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ON FATIGUE WEAR OF ELASTICO-PLASTIC PLATE AT PULSING TEMPERATURE ACTION

Abstract

The tensors components of stresses and deformations, the remainder stresses and deformations, the intensivity of remainder deformations are determined in the plate at its elastico-plastic deformation and the next full elastic unloading at every cycle in the case of pulsing temperature loading. The analytic formulas for the time and for the temperature cycle have been got at which in result of thermic fatigue wear from the surface layers of plate distracting the material of given thickness is determined.

Let's consider an elastico-plastic with linear hardening plate of thickness h in any form, which in a plane is free of external loadings. Let's apply the rectangular cartesion systems of the coordinate (x_1, x_2, x_3). Let's arrange the axes x_2 and x_3 in the middle of the surface. It is clear that in this case the axis x_1 will be perpendicular to this surface. On the both boundary surfaces of plate is realized the absorption of the heat $q(t)$, where t is time. Moreover it is considered that the heat stream $q(t)$ quite slowly changes by the time by the force of pulsing cycles. At $t=0$ we suppose $q(0)=0$. We'll consider the domains of plate on sufficiently deleting it from its edges. We suppose that all constants of materials don't depend on temperature. Subject to above noted distribution of the temperature $T(x,t)=T(x_1,t)$ will be symmetric with respect to the middle surface $x_1=0$. From this and according to [1,2] we'll suppose the temperature field of the plate in the following form

$$T(x,t)=T(x_1,t)=\frac{2q(t)x_1^2}{\chi h^2}, \tag{1}$$

where χ is a heat condition coefficients.

We'll denote continuity of every temperature cycle by t_*, the time before the destruction of plate by $t_c(x_1)$. In addition the number of temperature cycle will be $N_c(x_1)=t_c(x_1)/t_*$ an which will happen the destruction of plate will begin from the surface layers $x_1=\pm\frac{h}{2}$ and will be extended in direction x_1. It means that the separation of materials will begin from the surface layers, i.e. the process of thermal fatigue wear will begin. However the process of wear won't get the surface $x_1=0$. Because the plastic deformation occurs when $x_1=\pm\frac{h}{2}$ and at any cycle the central plastic zone won't happen.

Let's define the elastico-plastic stress-deformated state (also the remainder stresses and deformation's) of the plate at any cycle of temperature action for getting the $t_c(x_1)$ (or $N_c(x_1)$). By the conditions of problem every temperature cycle consists of temperature loading during the time $t_*/2$ and full temperature unloading during the same time.

First of all let's consider the elastico-plastic deformation problem of the investigated plate from the natural state at temperature loading (1) in interval time

$[0, t_*/2]$. The solution of this problem in that or other degree is considered in [1,2,3]. Let's use some results from these papers. In domain of elastic deformations the following relations [4] hold:

$$s_{ij} = 2Ge_{ij}, \tag{2}$$

$$\theta = 3\alpha T. \tag{3}$$

Here $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$, $e_{ij} = \varepsilon_{ij} - \varepsilon\delta_{ij}$ are tensors of the deviator of stress σ_{ij} and deformation ε_{ij}, $\sigma = \sigma_{ij}\delta_{ij}/3$, $\varepsilon = \varepsilon_{ij}\delta_{ij}/3$ are mean stresses and deformations; δ_{ij} are Kronecker symbols, G is modulus of shift, $\theta = 3\varepsilon$ is relative change of volume, α is coefficient of linear temperature extension. The plate material is supposed to be mechanico-incompressible ($\nu = \frac{1}{2}$, where ν is a Poisson coefficient).

In domain of plastic deformations the relations of small-elastico-plastic deformation theory of A.A.Ilyushin with the linear hardening [5] are fulfilled

$$s_{ij} = \frac{2\sigma_+}{3\varepsilon_+} e_{ij}, \tag{4}$$

$$\sigma_+ = \lambda\sigma_s + 3G(1-\lambda)\varepsilon_+, \tag{5}$$

$$\theta = 3\alpha T. \tag{6}$$

Here $\sigma_+ = \left(\frac{3}{2} s_{ij} s_{ij}\right)^{\frac{1}{2}}$ – is stress intensity, $\varepsilon_+ = \left(\frac{2}{3} e_{ij} e_{ij}\right)^{\frac{1}{2}}$ is deformation intensity, λ is coefficient of hardening $0 \le \lambda \le 1$, σ_s is limit of fluctuation by stresses, which is connected with the limit of fluctuation on the formations ε_s by the relations $\sigma_s = 3G\varepsilon_s$.

For our problem we have [1,2,3]:

$$\sigma_{11} = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0;\ \sigma_{22} = \sigma_{33} = \sigma_0 \ne 0;$$

$$\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0;\ \varepsilon_{22} = \varepsilon_{33} = \varepsilon_0 = \varepsilon_0(t) \ne 0;\ \varepsilon_{11} = \varepsilon_{11}(x_1, t) \ne 0.$$

Consequently the problem at the temperature loading (1) is reduced to the definition $\sigma_0 = \sigma_0(x_1, t)$, deformations $\varepsilon_0 = \varepsilon_0(t), \varepsilon_{11} = \varepsilon_{11}(x_1, t)$ and some functions $\xi_s(t)$ determining the boundary of elastic and plastic domain of plate.

For the quantities σ_0, ε_0 in [3] using the relations (2)-(6) and conditions on the absence of resulting intensifications on the edge of a plate

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_0(x_1, t)\,dx_1 = 0 \tag{7}$$

the following relations are got:

$$\sigma_0 = 6G\left(\varepsilon_0 - \frac{\alpha q(t)}{2\chi}\xi^2\right) \quad \text{when} \quad \xi \le \xi_s; \tag{8}$$

$$\sigma_0 = 6G(1-\lambda)\left(\varepsilon_0 - \frac{\alpha q(t)}{2\chi}\xi^2\right) - \lambda \quad \text{when} \quad \xi > \xi_s; \tag{9}$$

$$\varepsilon_0 = \frac{\alpha q(t)}{2\chi}\xi_s^2(t) - \frac{1}{2}, \ (\lambda \ne 0); \tag{10}$$

where

$$\frac{2x_1}{h}=\xi\,. \tag{11}$$

In addition in [3] the quantity ξ_s was not determined, only in [3] it the following equation for its determination is led:

$$\frac{2}{3}\varepsilon_T^0(t)=\frac{1}{2\lambda\xi_s^3(t)-(1-\lambda)\left(1-3\xi_s^2(t)\right)}. \tag{12}$$

Here we accept the notation $\varepsilon_T^0=\dfrac{\alpha q(t)}{2\chi\varepsilon_s}$.

Let's determine $\xi_s(t)$ from the equation (12). The analysis shows that in the interval $t\in\left[0,\frac{t_*}{2}\right]$ and $x_1\in\left(0,\frac{h}{2}\right]$ or $\xi\in(0,1]$ at conditions $0<\lambda\le 1$ and $2\ge 3/2\varepsilon_T^0(t)$ it has a unique real root, which is represented in the following form:

$$\xi_s(t)=\frac{n^2}{f\left(n,\varepsilon_T^0(t)\right)}+f\left(n,\varepsilon_T^0(t)\right)-n\,, \tag{13}$$

where noted

$$n=\frac{1-\lambda}{2\lambda}, \tag{14}$$

$$f\left(n,\varepsilon_T^0(t)\right)=\left\{\frac{3(2n+1)}{8\varepsilon_T^0(t)}-\frac{n(3n^2-1)}{2}+\left[\left(\frac{n(3n^2-1)}{2}-\frac{3(2n+1)}{8\varepsilon_T^0(t)}\right)^2-n^6\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}}. \tag{15}$$

The solution (13) subject to (14) and (15) is the exact solution of the equation (12). From (14) it follows that when $\lambda=1$ we have $n=0$. This case corresponds to the real plastic state of material. In this case from (15) will get

$$f\left(0,\varepsilon_T^0(t)\right)=\left(\frac{3}{4\varepsilon_T^0(t)}\right)^{\frac{1}{3}}.$$

Subject to this from the solution (13) $\lambda=1\ (n=0)$. We have

$$\xi_S(t)=\left(\frac{3}{4\varepsilon_T^0(t)}\right)^{\frac{1}{3}}.$$

This formula directly follows from the equation (12) when $\lambda=1$.

The state of plasticity of surface layers of plate corresponds to $\xi_s=1$. The time t_p at which the surface layers pass to the state of plasticity is determined from (12) when $\xi_c=1$. As shows the calculations the time t_p doesn't depends on λ and is determined from the equation

$$\varepsilon_T^0(t_p)=\frac{\alpha q(t_p)}{2\chi\varepsilon_s}=\frac{3}{4}. \tag{16}$$

Thus determining the function $\xi_c(t)$ by the formula (13) using (4) and (15) we find the stress σ_0 by the formula (8), (9) and the elastico-plastic deformation ε_0 by the formula (10). The deformation ε_{11} is easily determined from (6):

$$\varepsilon_{11} = 3\alpha T(x_1,t) - 2\varepsilon_0 . \tag{17}$$

Let's denote that the formulas (8), (9), (10), (17) subject to (13)-(15) hold in temperature loading process which happens in interval time $\left[0,\frac{t_*}{2}\right]$. In this case the heat stream $q(t)$ monotonically increases from zero to its maximal value $q_m = q\left(\frac{t_*}{2}\right)$, then in interval time $\left[\frac{t_*}{2},t_*\right]$ monotonically decreases to zero. In addition when $t=\frac{t_*}{2}$ the central plastic zone doesn't arise. Since we suppose that at interval time $\left[\frac{t_*}{2},t_*\right]$ the full elastic unloading happens then at $t=t_*$ we have $q=0$, $\varepsilon_T^0 = 0$. In this case the remainder deformation ε_0^0 and the remainder stress σ_0^0 are determined in [3]:

$$\varepsilon_{22}^0 = \varepsilon_{33}^0 = \varepsilon_0^0 = \varepsilon_0\left(\frac{t_*}{2}\right) - \frac{\alpha q\left(\frac{t_*}{2}\right)}{6\chi}, \tag{18}$$

$$\sigma_{22}^0 = \sigma_{33}^0 = \sigma_0^0(x_1) = \sigma_0\left(x_1,\frac{t_*}{2}\right) + \frac{G\alpha q\left(\frac{t_*}{2}\right)}{\chi}\left(\frac{12x_1^2}{h^2} - 1\right). \tag{19}$$

Now let's find the remainder deformation ε_{11}^0 since it isn't determined in [3]. Let's use the theory on elastic unloading of A.A.Ilyushin [5]. Following this theorem

$$\varepsilon_{11}^0 = \varepsilon_{11}' - \varepsilon_{11}^F, \tag{20}$$

where the quantity ε_{11}' is deformation, existing before the beginning of unloading

$$\varepsilon_{11}' = 3\alpha T\left(x_1,\frac{t_*}{2}\right) - 2\varepsilon_0\left(\frac{t_*}{2}\right), \tag{21}$$

the quantity ε_{11}^F is deformation which should arise in considered plate at its elastic deformation from the natural state, by the temperature field $T\left(x_1,\frac{t_*}{2}\right)$. For its determining first of all let's determine the elastic deformation ε_0^e, which arises in the plate at loading of temperature field $T\left(x_1,\frac{t_*}{2}\right)$. Let's use the relations (7) in addition (8). We'll get

$$\varepsilon_0^e = \frac{\alpha q\left(\frac{t_*}{2}\right)}{6\chi}.$$

In addition with using (21) we have

$$\varepsilon_{11}^F = 3\alpha T\left(x_1,\frac{t_*}{2}\right) - \frac{\alpha q\left(\frac{t_*}{2}\right)}{3\chi}. \tag{22}$$

From (20) subject to (21) and (22) we'll denote the remainder deformation ε_{11}^0

$$\varepsilon_{11}^{0}=\frac{\alpha q\left(\frac{t_*}{2}\right)}{3\chi}-2\varepsilon_0\left(\frac{t_*}{2}\right). \tag{23}$$

Let's denote that here $\varepsilon_0\left(\frac{t_*}{2}\right)$ is determined by the formulas (10) at using (13)-(15), when $t=\frac{t_*}{2}$.

Now let's determine the remainder intensity of the déformations ε_+^0 which is expressed by the formula in our case

$$\varepsilon_+^0=\frac{\sqrt{2}}{3}\left[\left(\varepsilon_{11}^0-\varepsilon_{22}^0\right)^2+\left(\varepsilon_{22}^0-\varepsilon_{33}^0\right)^2+\left(\varepsilon_{33}^0-\varepsilon_{11}^0\right)^2\right]^{\frac{1}{2}}. \tag{24}$$

Allowing for (18) and (23) in (24) we'll get

$$\varepsilon_+^0=\frac{2}{3}\left|3\varepsilon_0\left(\frac{t_*}{2}\right)-\frac{1}{2}\frac{\alpha q\left(\frac{t_*}{2}\right)}{\chi}\right|. \tag{25}$$

Consequently at the first cycle of temperature of pulsing loading of plate at its elastico-plastic deformation and next full elastic unloading components of tensors stresses and deformations the remainder stresses and deformations, and the remâinder intensity of deformation are also determined. Since the loading of plate is pulsing then every cycle of the heat stream $q(t)$ doesn't differ from one another and at this it has the same maximum $q_m=q\left(\frac{t_*}{2}\right)$. Following from this and following [4], w can conclude that in the cycles next after the first, the remainder stresses and deformations will not change and will be expressed by the corresponding formulas (18), (19), (23), the remainder intensity of deformations by the formula (25).

Now let's determine the thermal fatigue wear of plate at obtained data. Let's use the criterion represented in [6].

$$\int_0^{t_c}\left(\varepsilon_+^0(t)\right)^\beta\left(\frac{T(t)}{T_c}\right)^\delta dt=\varepsilon_c^\beta t_*. \tag{26}$$

Here $t_c=t_c(x,t)$ is the time before the destruction, ε_+^0 is remainder intensity of deformations, t_* is the time of duration of every cycle, T_c and ε_c - are experimentally determined critical temperature α deformations at monotonically loading during the time t_*, β and δ - are experimentally determined constants of material.

Let the heat stream $q(t)$ be represented in the following form

$$q(t)=q_m\sin^2\pi\frac{t}{t_*}. \tag{27}$$

Let's denote that in this case the time of appearance of plastic deformations in the first cycle on the surface layers of plate following to (16) is determined by the following formula:

$$t_p = \frac{t_*}{\pi} \arcsin \sqrt{\frac{3\chi\varepsilon_s}{2\alpha q\left(\frac{t_*}{2}\right)}}.$$

The temperature field of plate according to the formula (1) will be

$$T(x_1, t) = \frac{2x_1^2}{\chi h^2} q_m \sin^2 \pi \frac{t}{t_*}. \quad (28)$$

Allowing for (25) and (28) in (26) we'll determine the time t_c

$$t_c = \left(\frac{\varepsilon_c}{\varepsilon_+^0}\right)^\beta \left(\frac{\chi h^2 T_c}{2x_1^2 q_m}\right)^\delta \frac{\Gamma(\delta+1)}{\Gamma\left(\delta+\frac{1}{2}\right)} \pi^{\frac{1}{2}} t_*. \quad (29)$$

Here $\Gamma(\delta)$ is a gamma function. The quantity ε_+^0 is represented by the formula (25) at using the (10), (13)-(15). The number of the temperature cycles before the destruction N_c is determined from (29):

$$N_c = \left(\frac{\varepsilon_c}{\varepsilon_+^0}\right)^\beta \left(\frac{\chi h^2 T_c}{2x_1^2 q_m}\right)^\delta \frac{\Gamma(\delta+1)}{\Gamma\left(\delta+\frac{1}{2}\right)} \pi^{\frac{1}{2}}. \quad (30)$$

As we see quantities t_c and N_c depend on x_1. When $x_1 = \pm\frac{h}{2}$ from the formulas (29) and (30) we find the time t_{1c}, and the number of cycles N_{1c} at which begins the destruction of surface layers:

$$t_{1c} = \left(\frac{\varepsilon_c}{\varepsilon_+^0}\right)^\beta \left(\frac{2\chi T_c}{q_m}\right)^\delta \frac{\Gamma(\delta+1)}{\Gamma\left(\delta+\frac{1}{2}\right)} \pi^{\frac{1}{2}} t_*, \quad (31)$$

$$N_{1c} = \left(\frac{\varepsilon_c}{\varepsilon_+^0}\right)^\beta \left(\frac{2\chi T_c}{q_m}\right)^\delta \frac{\Gamma(\delta+1)}{\Gamma\left(\delta+\frac{1}{2}\right)} \pi^{\frac{1}{2}}. \quad (32)$$

The process of thermal fatigue wear begins at the time t_{1c} or the number of cycles N_{1c}, determined by the formula (31) and (32) from the surface layers of plate and flow past to deep in the direction of the axis x_1. In theory of wear gives the limit of wear, at increasing of which the construction becomes unfit for the used aim [7]. Let $\frac{h_1}{2}$ be the admissible thickness separated in finally destruction of materials of every surface of plate. At this supposing $x_1 = \pm\frac{h}{2} \mp \frac{h_1}{2}$, from the formulas (29) and (30) we get the final time t_{2c} and the cycle number N_{2c} of wear

$$t_{2c} = \left(\frac{\varepsilon_0}{\varepsilon_+^0}\right)^\beta \left(\frac{2\chi h^2 T_c}{\left(h^2 - 2hh_1 + h_1^2\right) q_m}\right)^\delta \frac{\Gamma(\delta+1)}{\Gamma\left(\delta+\frac{1}{2}\right)} \pi^{\frac{1}{2}} t_*, \quad (33)$$

$$N_{2c} = \left(\frac{\varepsilon_0}{\varepsilon_+^0}\right)^{\beta} \left(\frac{2\chi h^2 T_c}{\left(h^2 - 2hh_1 + h_1^2\right)q_m}\right)^{\delta} \frac{\Gamma(\delta+1)}{\Gamma\left(\delta+\frac{1}{2}\right)} \pi^{\frac{1}{2}}. \tag{34}$$

Thus at the time t_{2c} and the number of cycles N_{2c}, determined by the analytic formulas (33) and (34) in addition (25), (10), (13)-(15), in case of acting of heat stream (27), from the surface layers of plate distracting separates the material every thickness $\frac{h_1}{2}$ and the plate becomes unfit for the used aim.

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ON TWO APPROXIMATED METHODS FOR SOLUTION OF ONE BOUNDARY VALUE PROBLEM FOR A DIFFERENTIAL EQUATION OF THE FOURTH ORDER

Abstract

A boundary value problem is considered for the fourth order differential equation. This equation is reduced to the equivalent Volterra-Fredholm integral equation. The equation is solved by two iteration methods.

The boundary value problem for a differential equation of the fourth order is considered. It is substituted by the integral equation of Volter-Fredholm and the last is solved by two iteration methods.

Let's consider the following boundary value problem for a differential equation of the fourth order

$$\frac{d^4x(t)}{dt^4}+a(t)x(t)=f(t) \quad (0\le t\le 1), \tag{1}$$

$$\left.\begin{aligned} &x(0)=x_0,\ x'(0)=\dot{x}(0),\ x''(0)=\ddot{x}_0,\\ &x(1)=\alpha x(c)+\beta,\ \alpha c^3\ne 1,\ (0<c<1)\end{aligned}\right\} \tag{2}$$

Such problem is met, for example in the sections of construction mechanics – in the problem on equilibrium of beam on elastic base [1-4], in some problems of the theory of cylindrical shells [4].

For approximated solution of the problem (1), (2) the methods given at papers [5], [6] are applied.

Let's suppose that $a(t)$, $f(t)$, $(0\le t\le 1)$ are continuous. It is easily proved that we can substitute the problem (1)-(2) by the equivalent integral equation

$$\begin{aligned} x(t)=f^*(t)+\int_0^1\frac{t^3(1-s)^3}{6(1-\alpha c^3)}a(s)x(s)ds-\\ -\alpha\frac{t^3}{6(1-\alpha c^3)}\int_0^c(c-s)^3a(s)x(s)ds-\int_0^t\frac{(t-s)^3}{6}a(s)x(s)ds,\end{aligned} \tag{3}$$

where

$$\begin{aligned} f^*(t)=\frac{t^3}{1-\alpha c^3}\beta+x_0\left(1-\frac{t^3(1-\alpha)}{1-\alpha c^3}\right)+\dot{x}_0\left(t-\frac{t^3(1-\alpha c)}{1-\alpha c^3}\right)+\ddot{x}_0\left(\frac{t^2}{2}-\frac{t^3(1-\alpha c^2)}{2(1-\alpha c^3)}\right)-\\ -\int_0^1\frac{t^3(1-s)^3}{6(1-\alpha c^3)}f(s)ds+\frac{\alpha t^3}{6(1-\alpha c^3)}\int_0^1(c-s)^3f(s)ds+\frac{1}{6}\int_0^t(t-s)^3f(s)ds,\end{aligned} \tag{4}$$

or

$$x(t)=f^*(t)+\varphi(t)Fx+Vx, \tag{5}$$

where