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ON THE EQUATION OF FATIGUE WEAR AT EXTERNAL FRICTION TAKING INTO ACCOUNT THE INFLUENCE OF LOADING HISTORY

Abstract

The analytical equation accessible to engineering account fatigue wear of surface layers is received at contact interaction of bodies in case of external friction which takes into account the influence of history of non-stationary cyclic loading and temperature. It defines the time and number of cycles before the destruction of surface layer at fatigue wear.

As known the wear is a process of body's gradual changing as a result of friction which is developed in separation from the surface of material friction and its residual deformation [1]. There are some kind of classifications of wear. The fatigue wear takes especial place among them. It is modified with multi-repeated deformations of surface-layers in case of contact interaction of solids at external friction and it reduces to formation of damage and at the end party to the disturbance of surface layers strength continuity [2]. Therefore the investigation of the fatigue wear process of surface layers interacting at external body loading from the theory of damage accumulation [3-7] has a real meaning. It should be noted that in numerous investigations [8-11] the experimental proof of fatigue nature of wear have been got. In this direction good effort has been done at paper [12], where on the base of Kachanov-Rabotnov's damage concept some generality of analytical dependence is verified which describes the material distribution at usual fatigue and at friction. At paper [12] the equation of fatigue wear at external friction is got which corresponds to the principle of linear summation of damage. Denoted equation doesn't consider important physical influence of deformation history to the increment of damage at given moment and also such a real fact as a temperature field of surface layer. The given paper is realized with the aim to fill up this gap. The account of the influence of deformation history at determination of fatigue wear is especially important for contacting bodies which at interaction show the hereditary properties. As is known for example: polymers, metals has these properties at high temperatures.

Following [5-7], let's introduce some scalar $0 \le \Pi \le 1$. The value Π describes the damage of the surface layer of material, it increases with increasing of continuous parameter of loading, for which may be accepted the time t or a number of cycle loading of surface layer. If the beginning of k-th cycle w'll denote by t_k $(t_0 = 0)$, then extent of k-th cycle will be $t_k - t_{k-1}$, there $k = \overline{1, N_c}$. Here N_c is a critical number of cycle of loading corresponding to the destruction of surface layer. At that the value N_c is connected with the critical time t_c before the destruction of surface layer with relation

$$t_c = \sum_{k=1}^{N_c} (t_k - t_{k-1}) \quad (t_0 = 0). \tag{1}$$

Let's take that at initial real state $\Pi = 0$ at moment of fatigue destruction of surface layer at external loading t_c Π has the value $\Pi(t_c) = 1$.

The kinetic equation for $\Pi(t)$ we'll write as in [6]

$$\frac{d\Pi}{dt} = \Phi(\Pi(t), \Omega(t)), \tag{2}$$

where $\Omega(t)$ is some functional, continuously depending from the factors which has a influence for the considered process of wear with some function of influence $F(t-\tau)$. These factors are listed in the approximate references for example [1, 2]. For receiving at the end parity relations whose parameters are defined by simple enough experiments and which at the same time will be accessible to product of engineering designs, we'll use only primary factors playing an important role in process of fatigue wear. The analysis of structural changes of surface layer at external loading given in [11], has established that the basic influence on process of fatigue wear has a residual deformation in a surface layer of a material. Hence as well as in [12] as one determining parameter influencing on process of fatigue wear we'll accept residual intensity $\varepsilon_{+p}(t,x)$ in a surface layer at external loading. As other determining parameter we'll choose temperatures $T(t,x)$ where $(x)=(x_1,x_2,x_3)$, $t\in[0,t_c]$ of surface layer; counted from initial temperature. At it $\varepsilon_{+p}(t,x)$ and $T(t,x)$ (in the further records we will omit the argument (x)) can be variable parameters both inside each cycle and from cycle to cycle. Other parameters having influence on fatigue wear we accept as independent.

Taking into account the above stated we'll represent the nonlinear functional $\Omega(t)$ in the next form

$$\Omega(t)=\int_0^t F(t-\tau)\varphi_1(\varepsilon_{+p}(\tau),T(\tau))d\tau, \tag{3}$$

where F and φ_1 are until unknown functions of indicated arguments.

Allowing independent influence on $d\Pi/dt$ of values Π and Ω the equations (2) we'll rewrite as

$$\frac{d\Pi}{dt}=f(\Pi)\Omega(t). \tag{4}$$

Taking into account the relation (3) in the equation (4) then considering the condition $\Pi(0)=0$, let's integrate (4). In result we'll receive

$$\int_0^\Pi \frac{d\Pi}{f(\Pi)}=\int_0^t d\xi\int_0^\xi F(\xi-\tau)\varphi_1(\varepsilon_{+p}(\tau),T(\tau))d\tau. \tag{5}$$

Using the condition $\Pi(t_c)=1$ introduce the notations

$$\int_0^1 \frac{d\Pi}{f(\Pi)}=A=const,\quad \frac{\varphi_1}{A}=\varphi_2,$$

We'll transform the relation (5) to the form

$$\int_0^{t_c} d\xi\int_0^\xi F(\xi-\tau)\varphi_2(\varepsilon_{+p}(\tau),T(\tau))d\tau=1. \tag{6}$$

As in [6] the function of influence $F(\xi)$ we'll choose as $F(\xi)=F_0\xi^{-n}$, where F_0,n are constants then we'll transform double integral. As a result we'll receive

$$\int_0^{t_c}(t_c-\tau)^m\varphi(\varepsilon_{+p}(\tau),T(\tau))d\tau=1. \tag{7}$$

Here $\varphi=F_0\varphi_2/(1-n)$, $m=1-n$.

For the function $\varphi(\varepsilon_{+p},T)$ let's use approximation in the next form

$$\varphi(\varepsilon_{+p},T)=B(\varepsilon_{+p})^{\beta}\left(\frac{T}{T_s}\right)^{\delta}, \tag{8}$$

where B,β,δ are constants, T_s - is some constant temperature of reduction which is chosen from range of temperature change $T(t)$.

Taking into account (8) the relation (7) let's copy in the next form

$$\int_0^{t_c}(t_c-\tau)^m \varepsilon_{+p}^{\beta}(\tau)\left(\frac{T(\tau)}{T_s}\right)^{\delta}d\tau=\frac{1}{B}. \tag{9}$$

Now let's define constants contained in (9). We'll consider these constants as universal. The noted hypothesis allows to define them from simple experiences whose productions and realizations are possible now. Starting from told the extent of each cycle we'll adopt equal among themselves. Thus for any cycle k we have $t_k-t_{k-1}=\lambda=const$ $\left(k=\overline{1,N_c}\right)$.

At $\varepsilon_{+p}=\varepsilon_{+p_0}=const$ and $T=T_0=const\neq T_s$ from (9) follows

$$\varepsilon_{+p_0}^{\beta}\left(\frac{T_0}{T_s}\right)^{\delta}\frac{t_0^{1+m}}{1+m}=\frac{1}{B}, \tag{10}$$

where t_0 is time before destruction of surface layer at external friction in the case $\varepsilon_{+p}=\varepsilon_{+p_0}=const$ and $T=T_0=const\neq T_s$.

Let now there exist some constants ε_*, T_*, we'll call them as some deformation and temperature which at the deformation given by us $t_a=\lambda N_a$ satisfy the equation (9). Thus we have

$$\varepsilon_*^{\beta}\left(\frac{T_*}{T_s}\right)^{\delta}\frac{t_a^{1+m}}{1+m}=\frac{1}{B}. \tag{11}$$

After the definite transformation, from the relations (10) and (11) we'll get

$$\left(\frac{t_0}{\lambda}\right)^{1+m}\varepsilon_{+p_0}^{\beta}\left(\frac{T_0}{T_s}\right)^{\delta}=\left(\frac{t_a}{\lambda}\right)^{1+m}\varepsilon_*^{\beta}\left(\frac{T_*}{T_s}\right)^{\delta}.$$

That same

$$N_0^{\frac{1+m}{\beta}}\varepsilon_{+p_0}=N_a^{\frac{1+m}{\beta}}\varepsilon_*\left(\frac{T_*}{T_0}\right)^{\frac{\delta}{\beta}}. \tag{12}$$

Now let's use the result of experiences represented by the relations [11, 13, 14]:

$$N_0^{\alpha}\varepsilon_{+p_0}=C, \tag{13}$$

where N_0 is a number of cycles before the beginning of fatigue wear of surface layer. The value α we'll suppose not depending on the temperature and the value C generally speaking is the function from the temperature $T=const$, i.e. $C=C(T)$. For example, in [11] the next equation of frictional fatigue for the steel of the mark $45:\varepsilon_{+p_0}N_0^{0,4}=0,06$ has benn experimentally established.

The value C which is contained in the relation (13) in most cases in references for example in [6] is represented as

$$C(T)=C_0\left(\frac{T_s}{T}\right)^{\gamma}\quad (C_0,\gamma-const). \tag{14}$$

Comparing the relation (12) with the experimental relations (13) and taking into account (14) we have

$$\frac{1+m}{\beta}=\alpha,\quad N_a^{\frac{1+m}{\beta}}\varepsilon_*=C_0;\quad \left(\frac{T_*}{T_0}\right)^{\frac{\delta}{\beta}}=\left(\frac{T_s}{T}\right)^{\gamma}.$$

So, we have

$$\varepsilon_*=C_0N_a^{-\frac{1+m}{\beta}}=C_0N_a^{-\alpha}=C_0\left(\frac{t_a}{\lambda}\right)^{-\alpha}. \qquad (15)$$

As the value C_0 and α are defined from the experiment (13) then at the given t_a (or N_a) by the formula (15) it is defined one of above noted values ε_*. The other value T_* can be chosen as $T_*=T_s$, if we accept $\delta/\beta=\gamma$ and $T=T_0$. Besides we have $1+m=\alpha\beta$ and $\delta=\beta\gamma$. It follows that $\beta=\frac{1+m}{\alpha}$, $\delta=\frac{\gamma}{\alpha}(1+m)$. Therefore the constants β and δ are within to one until unknown multiplier $1+m$. About its determinations we shall speak below.

As the found constant are considered as universal constants of material then equating the relations (9), (11) and allowing for $T_*=T_s$ we'll get

$$(1+m)\int_0^{t_c}(t_c-\tau)^m\varepsilon_{+p}^{\beta}(\tau)\left(\frac{T(\tau)}{T_s}\right)^{\delta}d\tau=\varepsilon_*^{\beta}t_a^{1+m}. \qquad (16)$$

The relation (16) is the equation of fatigue wear of surface layer at contact interaction of bodies in case of external friction. It defines the destructions t_c of surface layer at fatigue wear.

For determination the remaining only unknown constant m the equation (16) we'll copy as

$$(1+m)\int_0^{t_c}(t_c-\tau)^m\varepsilon_{+p}^{\frac{1+m}{\alpha}}(\tau)\left(\frac{T(\tau)}{T_s}\right)^{\frac{\gamma(1+m)}{\alpha}}d\tau=\varepsilon_*^{\frac{1+m}{\alpha}}t_a^{1+m}. \qquad (17)$$

For determination the constant m let's use the results of the next additional experiment. Let in experiences spent in isothermal conditions $T=T_s$ the residual intensity of deformation of surface layer changes in each cycle with constant speed $\dot{\varepsilon}_{+p}=const$ and let thus critical residual intensity of deformation of surface layer is known at external friction $\varepsilon_{+c}=const$. Statement and results of similar experiences are given for example in [15]. Thus $\varepsilon_{+p}(\tau)=\dot{\varepsilon}_{+p}\tau$ and $\varepsilon_+(t_c)=\dot{\varepsilon}_{+p}t_c=\varepsilon_{+c}=const$ and from (17) we'll get the equation for determination the unknown constant m (the constants α and γ are defined above)

$$(1+m)(\dot{\varepsilon}_{+p})^{\frac{1+m}{\alpha}}\int_0^{\varepsilon_{+c}/\dot{\varepsilon}_{+p}}\left(\frac{\varepsilon_{+c}}{\dot{\varepsilon}_{+p}}-\tau\right)^m\tau^{\frac{1+m}{\alpha}}d\tau=\varepsilon_*^{\frac{1+m}{\alpha}}t_a^{1+m}. \qquad (18)$$

Let's denote that the determination of constant m according to the equation (18) is not unique.

Consequently all necessary parameters included in the relation (16) are determined.

Thus the relation (16) as the equation of fatigue wear at external friction allows at the given residual intensity of deformations ε_{+p} and the temperature of field T of surface layer to define time of destruction of the considered layer. Let's denote that in common case the number of cycles before the dectruction and the time before the destruction are connected by the formula (1).

As the equation (16) contain also the coordinates $(x)=(x_1,x_2,x_3)$ of surface layer (for simplisity we'll omit them), then this equation defines also the way of gradually wear of surface layer in depth.

Now let's consider some partial cases of equation (16). As we see the value t_a (or number of cycles N_a) given by us and determined by the formula (15) the value ε_* are connected among themselves. The connections of these values follows also from the physical consideration. From the relation (15) we have

$$\varepsilon_*^{\beta} t_a^{1+m} = C_0^{\beta} \lambda^{1+m}. \tag{19}$$

Let's denote that this formula holds in case when extent of every cycle are equal among themselves on value λ.

If to take into account that the value λ corresponds to the extents of one cycle of loading then the value C_0 will be critical intensity of material deformation at external friction after the single loading during the time λ. Denoting $C_0 = \varepsilon_c$ the equation (16) with the account (19) we'll write so

$$(1+m)\int_0^{t_c} (t_c-\tau)^m \varepsilon_{-p}^{\beta}(\tau)\left(\frac{T(\tau)}{T_s}\right)^{\delta} d\tau = \varepsilon_c^{\beta} \lambda^{1+m}. \tag{20}$$

If we'll consider the temperature field of surface layer as constant $(T = T_s = const)$ then from (16) and (20) correspondingly follows

$$(1+m)\int_0^{t_c} (t_c-\tau)^m \varepsilon_{+p}^{\beta}(\tau) d\tau = \varepsilon_*^{\beta} t_a^{1+m} \tag{21}$$

$$(1+m)\int_0^{t_c} (t_c-\tau)^m \varepsilon_{+p}^{\beta}(\tau) d\tau = \varepsilon_c^{\beta} \lambda^{1+m}. \tag{22}$$

It is clear that the equation (16) and (20) at the same time (21) and (22) differ among themselves by that if the equations (20) and (22) hold in the case when the time segment of each cycle are equal among themselves whereas the equations (16) and (21) represent more general case of length of segments of each cycle on time may be various.

The above presented equations of fatigue wear at external friction we'll get assuming that the speed of accumulated damages of surface layer at fatigue wear in case of external friction with the increasing of time number of cycles of loading depends on all change of temperature and residual intensity of deformations of researched layer both within the limits of one cycle and from cycle to cycle. If we refuse this assumption then in the equations (16), (20)-(22) we should accept $m=0$. In this case the equations (16), (20), (21), (22) correspondingly pass to the next relations:

$$\int_0^{t_c} \varepsilon_{+p}^{\beta}(\tau)\left(\frac{T(\tau)}{T_s}\right)^{\delta} d\tau = \varepsilon_*^{\beta} t_a, \tag{23}$$

$$\int_0^{t_c} \varepsilon_{+p}^{\beta}(\tau)\left(\frac{T(\tau)}{T_s}\right)^{\delta} d\tau = \varepsilon_c \lambda, \tag{24}$$

$$\int_0^{t_c} \varepsilon_{+p}^{\beta}(\tau)d\tau = \varepsilon_*^{\beta} t_a \ , \tag{25}$$

$$\int_0^{t_c} \varepsilon_{+p}^{\beta}(\tau)d\tau = \varepsilon_c^{\beta}\lambda \ . \tag{26}$$

Let's denote that the relation (26) coincides with the equation of fatigue wear at external friction received in [12].

Let's reduce some comparisons. Let's suppose that the next form of loading holds

$$\varepsilon_{+p}(\tau) = \varepsilon_{+p_1} = const \ \text{ when } \ 0 < t \le t_1 < t_c \ , \tag{27}$$

$$\varepsilon_{+p}(\tau) = \varepsilon_{+p_2} = const \ \text{ when } \ t_1 < t \le t_c \ .$$

In case of work [12] (formula (26)) considered program of loading gives

$$\varepsilon_{+p_1}^{\beta} t_1 + \varepsilon_{+p_2}^{\beta}(t_c - t_1) = \varepsilon_c^{\beta}\lambda \ . \tag{28}$$

The formula (22) at the loading (27) has the form

$$\varepsilon_{+p_1}^{\beta}\left[t_c^{1+m} - (t_c - t_1)^{1+m}\right] + \varepsilon_{+p_2}^{\beta}(t_c - t_1)^{1+m} = \varepsilon_c^{\beta}\lambda \ . \tag{29}$$

Comparing the formulas (28) and (29), we arrive at conclusion that the time of wear t_c, computed by the formula (12) differs from corresponding time which is computed by the formula (26). Thus the evasion can be in this or that part depending on values ε_{+p_1}, ε_{+p_2}, m. Here, we note that if it is possible to measure the time t_c from the experiment, corresponding to the program (27), then the relation (29) may be used to define the constant m.

Finally, let's denote that the equation (16) and its special cases can be used as criterion of multicycle wear and especially thermal fatigue for determination of time (number of cycles) before the damage (longevity) of structural elements (especially hereditary properties) at corresponding cycle loading.

References

[1]. Kramorenko G.V. *The technical exploitation of automobiles.* «Transport», 1972, 440p. (in Russian)

[2]. Kragelskiy I.V., Muckin N.M. *Bundle of friction of machines.* Reference book. -M., Mechanical engineering, 1984, 280p. (in Russian)

[3]. Bolotin V.V. *Forecasting of machine resource and designs.* -M., Mechanical engineering, 1984, 312p. (in Russian)

[4]. Ilyushin A.A. *On one theory of long durability.* Engineering journal, MTT, 1967, №3, p.21-35. (in Russian)

[5]. Kachakov L.M. *Bases of mechanics of destruction.* -M., Nauka, 1974, 312p. (in Russian)

[6]. Moskvitin V.V. *Cycle loading of structural elements.* -M., Nauka, 1981, 344p. (in Russian)

[7]. Rabotnov Ya.N. *On mechanism of long destruction.* Questions of materials durability and designs. M., Pub ANSSSR, 1959, p.5-7. (in Russian)

[8]. Kragelskiy I.V., Nepomnyashiy E.F. *the theory of high-elastic materials wear.*, M., Nauka, 1965, p.49-56. (in Russian)

[9]. Marchenko E.A., Nepomnyashiy E.F., Kharach G.M. *Cyclic character of accumulation of distortions of second sort in surface layer as physical confirmation of fatigue nature of wear.* DAN SSSR, 1968, v.181, №5, p.1103-1104. (in Russian)

[10].Kimura Y. *An interpretation of wear as a fatigue process.* ISLE-ASLE. Tokya, Inter.Jubs.Conf., 1975, p.89-95.

[11].Frolov K.V. *Methods of machines perfection modern problems of mechanical engineering.* M., Mashinistroyeniye, 1984, 224p. (in Russian)

[12].Matvienko Yu.G. *The kinetics of damages at contact weariness fatigue.* Physicochemical mechanics of materials, 1987, v.27, №3, p.66-68. (in Russian)
[13].Coffin L.F. *Study of the effect of cyclic thermal stresses on a ductile metals.* -Trans.ASME, 1954, v.76, p.931-950.
[14].Coffin L.F. *Desighn aspects of high temperature fatigue with particular reference to termal stresses.* -Trans. ASME, 1956, v.78, p.527-224.
[15].Aynbinder AS.B., Tyunina E.L. *Introduction to the theory of friction of polymers.* Riga, «Zinatne», 1978, 224p. (in Russian)

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THE APPROXIMATION OF SOLUTION OF THE STATIONARY PROBLEM OF SALINE FINGERS AT HIGH SUPERCRITICALITY

Abstract

The problem of stationary binary finger convention has been considered. The article proved that the conception of paralysis layer is quite just. The solution of this problem is described by means of simple and the algebraic equation. In the layers attaching the ends of saline fingers the problem leads off to the solution of non-linear regional sum for ordinary equation with the parameter depending on the square function.

1. Saline fingers very often appear in ocean at warm up and salinization of sea-water from above and create very intensive mechanism of heat and salt transport in top-layer of the ocean. The basic experimental and theoretical facts about saline fingers stated in the monograph [1]; point to the lag of the theoretical investigations of stabilization of saline fingers was clarified quite recently [2], and direct numerical analysis of the problem [3] was carried out at not do high supercriticaly in the parameter region far from the real oceanic conditions. In this connection in papers [4, 5] the mean field method for description of double convective diffusion (of heat and salt) in sea-water quite analogous to the mean field theory for the ordinary convection [6] was developed.

The single-mode theory of saline fingers allowing to reveal the qualitative regularities of the phenomenon was suggested in paper [7]. Papers [8, 9] are divoted to defining the limits of applicability of the single-mode approximation.

In particular, in these papers it was clarified that single-mode approximation is suitable for not high supercriticality and description of saline fingers was advanced to mild supercriticalities [8, 9].

It's important to note that the investigation of stationary situation for the free layers was carried out in papers [7-9], since in the all pointed out earlier papers the solution of non-stationary equations for saline fingers reach to stable stationary mode independent on initial data, and boundary conditions for velocity don't exert considerable influence on the solution of problem [1].

Papers [8, 9] showed that with increasing the supercriticality the number of generated modes sharply increases with connection of which at high supercriticality it's rational to refuse from the mode analysis of the problem which use the Fourier expansions of dynamic variables on the vertical coordinate.

2. It's easy to obtain dimensionless stationary correlations of non-linear theory of saline fingers in the approximation of mean field at large Luise numbers $\tau = {}^{\nu_\theta}/_{\nu_s} >> 1$ (ν_θ and ν_s are the coefficients of thermal diffusivity and salt diffusion) from the equations given in paper [7].

These correlations have the form

$$\left(\delta^2 D^2 - 1\right)^3 W = Q\left[W - \left(\delta^2 D^2 - 1\right)S\right], \qquad (1)$$

$$W\left(F + WS\right) = \left(\delta^2 D^2 - 1\right)S, \qquad (2)$$

$$D\bar{S} = F + WS - r, \qquad (3)$$

where

$$Q = Ra\left(\delta/_\pi\right)^4, \quad D = d/_{dz_1}, \qquad (4)$$