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BUCKLING AROUND THE TWO COLLINEAR MACRO-CRACKS IN THE CLAMPED COMPOSITE PLATE-STRIP

Abstract

Buckling (delamination) around the two collinear cracks in the composite plate-strip is investigated. For this purpose the approach based on the exact geometrically non-linear equations for anisiotropic body is used. It is assumed that the edges of the cracks have an initial insignificant imperfection and by employing boundary-form perturbation method the solution of the considered non-linear problem is reduced to the solution of the series linearized problems. Numerical results are obtained by employing FEM. According to these results, it is established that as a result of the interaction between the cracks the critical values of compressive force change insignificantly, but these values increase monotonically with decreasing a distance between the cracks.

1. Introduction. One of the major mechanisms of failure of unidirectional composites in compression along cracks is a stability loss of an equilibrium form around the cracks [5,6]. In the related investigations, which have been carried out in the framework of the Three-Dimensional Linearized Theory of Stability (TDLTS) of deformable solid body mechanics it has been assumed that a considered material occupies the infinite or semi-infinite region. The review of these investigations is given in [6, 7] and others. It follows from [6, 7] that up to now corresponding investigations, i.e. investigations carried out in the framework of TDLTS, for elements of constructions containing cracks are almost absent. It should be noted that in this field there are investigations [3, 8] in which the stability loss of the strip containing a macrocrack is studied. In [3] the stability loss of the simply supported strip containing a crack is investigated and it is assumed that this strip is fabricated from the isotropic, homogeneous linear elastic material. Moreover in [3] it is assumed that the crack is on the middle plane of the strip and corresponding eigen-value problem is studied by employing the finite-difference method. In this case the singularity order of the stresses and strains does not taken into account. However in [8] the stability loss of a clamped strip fabricated from the composite material and containing a macro-crack in a plane, which is parallel to the middle plane of the strip, is studied. In this study for solution of the corresponding mathematical problems the FEM is employed and by use of a special technique the order of the singularity the stresses and strains in the vicinity of the crack tips is taken into account.

In the present paper the investigations carried out in [8] is developed and the buckling around the two collinear macro-cracks in the clamped composite plate-strip is studied. The boundary- form perturbation technique and FEM are employed.

2. Formulation of the problem and the method of solution. Consider the strip fabricated from the composite material that is modelled as orthotropic material with normalized mechanical properties and contain two collinear cracks as shown in Fig.1a. Assume that the crack edges have initial insignificant imperfection in the form shown also in Fig.1a. We associate Lagrangian coordinate system $Ox_1x_2x_3$ and suppose that the principal elastic symmetry axes of the strip material are the Ox_1, Ox_2 and Ox_3 axes (Fig.1). Note that Ox_3 axis is perpendicular to the plane Ox_1x_2 and has not been shown in Fig.1. It is assumed that the plate-strip occupies the region $-\infty < x_3 < +\infty$.

In the considered system of coordinates the equations of the crack edges $S_1^{\pm}$ and $S_2^{\pm}$ can be selected as follows.

For $S_1^{\pm}$ (Fig.1a)

$$x_2^{\pm}=\frac{h}{2}-h_u \pm L\sin^2\left(\frac{\pi\left(x_1-\left(l_2+l_0\right)\right)}{l_0}\right),\ \ x_1\in\left(l_2,l_2+l_0\right). \tag{1}$$

For $S_2^{\pm}$ (Fig1a)

$$x_2^{\pm}=\frac{h}{2}-h_u \pm L\sin^2\left(\frac{\pi\left(x_1-\left(l_2+l_1+l_0\right)\right)}{l_0}\right),\ \ x_1\in\left(l_2+l_1+l_0,l_2+l_1+2l_0\right). \tag{2}$$

In (1) and (2) L is a maximum lifting of the crack edges from the plane $x_2=h/2-h_u$ and assume that $L<<l_0$. According to this assumption, we introduce dimensionless small parameter $\varepsilon=L/l_0$ and rewrite these equations as follows.

For $S_1^{\pm}$:

$$x_2^{\pm}=\frac{h}{2}-h_u \pm \varepsilon\, l_0\sin^2\left(\frac{\pi\left(x_1-\left(l_2+l_0\right)\right)}{l_0}\right),\ \ x_1=\left(l_2,l_2+l_0\right). \tag{3}$$

For $S_2^{\pm}$:

$$x_2^{\pm}=\frac{h}{2}-h_u \pm \varepsilon\, l_0\sin^2\left(\frac{\pi\left(x_1-\left(l_2+l_1+l_0\right)\right)}{l_0}\right),\ \ x_1\in\left(l_2+l_1+l_0,l_2+l_1+2l_0\right). \tag{4}$$

In the geometrical non-linear statement for the region occupied by the plate-strip we write the following formula equilibrium equation, mechanical and geometrical relations

$$\sigma_{12}=2A_{66}\varepsilon_{12}\,,\ \ \varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}+\frac{\partial u_n}{\partial x_j}\frac{\partial u_n}{\partial x_j}\right),\ i,j,n=1,2. \tag{5}$$

In (5) the conventional notation is used.

For the problem considered we have the following boundary conditions.

$$\left[\left(\delta_i^n+\frac{\partial u_i}{\partial x_n}\right)\sigma_{jn}\right]_{S_{1;2}^{\pm}} n_j^{\pm}=0, \tag{6}$$

$$\left[\left(\delta_i^n+\frac{\partial u_i}{\partial x_n}\right)\sigma_{2n}\right]_{x_2=\pm h/2}=0. \tag{7}$$

In (6) $n_j^{\pm}$ the ort-normal components of the surfaces $S_{1,2}^{\pm}$.

Furthermore, we assume that the plate-strip is compressed by the clamp along the Ox_1 axis with the normal forces by intensity p. In this case it is supposed that

$$u_1\left(0,x_2\right)=-u_1\left(l,x_2\right)=U\,,\ u_2\left(0,x_2\right)=u_2\left(l,x_2\right)=0. \tag{8}$$

Thus, in the framework of the foregoing we consider the plane strain state in the strip and investigate the development of the insignificant initial imperfection shown in Fig.1a. Not also we will investigate the delamination form of the stability loss shown in Fig.1b. For this purpose we use the criterion

$$u_2^{+}\Big|_{S_1^{+};x_1=l_2+l_0/2}=u_2^{+}\Big|_{S_2^{+};x_1=l_2+3l_0/2+l_1}\to+\infty \text{ and}$$

$$u_2^{-}\Big|_{S_1^{-};x_1=l_2+l_0/2}=u_2^{-}\Big|_{S_2^{-};x_1=l_2+3l_0/2+l_1}\to-\infty \text{ for } p\to p_{cr}. \tag{9}$$

Now we consider the solution procedure of the above formulated boundary-value problem. For this purpose we use the approach proposed in [1], according to which, the values characterizing the stress-strain state in the strip we represent in the series form in the parameter ε:

$$\{\sigma_{ij};\varepsilon_{ij};u_i\}=\sum_{q=0}^{\infty}\varepsilon^q\{\sigma_{ij}^{(q)};\varepsilon_{ij}^{(q)};u_i^{(q)}\}. \tag{10}$$

Substituting the equation (10) into (5) and comparing equal powers of ε to describe each approximation, we obtain the corresponding closed system of equations. Owing to the linearity of the constitutive relation they will be satisfied separately for each approximation in the equation (10). The remaining relation obtained from equation (5) for every q-th approximation contains the values of all previous approximations. Moreover, doing some operations described in [1], for the zeroth approximation we obtain the following equations.

$$\frac{\partial\sigma_{ij}^{(0)}}{\partial x_j}=0,\ \sigma_{11}^{(0)}=A_{11}\varepsilon_{11}^{(0)}+A_{12}\varepsilon_{22}^{(0)},\ \sigma_{22}^{(0)}=A_{12}\varepsilon_{11}^{(0)}+A_{22}\varepsilon_{22}^{(0)},$$

$$\sigma_{12}^{(0)}=2A_{66}\varepsilon_{12}^{(0)},\ \varepsilon_{ij}^{(0)}=\frac{1}{2}\left(\frac{\partial u_i^{(0)}}{\partial x_j}+\frac{\partial u_j^{(0)}}{\partial x_i}\right),$$

$$\sigma_i^{(0)}\Big|_{x_2=\pm h/2}=0,\ u_1^{(0)}(0,x_2)=-u_1^{(2)}(l,x_2)=U,\ u_2^{(0)}(0,x_2)=u_2^{(2)}(l,x_2)=0,$$

$$\sigma_{i2}^{(0)}\Big|_{\substack{x_2=(h/2-h_u)\pm 0\\ x_1\in(l_2,l_2+l_0)\cup(l_2+l_1+l_0,l_2+l_1+2l_0)}}=0. \tag{11}$$

For the values of the first approximation we derive the following equations.

$$\frac{\partial}{\partial x_j}\left[\sigma_{ij}^{(1)}+\sigma_{jn}^{(0)}\frac{\partial u_i^{(1)}}{\partial x_n}\right]=0,\ \sigma_{11}^{(1)}=A_{11}\varepsilon_{11}^{(1)}+A_{12}\varepsilon_{22}^{(1)},\ \sigma_{22}^{(1)}=A_{12}\varepsilon_{11}^{(1)}+A_{22}\varepsilon_{22}^{(1)},$$

$$\sigma_{12}^{(1)}=2A_{66}\varepsilon_{12}^{(1)},\ \varepsilon_{ij}^{(1)}=\frac{1}{2}\left(\frac{\partial u_i^{(1)}}{\partial x_j}+\frac{\partial u_j^{(1)}}{\partial x_i}\right),$$

$$\sigma_{i2}^{(1)}\Big|_{x_2=\pm h/2}=0,\ u_i^{(0)}(0,x_2)=u_i^{(2)}(l,x_2)=0,$$

$$\sigma_{i2}^{(1)}\Big|_{\substack{x_2=(h/2-h_u)\pm 0\\ x_1\in(l_2,l_2+l_0)}}=\pm\pi\sin\left(\frac{2\pi(x_1-(l_2+l_0))}{l_0}\right)\sigma_{11}^{(0)}\delta_i^1,$$

$$\sigma_{i2}^{(1)}\Big|_{\substack{x_2=(h/2-h_u)\pm 0\\ x_1\in(l_2+l_1+l_0,l_2+l_1+2l_0)}}=\pm\pi\sin\left(\frac{2\pi(x_1-(l_2+l_1+l_0))}{l_0}\right). \tag{12}$$

According to [1], the values of the second and subsequent approximations don't change the values of p_{cr}, therefore p_{cr}, can be determined from (11) and (12) in the framework of the zeroth and first approximations only. Note that in this case the values of p are determined by the expression $hp=\int_{-h/2}^{+h/2}\sigma_{11}^{(0)}\Big|_{x_1=l/2}dx_2$. Taking this situation into account the values of the zeroth and first approximation are determined by the use of FEM. The version of the FEM from which is used in the present investigation, is described in [1]. Moreover, in the present investigation for keeping the order of the singularities of the stresses and strains at the crack tips the spatial type finite element [1] are used.

3. Numerical results. We assume that the fabricated from a composite material consisting of the alternating layers of two isotropic homogeneous materials. The reinforcing layers will be assumed to be located in planes which are parallel to the plane Ox_1x_3 (Fig.1). Young module and Poisson coefficients of these materials we denote through $E^{(k)}$ and $\nu^{(k)}$ $(k=1,2)$ respectively. It is known that in the continuum approach the above layered composite material is taken as transversally isotropic material with normalized mechanical properties whose isotropy axis lies on the Ox_2 axis. Moreover, it is known that these normalized mechanical properties are determined through the following expressions described, for example, in [1] and elsewhere

$$A_{11}=\left(\lambda^{(1)}+2\mu^{(1)}\right)\eta^{(1)}+\left(\lambda^{(2)}+2\mu^{(2)}\right)\eta^{(2)}-$$
$$-\eta^{(1)}\eta^{(2)}\frac{\left(\lambda^{(1)}-\lambda^{(2)}\right)^2}{\left(\lambda^{(1)}+2\mu^{(1)}\right)\eta^{(1)}+\left(\lambda^{(2)}+2\mu^{(2)}\right)\eta^{(2)}},\ A_{66}=\frac{\mu^{(1)}\mu^{(2)}}{\mu^{(1)}\eta^{(2)}+\mu^{(2)}\eta^{(1)}},$$
$$A_{12}=\lambda^{(1)}\eta^{(1)}+\lambda^{(2)}\eta^{(2)}-\eta^{(1)}\eta^{(2)}\left(\lambda^{(1)}-\lambda^{(2)}\right)\frac{\left[\left(\lambda^{(1)}+2\mu^{(1)}\right)-\left(\lambda^{(2)}+2\mu^{(2)}\right)\right]}{\left(\lambda^{(1)}+2\mu^{(1)}\right)\eta^{(1)}+\left(\lambda^{(2)}+2\mu^{(2)}\right)\eta^{(2)}},$$
$$A_{22}=\left(\lambda^{(1)}+2\mu^{(1)}\right)\eta^{(1)}+\left(\lambda^{(2)}+2\mu^{(2)}\right)\eta^{(2)}-$$
$$-\eta^{(1)}\eta^{(2)}\frac{\left[\left(\lambda^{(1)}+2\mu^{(1)}\right)-\left(\lambda^{(2)}+2\mu^{(2)}\right)\right]^2}{\left(\lambda^{(1)}+2\mu^{(1)}\right)\eta^{(1)}+\left(\lambda^{(2)}+2\mu^{(2)}\right)\eta^{(2)}},$$
$$\lambda^{(k)}=\frac{\nu^{(k)}E^{(k)}}{\left(1+\nu^{(k)}\right)\left(1-2\nu^{(k)}\right)},\ \mu^{(k)}=\frac{E^{(k)}}{2\left(1+\nu^{(k)}\right)}. \qquad (13)$$

Assume that $h/l=0.15, \nu^{(1)}=\nu^{(2)}=0.3,\ \eta^{(2)}=0.5$, where $\eta^{(2)}$ is a filter concentration and $\eta^{(1)}=1-\eta^{(2)}$. Introduce the dimensionless parameters $E^{(2)}/E^{(1)}$, h_u/l_0, l_1/l, l_2/l and assume that $l_0/l=0.25$. Analyze the influence of these parameters to the values of $p_{cr.}/E^{(1)}$. First we consider the case where $E^{(2)}/E^{(1)}=1$ (isotropic plate). In this case the values of $p_{cr.}/E^{(1)}$ are given in Table 1. The values of $p_{cr.}/E^{(1)}$ for $E^{(2)}/E^{(1)}>1$ (anisotropic plate) are given in Table 2.

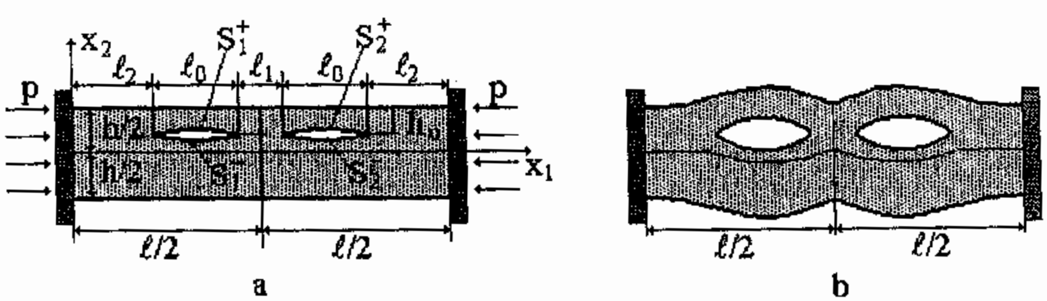

Fig. 1.

Table 1 and 2 show that for each selected h_u / l_0 and $E^{(2)} / E^{(1)}$ by decreasing the distance, i.e. by decreasing l_1 / l or by increasing l_2 / l, the values of $p_{cr.} / E^{(1)}$ change insignificantly and are very near to the corresponding ones obtained for a single crack in the considered plate-strip. Nevertheless, for $l_1 / l \geq 0.2$ the values of $p_{cr.} / E^{(1)}$ increase monotonically with l_1 / l. This situation agrees with the well-known mechanical consideration. Consequently, we can conclude that the values of $p_{cr.} / E^{(1)}$ increase monotonically by approaching of the collinear cracks to each other. However, this increasing is very insignificant and in the many cases under determination of $p_{cr.} / E^{(1)}$ the interaction of the collinear cracks can be neglected with the very high accuracy.

Table 1

$\frac{h_u}{l_0}$	l_1 / l					
	0.3	0.25	0.2	0.15	0.1	0.05
0.30	0.1105	0.1103	0.1102	0.1103	0.1118	0.1181
0.24	0.0901	0.0900	0.0900	0.0899	0.0901	0.0911
0.20	0.0725	0.0724	0.0724	0.0724	0.0728	0.0744
0.16	0.0546	0.545	0.0545	0.0546	0.0550	0.0560

Table 2

$E^{(2)} / E^{(1)}$	h_u / l_0	l_1 / l			
		0.3	0.15	0.1	0.05
10	0.30	0.3458	0.3482	0.3547	0.3679
	0.24	0.3013	0.3023	0.3053	0.3114
	0.20	0.2581	0.2588	0.2617	0.2686
	0.16	0.2071	0.2077	0.2100	0.2156
20	0.30	0.4460	0.4488	0.4549	0.4657
	0.24	0.4013	0.4030	0.4068	0.4137
	0.20	0.3567	0.3582	0.3622	0.3704
	0.16	0.2980	0.2993	0.3029	0.3108
50	0.30	0.5628	0.5645	0.5679	0.5739
	0.24	0.5255	0.5270	0.5299	0.5350
	0.20	0.4901	0.4919	0.4956	0.5025
	0.16	0.4329	0.4350	0.4395	0.4481

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ON THE EQUATION OF FATIGUE WEAR AT EXTERNAL FRICTION TAKING INTO ACCOUNT THE INFLUENCE OF LOADING HISTORY

Abstract

The analytical equation accessible to engineering account fatigue wear of surface layers is received at contact interaction of bodies in case of external friction which takes into account the influence of history of non-stationary cyclic loading and temperature. It defines the time and number of cycles before the destruction of surface layer at fatigue wear.

As known the wear is a process of body's gradual changing as a result of friction which is developed in separation from the surface of material friction and its residual deformation [1]. There are some kind of classifications of wear. The fatigue wear takes especial place among them. It is modified with multi-repeated deformations of surface-layers in case of contact interaction of solids at external friction and it reduces to formation of damage and at the end party to the disturbance of surface layers strength continuity [2]. Therefore the investigation of the fatigue wear process of surface layers interacting at external body loading from the theory of damage accumulation [3-7] has a real meaning. It should be noted that in numerous investigations [8-11] the experimental proof of fatigue nature of wear have been got. In this direction good effort has been done at paper [12], where on the base of Kachanov-Rabotnov's damage concept some generality of analytical dependence is verified which describes the material distribution at usual fatigue and at friction. At paper [12] the equation of fatigue wear at external friction is got which corresponds to the principle of linear summation of damage. Denoted equation doesn't consider important physical influence of deformation history to the increment of damage at given moment and also such a real fact as a temperature field of surface layer. The given paper is realized with the aim to fill up this gap. The account of the influence of deformation history at determination of fatigue wear is especially important for contacting bodies which at interaction show the hereditary properties. As is known for example: polymers, metals has these properties at high temperatures.

Following [5-7], let's introduce some scalar $0 \le \Pi \le 1$. The value Π describes the damage of the surface layer of material, it increases with increasing of continuous parameter of loading, for which may be accepted the time t or a number of cycle loading of surface layer. If the beginning of k-th cycle w'll denote by t_k $(t_0 = 0)$, then extent of k-th cycle will be $t_k - t_{k-1}$, there $k = \overline{1, N_c}$. Here N_c is a critical number of cycle of loading corresponding to the destruction of surface layer. At that the value N_c is connected with the critical time t_c before the destruction of surface layer with relation

$$t_c = \sum_{k=1}^{N_c} (t_k - t_{k-1}) \quad (t_0 = 0). \tag{1}$$

Let's take that at initial real state $\Pi = 0$ at moment of fatigue destruction of surface layer at external loading t_c Π has the value $\Pi(t_c) = 1$.

The kinetic equation for $\Pi(t)$ we'll write as in [6]

$$\frac{d\Pi}{dt} = \Phi(\Pi(t), \Omega(t)), \tag{2}$$