APPLIED PROBLEMS OF MATHEMATICS AND MECHANICS

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THE NECESSARY CONDITIONS OF OPTIMALITY IN THE PROBLEM FOR ONE CLASS SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS OF THE FIRST ORDER WITH ORDINARY AND GENERALIZED CONTROLS

Abstract

The integral and differential principles of maximum in the problem, controlled by system described for one class of the system of partial differential equations of the first order with generalized and ordinary control are derived at the paper.

Introduction. By investigating chemical-technological processes the problem of optimal control, described by the partial differential equations of the first order [1-5], appears. At these papers the necessary conditions of optimality are obtained and the theorem on the existence and uniqueness of solutions of initial boundary problems at fixed ordinary controls is proved. It must be to noted that in solving many applied problems it is natural to introduce the generalized controls too. In the given paper the necessary conditions of optimality both ordinary and generalized controls in the processes described by the partial differential equations of the first order, are concluded.

1. Statement of the problem. Let the controlled process be described by the system of equations:

$$\begin{aligned} \frac{\partial y}{\partial t} &= f^1(t,x,y,z,\omega(t,x)) + b^1(t,x)\frac{du}{dt}, \\ \frac{\partial z}{\partial x} &= f^2(t,x,y,z,\omega(t,x)) + b^2(t,x)\frac{dv}{dx}, \ (t,x)\in Q \end{aligned} \tag{1.1}$$

with the initial conditions

$$\begin{aligned} y(t_0,x) &= \varphi^1(x,\xi(x)), \ x\in[x_0,x_1], \\ z(t,x_0) &= \varphi^2(t,\eta(t)), \ t\in[t_0,t_1]. \end{aligned} \tag{1.2}$$

Here all the derivatives are understood in the meaning of generalized functions [6,7]; $Q=(t_0,t_1)\times(x_0,x_1)$; $f^i(t,x,y,z,\omega)$, $\varphi^i(s,r)$ - are given n_i - dimensional vector columns; $b^i(t,x) - n_i\times m_i$ are matrix functions; $(\omega(t,x),\xi(x),\eta(t),u(t),v(x))$ – are $r+r_1+r_2+m_1+m_2$ - dimensional controlling parameters.

Let's denote by $VB_m(a,b)$ the space of m-dimensional continuous from the left on (a,b) functions $u(t)$ of bound variation on $[a,b]$ with the norm $\|u\|_{VB_m(a,b)} = \|u(a)\| + Var_a^b\|u(t)\|$ [8], where $\|\cdot\|$ is a Euclidean norm.

Let $\Omega_0(\cdot), \Omega_1(\cdot), \Omega_2(\cdot), U(\cdot), V(\cdot)$ be convex sets in $L_2^r(Q), L_2^{r_1}(x_0,x_1)$, $L_2^{r_2}(t_0,t_1), VB_{m_1}(t_0,t_1), VB_{m_2}(x_0,x_1)$ respectively. The functions $(\omega(t,x),\xi(x),\eta(t),u(t),v(x))\in$ $\in U_\partial = \Omega_0(\cdot)\times\Omega_1(\cdot)\times\Omega_2(\cdot)\times U(\cdot)\times V(\cdot)$ are taken as admissible controls.

Further we'll denote by $VB\left((a,b),L_2^n(c,d)\right)$ a space of n-dimensional measurable functions $h(t,x)$ such that $\|h(t,\cdot)\|_{L_2^n(c,d)}\in VB(a,b)$. In this space the norm will be determined in the next form

$$\|h\|_{VB\left((a,b),L_2^n(c,d)\right)}=\|h(a,\cdot)\|_{L_2^n(c,d)}+Var_a^b\|h(t,\cdot)\|_{L_2^n(c,d)}.$$

Let's introduce the space

$$W(Q)=\left\{h=(y,z):y\in VB\left((t_0,t_1);L_2^{n_1}(x_0,x_1)\right),\ z\in VB\left((x_0,x_1):L_2^{n_2}(t_0,t_1)\right)\right\}$$

with the norm

$$\|h\|_{W(Q)}=\|y\|_{VB\left((t_0,t_1);L_2^{n_1}(x_0,x_1)\right)}+\|z\|_{VB\left((x_0,x_1);L_2^{n_2}(t_0,t_1)\right)}.$$

The vector-function $(y(t,x),z(t,x))\in W(Q)$ satisfying almost everywhere in Q the integral system

$$y(t,x)=\varphi^1(x,\xi(x))+\int_{t_0}^{t}f^1(\tau,x,y(\tau,x),z(\tau,x),\omega(\tau,x))d\tau+\int_{t_0}^{t}b^1(\tau,x)du(\tau),\qquad(1.3)$$

is called the weak solution of the problem (1.1), (1.2) at the control $(\omega(t,x),\xi(x),\eta(t),u(t),v(x))\in U_\partial$, where the last integrals are understood in the meaning of Lebesque-Stielties.

Let's consider the minimization of the functional

$$I=\iint_Q\Phi^0(t,x,y(t,x),z(t,x),\omega(t,x))dxdt+\int_{t_0}^{t_1}\Phi^1(t,z(t,x_1),u(t),\eta(t))dt+$$
$$+\int_{x_0}^{x_1}\Phi^2(x,y(t_1,x),v(x),\xi(x))dx,\qquad(1.4)$$

given on the set of the weak solutions of the problem (1.1), (1.2).

Let the next conditions be fulfilled:

1. The functions $f^i(t,x,y,z,\omega)$, $i=1,2$ are continuous together with the partial derivatives f_y^i,f_z^i,f_ω^i by $(y,z,\omega)\in R_{n_1+n_2+r}$ for almost everywhere $(t,x)\in Q$ measurable by (t,x) for all (y,z,ω) and satisfy the condition-Lipschitz

$$\left\|f^i(t,x,\tilde y,\tilde z,\tilde\omega)-f^i(t,x,y,z,\omega)\right\|\le M_i\left\{\|\tilde y-y\|+\|\tilde z-z\|+\|\tilde\omega-\omega\|\right\}\qquad(1.5)$$

for $(y,z,\omega)\in R_{n_1+n_2+r}$ and for almost everywhere $(t,x)\in Q$. Besides the derivatives f_y^i,f_z^i,f_ω^i locally satisfy the Lipschitz condition by (y,z,ω), $f^i(t,x,0,0,0)\in L_2^{n_i}(Q)$, $i=1,2$.

2. $b^1(t,x)\in C^1\left([t_0,t_1];L_2^{n_1\times m_1}(x_0,x_1)\right)$, $b^2(t,x)\in C^1\left([x_0,x_1];L_2^{n_2\times m_2}(t_0,t_1)\right)$ where $C^1\left([a,b];L_2^{n\times m}(c,d)\right)$ is a space of continuous differentiable mappings $[a,b]\to L_2^{n\times m}(c,d)$.

3. The function $\varphi^1(x,\xi)\left(\varphi^2(t,\eta)\right)$ is continuous together with the partial derivatives $\varphi_\xi^1(x,\xi)\left(\varphi_\eta^2(t,\eta)\right)$ by $\xi\in R_{m_2}$ for almost everywhere $x\in(x_0,x_1)$ (by $\eta\in R_{m_1}$ for almost everywhere $t\in(t_0,t_1)$) and measurable by x for all ξ (is measurable by t for

all n) and locally satisfies the Lipschitz condition by ξ (by η). Besides $\varphi^1(x,0)\in L_2^{n_1}(x_0,x_1)$ $\left(\varphi^2(t,0)\in L_2^{n_2}(t_0,t_1)\right)$.

4. The functions $\Phi^0(t,x,y,z,\omega),\Phi^1(t,z,u,\eta),\Phi^2(x,y,v,\xi)$ are continuous together with the partial derivatives $\Phi^0_y,\Phi^0_z,\Phi^0_\omega,\Phi^1_z,\Phi^1_u,\Phi^1_\eta,\Phi^2_y,\Phi^2_v,\Phi^2_\xi$ by $(y,z,\omega)\in R_{n_1+n_2+r}$, $(z,u,\eta)\in R_{n_2+m_1+r_1}$, $(y,v,\xi)\in R_{n_1+m_2+r_2}$ for almost all $(t,x)\in Q$, almost all $t\in[t_0,t_1]$, almost all $x\in[x_0,x_1]$ and measurable by $(t,x)\in Q,\ t\in[t_0,t_1],x\in[x_0,x_1]$ for all $(y,z,\omega),(z,u,\eta),(y,v,\xi)$ and locally satisfy the Lipschitz condition by $(y,z,\omega),(z,u,\eta),(y,v,\xi)$ respectively, $\Phi^0(t,x,0,0,0)\in L_2(Q)$, $\Phi^1(t,0,0,0)\in L_2(t_0,t_1)$, $\Phi^2(x,0,0,0)\in L_2(x_0,x_1)$.

At these conditions analogously [5] we can prove that the unique weak solution of the problem (1.1)-(1.2) exists at fixed controls and the solution is correct at variation of controls.

2. The conjugate system. Let's introduce the function

$$H(t,x,y,z,p,q,\omega)=pf^1(t,x,y,z,\omega)+qf^2(t,x,y,z,\omega)-\Phi^0(t,x,y,z,\omega) \qquad (2.1)$$

and let's consider the conjugate system

$$\begin{aligned}&\frac{\partial p}{\partial t}+H_y(t,x,y,z,p,q,\omega)=0,\\ &\frac{\partial q}{\partial x}+H_z(t,x,y,z,p,q,\omega)=0,\ (t,x)\in Q,\end{aligned} \qquad (2.2)$$

satisfying the conditions

$$p(t_1,x)+\Phi^2_y(x,y(t_1,x),v(x),\xi(x))=0,\ x[x_0,x_1], \qquad (2.3)$$

Here $p,q,H_y,H_z,\Phi^1_z,\Phi^2_y$ are assumed as a vector rows. Let's denote that the conjugate system doesn't contain the generalized effect, its solution $p(t,x)$ is absolutely continuous by $t\in[t_0,t_1]$ for every $x\in[x_0,x_1]$, and $q(t,x)$ is absolutely continuous by $x\in[x_0,x_1]$ for every $t\in[t_0,t_1]$.

3. The increment of functional. Let $(y(t,x),z(t,x))$ be a weak solution of the problem (1.1), (1.2) at control $(\omega(t,x),\xi(x),\eta(t),u(t),v(x))\in U_\partial$ and $(\tilde y(t,x),\tilde z(t,x))$ is a weak solution those problems at the control $\left(\tilde\omega(t,x),\tilde\xi(x),\tilde\eta(t),\tilde u(t),\tilde v(x)\right)\in U_\partial$. Then $\delta y(t,x)=\tilde y(t,x)-y(t,x),\delta z(t,x)=\tilde z(t,x)-z(t,x)$ are weak solutions of the system:

$$\begin{aligned}&\frac{\partial(\delta y)}{\partial t}=\Delta_{\tilde y,\tilde z,\tilde\omega}f^1(t,x,y(t,x),z(t,x),\omega(t,x))+b^1(t,x)\frac{d(\delta u(t))}{dt},\\ &\frac{\partial(\delta z)}{\partial x}=\Delta_{\tilde y,\tilde z,\tilde\omega}f^2(t,x,y(t,x),z(t,x),\omega(t,x))+b^2(t,x)\frac{d(\delta v(t))}{dx},\ (t,x)\in Q\end{aligned} \qquad (3.1)$$

with the initial conditions

$$\begin{aligned}&\delta y(t_0,x)=\Delta_{\tilde\xi}\varphi^1(x,\xi(x)),\ \ x\in[x_0,x_1],\\ &\delta z(t,x_0)=\Delta_{\tilde\eta}\varphi^2(t,\eta(t)),\ \ t\in[t_0,t_1],\end{aligned} \qquad (3.2)$$

where

$$\delta u(t)=\tilde{u}(t)-u(t),\ \delta v(x)=\tilde{v}(x)-v(x);\ \Delta_{\tilde{y},\tilde{z},\tilde{\omega}}f^i(t,x,y(t,x),z(t,x),\omega(t,x))=$$
$$=f^i(t,x,\tilde{y}(t,x),\tilde{z}(t,x),\tilde{\omega}(t,x))-f^i(t,x,y(t,x),z(t,x),\omega(t,x)),\quad i=1,2, \tag{3.3}$$
$$\Delta_\xi\varphi^1(x,\xi(x))=\varphi^1(x,\tilde{\xi}(x))-\varphi^i(x,\xi(x)),\ \Delta_\eta\varphi^2(t,\eta(t))=\varphi^2(t,\tilde{\eta}(t))-\varphi^2(t,\eta(t)).$$

According to the definition of a weak solution of the problem (3.1), (3.2) we have:

$$\delta y(t,x)=\Delta_{\tilde{\xi}}\varphi^1(x,\xi(x))+$$
$$+\int_{t_0}^{t}\Delta_{\tilde{y},\tilde{v},\tilde{\omega}}f^1(\tau,x,y(\tau,x),z(\tau,x),\omega(\tau,x))d\tau+\int_{t_0}^{t}b^1(\tau,x)d(\delta u(\tau)), \tag{3.4}$$

$$\delta z(t,x)=\Delta_{\tilde{\eta}}\varphi^2(t,\eta(t))+$$
$$+\int_{x_0}^{x}\Delta_{\tilde{y},\tilde{z},\tilde{\omega}}f^2(t,s,y(t,s),z(t,s),\omega(t,s))ds+\int_{x_0}^{x}b^2(t,s)d(\delta v(s)). \tag{3.5}$$

Using the notation (2.1) and the system (3.1) we obtain the expression for the increment of the functional (1.4) at the variation of control

$$\delta I=\iint_Q\left\{p(t,x)\frac{\partial(\delta y)}{\partial t}+q(t,x)\frac{\partial(\delta z)}{\partial x}-\Delta_{\tilde{y},\tilde{z},\tilde{\omega}}H(t,x,y(t,x),z(t,x),p(t,x),q(t,x),\omega(t,x))\right\}dxdt+$$
$$+\int_{t_0}^{t_1}\Delta_{\tilde{z},\tilde{u},\tilde{\eta}}\Phi^1(t,z(t,x_1),u(t),\eta(t))dt+\int_{x_0}^{x_1}\Delta_{\tilde{y},\tilde{v},\tilde{\xi}}\Phi^2(x,y(t_1,x),v(x),\xi(x))dx-$$
$$-\int_{t_0}^{t_1}\left(\int_{x_0}^{x_1}p(t,x)b^1(t,x)dx\right)d(\delta u(t))-\int_{x_0}^{x_1}\left(\int_{t_0}^{t_1}q(t,x)b^2(t,x)dt\right)d(\delta v(x)). \tag{3.6}$$

Applying Lagrange theorem to the difference and using continuity of partial derivatives we'll get:

$$\Delta_{\tilde{y},\tilde{z},\tilde{\omega}}H(t,x,y(t,x),z(t,x),p(t,x),q(t,x),\omega(t,x))=\mathcal{H}_y(t,x)\delta y(t,x)+\mathcal{H}_z(t,x)\delta z(t,x)+$$
$$+\mathcal{H}_\omega(t,x)\delta\omega(t,x)+\gamma_1\delta y(t,x)+\gamma_2\delta z(t,x)+\gamma_3\delta\omega(t,x);$$
$$\Delta_{\tilde{z},\tilde{u},\tilde{\eta}}\Phi^1(t,z(t,x_1),u(t),\eta(t))=\Phi^1_z(t,z(t,x_1),u(t),\eta(t))\delta z(t,x_1)+$$
$$+\Phi^1_u(t,z(t,x_1),u(t),\eta(t))\delta u(t)+\Phi^1_\eta(t,z(t,x_1),u(t),\eta(t))\delta\eta(t)+$$
$$+\gamma_4\delta z(t,x_1)+\gamma_5\delta u(t)+\gamma_6\delta\eta(t),$$
$$\Delta_{\tilde{y},\tilde{v},\tilde{\xi}}\Phi^2(x,y(t_1,x),v(x),\xi(x))=\Phi^2_y(x,y(t_1,x),v(x),\xi(x))\delta y(t_1,x)+$$
$$+\Phi^2_v(x,y(t_1,x),v(x),\xi(x))\delta v(x)+\Phi^2_\xi(x,y(t_1,x),v(x),\xi(x))\delta\xi(x)+$$
$$+\gamma_7\delta y(t_1,x)+\gamma_8\delta v(x)+\gamma_9\delta\xi(x),$$
$$\Delta_{\tilde{\xi}}\varphi^1(x,\xi(x))=\varphi^1_\xi(x,\xi(x))\delta\xi(x)+\gamma_{10}\delta\xi(x),$$
$$\Delta_\eta\varphi^2(t,\eta(t))=\varphi^2_\eta(t,\eta(t))\delta\eta(t)+\gamma_{11}\delta\eta(t), \tag{3.7}$$

where $\mathcal{H}_y(t,x)=H_y(t,x,y(t,x),z(t,x),p(t,x),q(t,x),\omega(t,x)),\ldots$ are the values of γ_i tends to zero, when $\delta y,\delta z,\delta\omega,\delta u,\delta v,\delta\xi,\delta\eta$ tends to zero, $i=1,2,\ldots,11$.

From (3.6), using the expression (3.7) and taking into account that $(\delta y(t,x),\delta z(t,x))$ and $(p(t,x),q(t,x))$ are the weak solutions of the problem (3.1), (3.2) and the conjugate problem (2.2), (2.3) we can write the increment of functional at variation of control in the form:

$$\delta I = -\iint_Q \mathcal{H}_\omega(t,x)\delta\omega(t,x)dxdt + \int_{t_0}^{t_1}\{\Phi_u^1(t,z(t,x_1),u(t),\eta(t))\delta u(t) +$$

$$+\left[\Phi_\eta^1(t,z(t,x_1),u(t),\eta(t)) - q(t,x_0)\varphi_\eta^2(t,\eta(t))\right]\delta\eta(t)\}dt + \int_{x_0}^{x_1}\{\Phi_v^2(x,y(t_1,x),v(x),\xi(x))\delta v(x) +$$

$$+\left[\Phi_\xi^2(x,y(t_1,x),v(x),\xi(x)) - p(t_0,x)\varphi_\xi^1(x,\xi(x))\right]\delta\xi(x)\}dx -$$

$$-\int_{t_0}^{t_1}\left(\int_{x_0}^{x_1} p(t,x)b^1(t,x)dx\right)d(\delta u(t)) - \int_{x_0}^{x_1}\left(\int_{t_0}^{t_1} q(t,x)b^2(t,x)dt\right)d(\delta v(x)) + \eta, \qquad (3.8)$$

where

$$\eta = -\iint_Q(\gamma_1\delta y(t,x) + \gamma_2\delta z(t,x) + \gamma_3\delta\omega(t,x))dxdt +$$

$$+\int_{t_0}^{t_1}[\gamma_4\delta z(t,x_1) + \gamma_5\delta u(t) + (\gamma_6 - q(t,x_0)\gamma_{11})\delta\eta(t)]dt + \qquad (3.9)$$

$$+\int_{x_0}^{x_1}[\gamma_7\delta y(t_1,x) + \gamma_8\delta v(x) + (\gamma_9 - p(t_0,x)\gamma_{10})\delta\xi(x)]dx.$$

Integrating by part from (3.8) and using the system (2.2) we obtain

$$\delta I = -\iint_Q \mathcal{H}_\omega(t,x)\delta\omega(t,x)dxdt -$$

$$-\int_{t_0}^{t_1}\left\{\int_{x_0}^{x_1}\left[\mathcal{H}_y(t,x)b^1(t,x) - p(t,x)b_t^1(t,x)\right]dx - \Phi_u^1(t,z(t,x_1),u(t),\eta(t))\right\}\delta u(t)dt -$$

$$-\int_{x_0}^{x_1}\left\{\int_{t_0}^{t_1}\left[\mathcal{H}_z(t,x)b^2(t,x) - q(t,x)b_x^2(t,x)\right]dt - \Phi_v^1(x,y(t_1,x),v(x),\xi(x))\right\}\delta v(x)dx -$$

$$-\int_{t_0}^{t_1}\left(q(t,x_0)\varphi_\eta^2(t,\eta(t)) - \Phi_\eta^1(t,z(t,x_1),u(t),\eta(t))\right)\delta\eta(t)dt -$$

$$-\int_{x_0}^{x_1}\left(p(t_0,x)\varphi_\xi^1(x,\xi(x)) - \Phi_\xi^2(x,y(t_1,x),v(x),\xi(x))\right)\delta\xi(x)dx -$$

$$-\int_{x_0}^{x_1} p(t,x)b^1(t,x)\delta u(t)dx\Big|_{t_0}^{t_1} - \int_{t_0}^{t_1} q(t,x)b^2(t,x)\delta v(x)dt\Big|_{x_0}^{x_1} + \eta. \qquad (3.10)$$

Let's introduce the functions

$$h(t) = \int_{x_0}^{x_1}\left[\mathcal{H}_y(t,x)b^1(t,x) - p(t,x)b_t^1(t,x)\right]dx - \Phi_u^1(t,z(t,x_1),u(t),\eta(t)),$$

$$g(x) = \int_{t_0}^{t_1}\left[\mathcal{H}_z(t,x)b^2(t,x) - q(t,x)b_x^2(t,x)\right]dt - \Phi_v^2(x,y(t_1,x),v(x),\xi(x)),$$

$$r(t) = q(t,x_0)\varphi_\eta^2(t,\eta(t)) - \Phi_\eta^1(t,z(t,x_1),u(t),\eta(t)),$$
$$\lambda(x) = p(t_0,x)\varphi_\xi^1(x,\xi(x)) - \Phi_\xi^2(x,y(t_1,x),v(x),\xi(x)). \quad (3.11)$$

Using these notations from (3.10) we finally get the expression for the increment of the functional (1.4) at the variation of the control:

$$\delta I = -\iint_Q \mathcal{H}_\omega(t,x)\delta\omega(t,x)dxdt - \int_{t_0}^{t_1}(h(t)\delta u(t) + r(t)\delta\eta(t))dt -$$
$$-\int_{x_0}^{x_1}(g(x)\delta v(x) + \lambda(x)\delta\xi(x))dx - \int_{t_0}^{t_1} q(t,x)b^2(t,x)\delta v(x)dt\Big|_{x_0}^{x_1} - \quad (3.12)$$
$$-\int_{x_0}^{x_1} p(t,x)b^1(t,x)\delta u(t)dx\Big|_{t_0}^{t_1} + \eta .$$

4. The estimation of remainder term. To get the remainder term η in the formula (3.9) first of all we'll get the estimation for the solution $(\delta y(t,x),\delta z(t,x))$ of the problem (3.1), (3.2). From (3.4) and (3.5) we'll get:

$$\|\delta y(t,\cdot)\|_{L_2^{m_1}(x_0,x)} \le \left\|\Delta_{\tilde\xi}\varphi^1\right\|_{L_2^{m_1}(x_0,x_1)} + \left\|\int_{t_0}^{t}\Delta_{\tilde y,\tilde z,\tilde\omega} f^1(\tau,\ldots)d\tau\right\|_{L_2^{m_1}(x_0,x)} +$$
$$+\int_{t_0}^{t}\left\|b^1(t,\cdot)\right\|_{L_2^{n_1\times m_1}(x_0,x_1)}\|d(\delta u(t))\|, \quad (4.1)$$

$$\|\delta z(\cdot,x)\|_{L_2^{n_2}(t_0,t)} \le \left\|\Delta_{\tilde\eta}\varphi^2\right\|_{L_2^{n_2}(t_0,t_1)} + \left\|\int_{tx_0}^{x}\Delta_{\tilde y,\tilde z,\tilde\omega} f^2(\cdot,s,\ldots)ds\right\|_{L_2^{n_2}(t_0,t)} +$$
$$+\int_{x_0}^{x}\left\|b^2(\cdot,x)\right\|_{L_2^{n_2\times m_2}(t_0,t_1)}\|d(\delta\, v(x))\|. \quad (4.2)$$

From here using the Holder inequality [9] and the inequality $(a+b+c)^2 \le 3(a^2+b^2+c^2)$ we'll get:

$$\|\delta y(t,\cdot)\|^2_{L_2^{m_1}(x_0,x)} \le$$
$$\le 3\left\{\left\|\Delta_{\tilde\xi}\varphi^1\right\|^2_{L_2^{m_1}(x_0,x_1)} + (t-t_0)\int_{x_0}^{x}\int_{t_0}^{t}\left\|\Delta_{\tilde y,\tilde z,\tilde\omega} f^1(\tau,s,y(\tau,s),z(\tau,s),\omega(\tau,s))\right\|^2 d\tau ds + \right.$$
$$\left. + \left(\int_{t_0}^{t_1}\left\|b^1(t,\cdot)\right\|_{L_2^{n_1\times m_1}(x_0,x_1)}\|d(\delta u(t))\|\right)^2\right\},$$

$$\|\delta z(\cdot,x)\|^2_{L_2^{n_2}(t_0,t)} \le$$
$$\le 3\left\{\left\|\Delta_{\tilde\eta}\varphi^2\right\|^2_{L_2^{n_2}(t_0,t_1)} + (x-x_0)\int_{x_0}^{x}\int_{t_0}^{t}\left\|\Delta_{\tilde y,\tilde z,\tilde\omega} f^2(\tau,s,y(\tau,s),z(\tau,s),\omega(\tau,s))\right\|^2 d\tau ds + \right.$$

$$+\left(\int_{x_0}^{x_1}\left\|b^2(\cdot,x)\right\|_{L_2^{n_2\times m_2}(t_0,t_1)}\left\|d(\delta v(x))\right\|\right)^2\Bigg\}. \qquad (4.3)$$

Let's introduce the functions

$$Y(t,x)=\left\|\delta y(t,\cdot)\right\|^2_{L_2^{n_1}(x_0,x)},\ Z(t,x)=\left\|\delta z(\cdot,x)\right\|^2_{L_2^{n_2}(t_0,t)} \qquad (4.4)$$

and using the Lipschitz condition (1.5) from (4.3) we'll obtain

$$Y(t,x)\le 9M_1^2(t_1-t_0)\int_{t_0}^{t}Y(\tau,x)d\tau+9M_1^2(t_1-t_0)\int_{x_0}^{x}Z(t,s)ds+h_1(t_1,x_1), \qquad (4.5)$$

$$Z(t,x)\le 9M_2^2(x_1-x_0)\int_{x_0}^{x}Z(t,s)ds+9M_2^2(x_1-x_0)\int_{t_0}^{t}Y(\tau,x)d\tau+h_2(t_1,x_1), \qquad (4.6)$$

where

$$h_1(t_1,x_1)=3\Bigg\{\left\|\Delta_{\tilde{\xi}}\varphi^1\right\|^2_{L_2^{n_1}(x_0,x_1)}+3M_1^2(t_1-t_0)\iint_Q\left\|\delta\omega(t,x)\right\|^2dxdt+$$
$$+\left(\int_{t_0}^{t_1}\left\|b^1(t,\cdot)\right\|_{L_2^{n_1\times m_1}(x_0,x_1)}\left\|d(\delta u(t))\right\|\right)^2\Bigg\},$$

$$h_2(t_1,x_1)=3\Bigg\{\left\|\Delta_{\tilde{\eta}}\varphi^2\right\|^2_{L_2^{n_2}(t_0,t_1)}+3M_2^2(x_1-x_0)\iint_Q\left\|\delta\omega(t,x)\right\|^2dxdt+$$
$$+\left(\int_{x_0}^{x_1}\left\|b^2(\cdot,x)\right\|_{L_2^{n_2\times m_2}(t_0,t_1)}\left\|d(\delta v(x))\right\|\right)^2\Bigg\}.$$

Applying to (4.5) for fixed x the Cronwall lemma for $Y(t,x)$ as function of t [10] we'll obtain:

$$Y(t,x)\le\left(h_1(t_1,x_1)+9M_1^2(t_1-t_0)\int_{x_0}^{x}Z(t,s)ds\right)\exp\left(9M_1^2(t_1-t_0)^2\right),\ (t,x)\in Q. \qquad (4.7)$$

Analogously from (4.6) we have:

$$Z(t,x)\le\left(h_2(t_1,x_1)+9M_2^2(x_1-x_0)\int_{t_0}^{t}Y(\tau,x)d\tau\right)\exp\left(9M_2^2(x_1-x_0)^2\right),\ (t,x)\in Q. \qquad (4.8)$$

Substituting (4.8) in (4.7) we have:

$$Y(t,x)\le h_3(t_1,x_1)+M_3\int_{t_0}^{t}\int_{x_0}^{x}Y(\tau,s)dsd\tau\ , \qquad (4.9)$$

where

$$M_3=81M_1^2M_2^2(t_1-t_0)(x_1-x_0)\exp\left(9M_2^2(x_1-x_0)^2\right),$$

$$h_3(t_1,x_1)=h_1(t_1,x_1)\exp\left(9M_1^2(t_1-t_0)^2\right)+$$
$$+9M_1^2(x_1-x_0)\exp\left(9M_2^2(x_1-x_0)^2\right)\int_{x_0}^{x_1}h_2(t_1,s)ds.$$

In the inequality (4.9) again let's apply the Crownwall lemma for $Y(t,x)$, but now as a function of two arguments (t,x) [10] we'll obtain:

$$Y(t,x)\le h_3(t_1,x_1)\exp(M_3(t_1-t_0)(x_1-x_0)),\ (t,x)\in Q. \tag{4.10}$$

By analogous substituting (4.7) into (4.8) the inequality:

$$Z(t,x)\le h_4(t_1,x_1)\exp(M_4(t_1-t_0)(x_1-x_0)),\ (t,x)\in Q, \tag{4.11}$$

where

$$M_4=81M_1^2M_2^2(t_1-t_0)(x_1-x_0)\exp\left(9M_1^2(t_1-t_0)^2\right),$$
$$h_4(t_1,x_1)=h_2(t_1,x_1)\exp\left(9M_2^2(x_1-x_0)^2\right)+$$
$$+9M_2^2(t_1-t_0)\exp\left(9M_1^2(t_1-t_0)\right)\int_{t_0}^{t_1}h_1(t,x_1)dt$$

is proved.

From (4.10), (4.11) taking into account the notation (4.4) we have:

$$\begin{aligned}&\|\delta y(t,\cdot)\|_{L_2^{n_1}(x_0,x_1)}\le\left(h_3(t_1,x_1)\exp(M_3(x_1-x_0)(t_1-t_0))\right)^{\frac{1}{2}},\ t\in[t_0,t_1],\\&\|\delta z(\cdot,x)\|_{L_2^{n_2}(t_0,t_1)}\le\left(h_4(t_1,x_1)\exp(M_4(x_1-x_0)(t_1-t_0))\right)^{\frac{1}{2}},\ x\in[x_0,x_1].\end{aligned} \tag{4.12}$$

From the equality (3.4) at subdivision of the segment $[t_0,t_1]$ by the points $t_0=\tau_0<\tau_1<\tau_2<...<\tau_n=t_1$, we'll get:

$$\sum_{i=0}^{n-1}\|\delta y(\tau_{i+1},\cdot)-\delta y(\tau,\cdot)\|_{L_2^{n_1}(x_0,x_1)}\le\left\|\int_{t_0}^{t_1}\left\|\Delta_{\tilde y,\tilde z,\tilde\omega}f^1(t,...)\right\|dt\right\|_{L_2(x_0,x_1)}+$$
$$+\int_{t_0}^{t_1}\left\|b^1(t,\cdot)\right\|_{L_2^{n_1\times m_1}(x_0,x_1)}\|d(\delta u(t))\|,$$
$$\|\delta y(t_0,\cdot)\|_{L_2^{n_1}(x_0,x_1)}=\left\|\Delta_{\tilde\xi}\varphi^1\right\|_{L_2^{n_1}(x_0,x_1)}.$$

From here using the Lipschitz conditions for the function $f^i(t,x,y,z,\omega),\varphi^1(x,v),\varphi^2(t,u)$ and inequality (4.12) we have:

$$\begin{aligned}&\|\delta y\|_{VB((t_0,t_1);L_2^{n_1}(x_0,x_1))}=\\&=O\left(\|\delta\omega\|_{L_2^r(Q)}+\|\delta\xi\|_{L_2^{n_1}(x_0,x_1)}+\|\delta\eta\|_{L_2^{n_2}(t_0,t_1)}+\|\delta u\|_{VB_{m_1}(t_0,t_1)}+\|\delta v\|_{VB_{m_2}(x_0,x_1)}\right).\end{aligned} \tag{4.13}$$

Analogously from (3.5) we can get that

$$\begin{aligned}&\|\delta z\|_{VB((x_0,x_1);L_2^{n_2}(t_0,t_1))}=\\&=O\left(\|\delta\omega\|_{L_2^r(Q)}+\|\delta\xi\|_{L_2^{n_1}(x_0,x_1)}+\|\delta\eta\|_{L_2^{n_2}(t_0,t_1)}+\|\delta u\|_{VB_{m_1}(t_0,t_1)}+\|\delta v\|_{VB_{m_2}(x_0,x_1)}\right).\end{aligned} \tag{4.14}$$

From (4.13) and (4.14) it follows that the solution of the problem (1.1), (1.2) is correct at the variation of the controls. From the obtained estimation (4.13) and (4,14) using the explicit form (3.9) of the remainder term η of functional, we can get the estimation:

$$\eta = o\left(\|\delta\omega\|_{L_2^r(Q)} + \|\delta\xi\|_{L_2^{r_1}(x_0,x_1)} + \|\delta\eta\|_{L_2^{r_2}(t_0,t_1)} + \|\delta u\|_{VB_{m_1}(t_0,t_1)} + \|\delta v\|_{VB_{m_2}(x_0,x_1)} \right). \quad (4.15)$$

5. The necessary conditions. The necessary conditions for the given problems we'll formulate as theorem

Theorem 1. *Let* $(\omega(t,x), \xi(x), \eta(t), u(t), v(x)) \in U_\partial$ *be optimal controls in the problem (1.1), (1.2), (1.4), and* $(y(t,x), z(t,x))$ *and* $(p(t,x), q(t,x))$ *be the solutions of the problems (1.1), (1.2) and (2.2),(2.3) corresponding to these controls. Then the next necessary conditions*

$$\max_{\tilde\omega \in \Omega_0(\cdot)} \iint_Q \mathcal{H}_\omega(t,x)\tilde\omega(t,x)dxdt = \iint_Q \mathcal{H}_\omega(t,x)\omega(t,x)dxdt, \quad (5.1)$$

$$\max_{\tilde\xi \in \Omega_1(\cdot)} \int_{x_0}^{x_1} \lambda(x)\tilde\xi(x)dx = \int_{x_0}^{x_1} \lambda(x)\xi(x)dx, \quad \max_{\tilde\eta \in \Omega_2(\cdot)} \int_{t_0}^{t_1} r(t)\tilde\eta(t)dt = \int_{t_0}^{t_1} r(t)\eta(t)dt, \quad (5.2)$$

$$\max_{\tilde u \in U(\cdot)} \int_{t_0}^{t_1} h(t)\tilde u(t)dt = \int_{t_0}^{t_1} h(t)u(t)dt, \quad \max_{\tilde v \in U(\cdot)} \int_{x_0}^{x_1} g(x)\tilde v(x)dx = \int_{x_0}^{x_1} g(x)v(x)dx, \quad (5.3)$$

$$\max_{\tilde u \in U(\cdot)} (-1)^{i+1} \int_{x_0}^{x_1} p(t_i,x)b^1(t_i,x)\tilde u(t_i)dx = (-1)^{i+1} \int_{x_0}^{x_1} p(t_i,x)b^1(t_i,x)u(t_i)dx, \quad (5.4)$$

$$\max_{\tilde v \in V(\cdot)} (-1)^{i+1} \int_{t_0}^{t_1} q(t,x_i)b^2(t,x_i)\tilde v(x_i)dt = (-1)^{i+1} \int_{t_0}^{t_1} q(t,x_i)b^2(t,x_i)v(x_i)dt, \ i = 0,1. \quad (5.5)$$

are fulfilled.

Proof. By virtue of optimality of the control $(\omega(t,x), \xi(x), \eta(t), u(t), v(x)) \in U_\partial$ for any controls $(\tilde\omega(t,x), \tilde\xi(x), \tilde\eta(t), \tilde u(t), \tilde v(x)) \in U_\partial$ the inequalities

$$\delta I = -\iint_Q \mathcal{H}_\omega(t,x)\delta\omega(t,x)dxdt - \int_{t_0}^{t_1} (h(t)\delta u(t) + r(t)\delta\eta(t))dt -$$
$$- \int_{x_0}^{x_1} (g(x)\delta v(x) + \lambda(x)\delta\xi(x))dx - \int_{t_0}^{t_1} q(t,x)b^2(t,x)\delta v(x)dt\Big|_{x_0}^{x_1} - \quad (5.6)$$
$$- \int_{x_0}^{x_1} p(t,x)b^1(t,x)\delta u(t)dx\Big|_{t_0}^{t_1} + \eta \geq 0,$$

where

$$\delta\omega(t,x) = \tilde\omega(t,x) - \omega(t,x), \quad \delta\xi(x) = \tilde\xi(x) - \xi(x), \ \delta\eta(t) = \tilde\eta(t) - \eta(t),$$
$$\delta u(t) = \tilde u(t) - u(t), \ \delta v(x) = \tilde v(x) - v(x)$$

are fulfilled.

Let's suppose that the condition (5.1) isn't fulfilled. Then exist $\tilde\omega(t,x) \in \Omega_0(\cdot)$ such that for $\delta\omega(t,x) = \tilde\omega(t,x) - \omega(t,x)$ the inequality

$$\iint_Q \mathcal{H}_\omega(t,x)\delta\omega(t,x)dxdt > 0 \quad (5.7)$$

is fulfilled.

From the convexity of the set $\Omega_0(\cdot)$ it follows that for $0<\varepsilon\le 1$ the control $\omega_\varepsilon(t,x)=\omega(t,x)+\varepsilon\delta\omega(t,x)\in\Omega_0(\cdot)$. For the control $(\omega_\varepsilon(t,x),\xi(x),\eta(t),u(t),v(x))\in U_\partial$ from (3.12), (4.15) we'll get:

$$\delta I=-\varepsilon\iint_Q \mathcal{H}_\omega(t,x)\delta\omega(t,x)dxdt+o(\varepsilon).$$

Hence taking into account the condition (5.7), at sufficiently small number $\varepsilon>0$ we get that $\delta I<0$. But this contradicts to the condition (5.6). Consequently the condition (5.1) is fulfilled. Analogously we can prove the conditions (5.2), (5.3).

Let's prove the condition (5.4) for $i=1$. If this condition isn't fulfilled then there is the control $\tilde{u}(t)\in U(\cdot)$ such that for the functions $\delta u(t)=\tilde{u}(t)-u(t)$ the inequality

$$\left(\int_{x_0}^{x_1} p(t_1,x)b^1(t_1,x)dx\right)\delta u(t_1)>0 \tag{5.8}$$

is fulfilled.

Then by the virtue of convexity of the set $U(\cdot)$ for $0<\varepsilon\le 1,\ t_0<t_1-\varepsilon$ the functions $u_\varepsilon(t)=u(t)+\varepsilon\delta u(t)$ for $t\in(t_1-\varepsilon,t_1]$ and $u_\varepsilon(t)=u(t)$ for $t\in[t_0,t_1-\varepsilon]$ are admitted.

For the control $(\omega(t,x),\xi(x),\eta(t),u_\varepsilon(t),v(x))\in U_\partial$ from (3.12), (4.15) we get:

$$\delta I=-\varepsilon\int_{t_1-\varepsilon}^{t_1} h(t)\delta u(t)dt-\varepsilon\left(\int_{x_0}^{x_1} p(t_1,x)b^1(t_1,x)dx\right)\delta u(t_1)+o(\varepsilon).$$

This equality we can write in the next form:

$$\delta I=-\varepsilon\left(\int_{x_0}^{x_1} p(t_1,x)b^1(t_1,x)dx\right)\delta u(t_1)+o(\varepsilon).$$

From here by virtue of the condition (5.8) for sufficiently small number $\varepsilon>0$ we get that $\delta I<0$. This contradicts to the condition (5.6) i.e. the condition (5.4) for $i=1$ is fulfilled. The correctness of other conditions is analogously considered.

The theorem is proved.

6. The differential maximum principle. For the class of admissible controls U_∂ we take the functions

$$(\omega(t,x),\xi(x),\eta(t),u(t),v(x))\in L_2^r(Q)\times L_2^{r_1}(x_0,x_1)\times L_2^{r_2}(t_0,t_1)\times VB_{m_1}(t_0,t_1)\times VB_{m_2}(x_0,x_1),$$

satisfying the limitations $\omega(t,x)\in\Omega_0$ almost everywhere $(t,x)\in Q$, $\xi(x)\in\Omega_1$, almost everywhere $x\in[x_0,x_1]$, $\eta(t)\in\Omega_2$, almost everywhere $t\in[t_0,t_1]$, $u(t)\in U, t\in[t_0,t_1]$, $v(x)\in V$, $x\in(x_0,x_1)$, where $\Omega_0,\Omega_1,\Omega_2,U,V$ are convex sets in $R_r,R_{r_1},R_{r_2},R_{m_1},R_{m_2}$.

Theorem 2. *Let* $(\omega(t,x),\xi(x),\eta(t),u(t),v(x))\in U_\partial$ *be an optimal control in problem (1.1), (1.2), (1.4), and* $(y(t,x),z(t,x))$ *and* $(p(t,x),q(t,x))$ *are the solutions of the problem (1.1), (1.2) and (2.2), (2.3) corresponding to these controls. Then the next conditions:*

$$\max_{\omega\in\Omega_0}\mathcal{H}_\omega(t,x)\omega=\mathcal{H}_\omega(t,x)\omega(t,x),\ \textit{almost everywhere}\ (t,x)\in Q, \tag{6.1}$$

$$\max_{\xi\in\Omega_1}\lambda(x)\xi=\lambda(x)\xi(x),\ \textit{almost everywhere}\ x\in[x_0,x_1],\ \max_{\eta\in\Omega_2}r(t)\eta=r(t)\eta(t)$$
$$\textit{almost everywhere}\ t\in[t_0,t_1] \tag{6.2}$$

$$\max_{u\in U} h(t)u = h(t)u(t),\ t\in[t_0,t_1],\ \max_{v\in V} g(x)v = g(x)v(x),\ x\in[x_0,x_1], \qquad (6.3)$$

$$\max_{u\in U}(-1)^{i+1}\int_{x_0}^{x_1} p(t_i,x)b^1(t_i,x)u\,dx = (-1)^{i+1}\int_{x_0}^{x_1} p(t_i,x)b^1(t_i,x)u(t_i)dx, \qquad (6.4)$$

$$\max_{v\in V}(-1)^{i+1}\int_{t_0}^{t_1} q(t,x_i)b^2(t,x_i)v\,dt = (-1)^{i+1}\int_{t_0}^{t_1} q(t,x_i)b^2(t,x_i)v(x_i)dt,\quad i=0,1 \qquad (6.5)$$

are fulfilled.

Proof. For proving the condition (6.1) by the Q_E let's denote the totality of the points Q which are the points of Lebesque for the functions $\omega(t,x)$ and $\mathcal{H}_\omega(t,x)$. Then evident the $mes(Q-Q_E)=0$ and for the point $(\theta,\rho)\in Q_E$:

$$\mathcal{H}_\omega(\theta,\rho)\omega(\theta,\rho)=\lim_{\varepsilon\to 0}\frac{1}{\pi\varepsilon^2}\iint_{K(\varepsilon)}\mathcal{H}_\omega(t,x)\omega(t,x)dxdt,$$

where $K(\varepsilon)$ is circle with the center at the point (θ,ρ) of radius ε, contained in Q.

Let's suppose the condition (6.1) at the point $(\theta,\rho)\in Q_E$ is not fulfilled. Then there exist $\tilde{\omega}\in\Omega_0$, $\alpha>0$ such that

$$\mathcal{H}_\omega(\theta,\rho)\tilde{\omega}=\mathcal{H}_\omega(\theta,\rho)\omega(\theta,\rho)+\alpha. \qquad (6.6)$$

Let's consider the variation control

$$\omega_\varepsilon(t,x)=\omega(t,x),\ (t,x)\notin K(\varepsilon),\ \omega_\varepsilon(t,x)=\omega(t,x)+\varepsilon(\tilde{\omega}-\omega(t,x)),\ (t,x)\in K(\varepsilon),\ 0<\varepsilon\le 1.$$

From the convexity of the set Ω_0 follows that $\omega_\varepsilon(t,x)\in\Omega$ almost everywhere $(t,x)\in Q$. For the control $(\omega_\varepsilon(t,x),\xi(x),\eta(t),u(t),v(x))\in U_\partial$ from (3.12), (4.15) we obtain:

$$\delta I=-\varepsilon\iint_{K(\varepsilon)}\mathcal{H}_\omega(t,x)(\tilde{\omega}-\omega(t,x))dxdt+o\left(\varepsilon\|\tilde{\omega}-\omega(t,x)\|_{L_2(K(\varepsilon))}\right).$$

From here by virtue of the equality (6.6) for sufficiently small $\varepsilon>0$ we get that $\delta I<0$. This is contradiction to the condition (5.6). The other conditions of the theorem are proved easily.

The theorem is proved.

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ON FATIGUE WEAR OF ELASTICO-PLASTIC PLATE AT PULSING TEMPERATURE ACTION

Abstract

The tensors components of stresses and deformations, the remainder stresses and deformations, the intensivity of remainder deformations are determined in the plate at its elastico-plastic deformation and the next full elastic unloading at every cycle in the case of pulsing temperature loading. The analytic formulas for the time and for the temperature cycle have been got at which in result of thermic fatigue wear from the surface layers of plate distracting the material of given thickness is determined.

Let's consider an elastico-plastic with linear hardening plate of thickness h in any form, which in a plane is free of external loadings. Let's apply the rectangular cartesion systems of the coordinate (x_1, x_2, x_3). Let's arrange the axes x_2 and x_3 in the middle of the surface. It is clear that in this case the axis x_1 will be perpendicular to this surface. On the both boundary surfaces of plate is realized the absorption of the heat $q(t)$, where t is time. Moreover it is considered that the heat stream $q(t)$ quite slowly changes by the time by the force of pulsing cycles. At $t = 0$ we suppose $q(0) = 0$. We'll consider the domains of plate on sufficiently deleting it from its edges. We suppose that all constants of materials don't depend on temperature. Subject to above noted distribution of the temperature $T(x,t) = T(x_1,t)$ will be symmetric with respect to the middle surface $x_1 = 0$. From this and according to [1,2] we'll suppose the temperature field of the plate in the following form

$$T(x,t) = T(x_1,t) = \frac{2q(t)x_1^2}{\chi h^2}, \tag{1}$$

where χ is a heat condition coefficients.

We'll denote continuity of every temperature cycle by t_*, the time before the destruction of plate by $t_c(x_1)$. In addition the number of temperature cycle will be $N_c(x_1) = t_c(x_1)/t_*$ an which will happen the destruction of plate will begin from the surface layers $x_1 = \pm\frac{h}{2}$ and will be extended in direction x_1. It means that the separation of materials will begin from the surface layers, i.e. the process of thermal fatigue wear will begin. However the process of wear won't get the surface $x_1 = 0$. Because the plastic deformation occurs when $x_1 = \pm\frac{h}{2}$ and at any cycle the central plastic zone won't happen.

Let's define the elastico-plastic stress-deformated state (also the remainder stresses and deformation's) of the plate at any cycle of temperature action for getting the $t_c(x_1)$ (or $N_c(x_1)$). By the conditions of problem every temperature cycle consists of temperature loading during the time $t_*/2$ and full temperature unloading during the same time.

First of all let's consider the elastico-plastic deformation problem of the investigated plate from the natural state at temperature loading (1) in interval time