exp(\alpha_G(1 - I_W)). \tag{3}$$

Stated mathematical expressions for mechanical parameters of settled grounds may be used for design formulate in engineering problems, static and quasi-static models of settled ground given in [1,2].

Consider now the regularities of dynamic deformation of settled ground with account of humidity. As it is known loess ground are media possessing plastic characteristics. Besides should be noted that in structurally disturbed skeleton of soil the connections between the particles are mutual compression and friction in contact points. That is why the value of finite mutual displacements of the particles at shear could not effect on arising stresses. The latter should be connected with deformation of current state, that is with tensor of deformation velocities [8]. Because of this the construction of the model of deformation is here based on principle rules of the theory of plastic flow [9] stating the dependence at shear between the deviator of stress tensor and deviator of tensor of deformation velocities.

The regularity of volume deformation according to [9] is taken by linear relation between hydrostatic pressure and volume deformation

$$P = K_{sat}\exp(\alpha_K(1 - I_W))\cdot\varepsilon\ . \tag{4}$$

Total deformation of shear e_{ij} in plastic zone in the theory of plasticity [9] may be presented by the sum of elastic e_{ij}^e and plastic e_{ij}^p components

$$e_{ij} = e_{ij}^e + e_{ij}^p \quad \text{or} \quad \dot{e}_{ij} = \dot{e}_{ij}^e + \dot{e}_{ij}^p \tag{5}$$

In elastic zone of deformation the determining relations are expressed by Hook's law:

$$e_{ij}^e = S_{ij}/(2G(I_W)) \quad \text{or} \quad \dot{e}_{ij}^e = \dot{S}_{ij}/(2G(I_W)). \tag{6}$$

Here it is necessary to note that the change of ground humidity (water spreading) occur slowly comparing with development of dynamic processes in loess grounds (propagation of elastic waves).

In plastic zone of ground deformation according to [9] tensor of velocities of plastic deformations of shear is taken as proportional to instant deviator of stresses. Thus we have the following relation

$$\dot{e}_{ij}^{p} = \lambda S_{ij} / (2G(I_W)), \tag{7}$$

here $\lambda > 0$ when the condition of plasticity

$$J_2 = F^2 / 3, \qquad J_2 = S_{ij} S_{ij} / 2 \tag{8}$$

is fulfilled, F - yield limit (function of plasticity) of ground.

Substituting the second relation (6) and (7) with (3) to a last one from relation (5) we obtain the following expression for the law of shear deformation of settled grounds

$$\frac{dS_{ij}}{dt} + \lambda S_{ij} = G_{sat} \exp(\alpha_G (1 - I_W)) \frac{de_{ij}}{dt}. \tag{9}$$

Relation (9) is applicable for plastic flow only, when the relation (8) and $\lambda > 0$ are fulfilled. If not $\lambda \equiv 0$. In the last case the relation (9) transforms to Hook's law (6).

Now we will define the value of λ. Multiplying (9) by S_{ij} and summing up we obtain

$$S_{ij} \cdot \frac{dS_{ij}}{dt} + \lambda S_{ij} S_{ij} = G_{sat} \exp(\alpha_G (1 - I_W)) \Omega, \qquad \Omega = S_{ij} \frac{de_{ij}}{dt}, \tag{10}$$

taking into consideration that

$$S_{ij} \cdot \frac{dS_{ij}}{dt} = \frac{1}{2} \frac{d(S_{ij} S_{ij})}{dt} = \frac{dJ_2}{dt}, \tag{11}$$

from (10) and (11) with (8) we obtain

$$\lambda = \left(2G_{sat} \exp(\alpha_G (1 - I_W)) \Omega - \frac{dJ_2}{dt} \right) \Big/ (2J_2).$$

We should state that in case of constant humidity of ground the law of shear deformation coincides with Grigorian's equation of state [8].

So complete mathematical model of deformation of settled loess grounds includes the law of deformation at volume compression (4), the conditions of plasticity (8) and relationships (9) characterizing the law of shear deformation. Plasticity function F in (8) in general case depends also on third invariant of stress tensor. Usually in the theory of plasticity [9] the independence of relations (8) on third invariant of stress is assumed. F could be taken in the form offered in [8] or as in experiments in [10]; in many cases plasticity function is taken in the form of linear dependence on the first invariant of stresses that is on pressure $F = c + \mu P$. Account the dependence of cohesion force and coefficient of the angle of inner friction on the content of water-saturation (2) we may conclude that the function of plasticity of settled ground is also a one-valued function on humidity (on the extent of saturation) :

$$F(P, I_W) = c(I_W) + \mu(I_W) P. \tag{12}$$

Basing on experimental values of cohesion force and coefficient of the angle of inner friction of grounds at different extent of water-saturation we may build the dependence of plasticity function on humidity (12) for different values of hydrostatic pressure. The regularity of changes of plasticity function depending on humidity is shown in fig.4.

Now consider the ways of definition of parameters and coefficients (1)-(3). At complete water-saturation of ground that is $I_W = 1$ from (1)-(3) follows $K = K_{sat}$, $c = c_{sat}$, $\mu = \mu_{sat}$ and $G = G_{sat}$ where the values $K_{sat}, c_{sat}, \mu_{sat}$ and G_{sat} are defined from experiments [1-5]. With dry arid ground $I_W = 0$ or ground with natural humidity $I_W = I_{W0}$ defining the values of modulus of compression and tension, cohesion

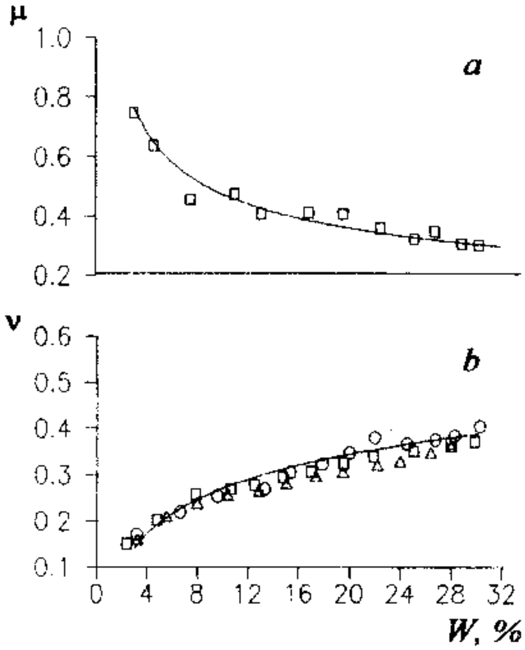


Fig.1

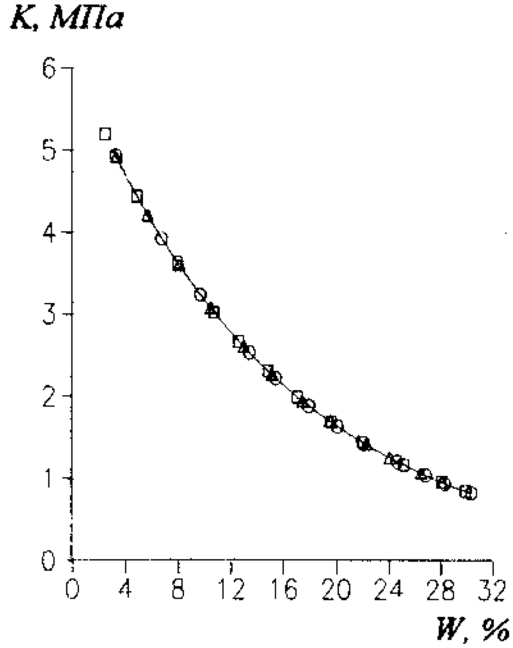
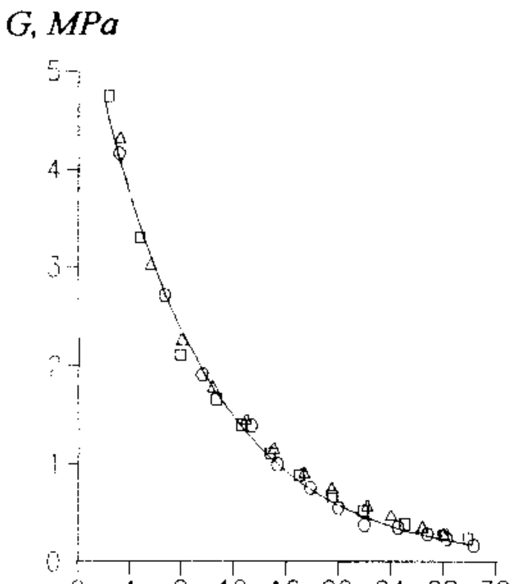
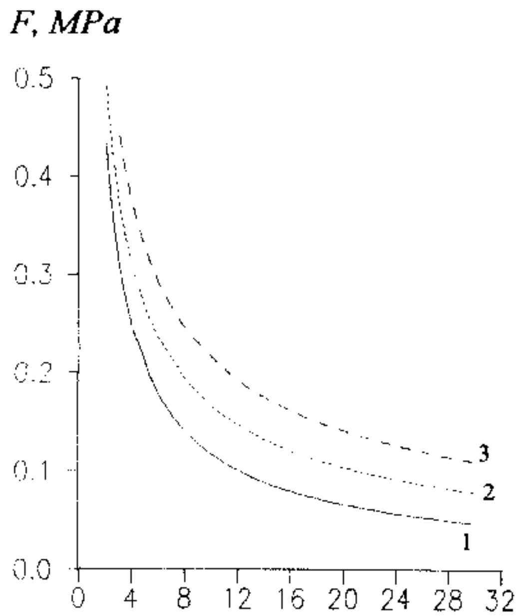


Fig.2



△ - ground of Mingechaur,
□ - Kazakh, o - Sumgait

Fig.3



1 - P=0.1 MPa, 2 - P=0.2 MPa,
3 - P=0.3 MPa

Fig.4

force and coefficients of friction from [1-5] the dimensionless coefficients $\alpha_K, \alpha_G, \beta$ and γ are calculated with (1)-(3). For experimentally tested settled grounds in Azerbaijan we have [1-3] $W_{sat} = 38\%$; $K_{sat} = 0{,}5\ MPa$; $G_{sat}\ 0{,}096\ MPa$; $c_{sat} = 0{,}005\ MPa$; $\mu_{sat} = 0{,}24$;

$$\alpha_K = 2{,}5;\ \alpha_G = 4{,}02;\ \beta = 3{,}92 \text{ and } \gamma = 1{,}09 .$$

So proposed model of deformation of settled grounds accounting non-uniform humidification of grounds allows theoretically to carry out calculations on definition of settlement of ground-foundations near water sources under its own weight under the action of different loads (cyclic loading from passing transport vehicles, work of powerful equipment, etc.) and to define theoretically the depth of penetration of precast piles foundations or their pulling out with account of weakening of strength characteristics of ground y watering.

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