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ON NORMAL IMPACT ON FLEXIBLE ELASTIC FILAMENT

Abstract

In the paper the construcıion of solving of the problem on normal impact with constant velocity by obtuse rigid wedge on elastic filament is given. It is assumed that the velocity of the breakpoint is less than the velocity of elastic wave in filament.

In works [1-4] the behaviour of flexible filament at transverse impact by rigid wedge is investigated when the flexure part of filament covers to check of wedge. In the present paper the solution of the problem on normal impact by rigid symmetric wedge having plane fore-part on flexible elastic filament is investigated. It's accepted that domain beyond breakpoints A and A_1 covers the surface of bombarding body, and velocity of the breakpoint A (and A_1) is less that velocity of elastic wave in the filament $b = Vctg\gamma < a_0$.

§1. Let the normal impact by symmetric wedge with plane fore-part with the constant velocity V be performed by infinite long flexible linear-elastic, rectilinear non-strained filament. After impact in filament four elastic waves whose fronts are N_1, C_1, C, N and two waves of strong break (break point) A and A_1 arise (pic.1). Denote the width BB_2 by $2L$. The behaviour of the filament in the domains $NABCO$ and $N_1A_1B_1C_1O$ are the same. The velocity of particles of filament in these domains are directed along the filament respectively. In the domains OC and OC_1 the filament is at rest to the zero time $t = \frac{L}{a_0}$ $\left(0 \le t \le \frac{L}{\alpha_0}\right)$ relative to "wedge". Since the impact is performed with constant velocity, then in originating domains the filaments determining the parameters are constant. It's assumed that the friction is absent in covering domain between the filament and bombarding body.

In fig.1 B_1 and B are stationary break points and the motion of filaments relative to these points are taken as motion via fixed block [5]. The following designations are accepted: ε is deformation, σ is stress, ϑ is velocity of particles of filament, $a_0 = \sqrt{E\rho^{-1}}$ is velocity of elastic wave; E is Young's modulus, ρ is density, γ is an angle between the initial position of filament and the check of wedge BA (and B_1A_1) (pic.1), t is time, x is Lagrangian coordinate.

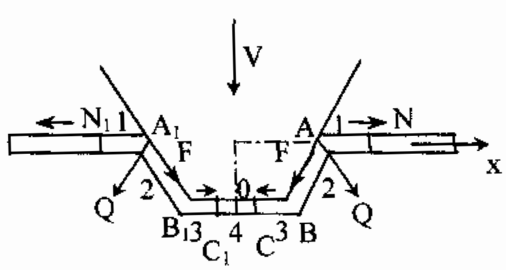

Fig. 1.

[Mutallimov Sh.M.]

The wave picture of motion of elastic filament after impact to the plane x,t is shown in fig.2. We'll supply the unknown parameters of motion of the filament $\varepsilon;\sigma;\vartheta$ originating in domains 1,2,3,4,5,11,21,31,51,... (fig.1, fig.2) with corresponding indices.

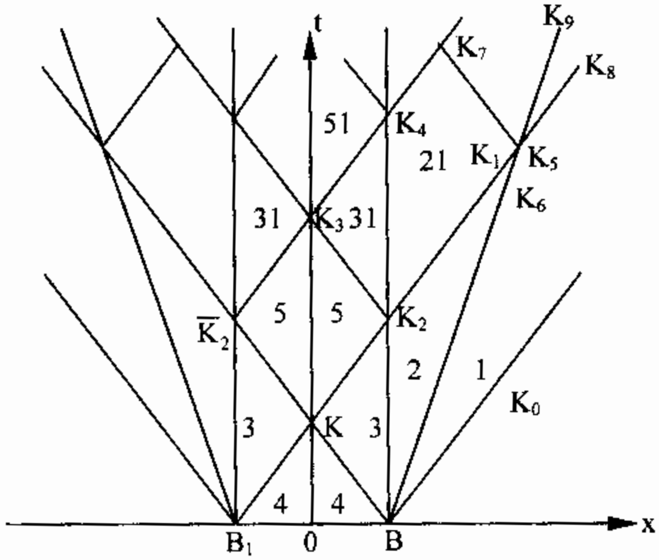

Fig. 2.

Note that the behaviour the filament in the domains $OBAN$ and $OB_1A_1N_1$ (fig.1) are same, i.e. the behaviour of the statement relative to the point O is symmetric and we'll consider the problem in right-hand side of the filament $(OBAN)$.

The material of filament is assumed to be elastic

$$\sigma = E\varepsilon \, . \tag{1.1}$$

We write the condition at the breakpoint A in the following form [2]

$$\frac{b-\vartheta_1}{1+\varepsilon_1} = \frac{b\sec\gamma * -\vartheta_2}{1+\varepsilon_2} = z \, , \tag{1.2}$$

$$z(\vartheta_2 - \vartheta_1\cos\gamma - V\sin\gamma) = \sigma_1\cos\gamma - \sigma_2 - F; \tag{1.3}$$

$$z(\vartheta\cos\gamma - V_2\sin\gamma) = \sigma_1\sin\gamma + Q \, . \tag{1.4}$$

Here $b = V\,ctg\gamma$; z is velocity of waves of strong break. All designations of [2] are also kept here in the plane V,γ (fig.3) the equation $V = a_0 tg\gamma$ corresponds to the line $OO_1\overline{C}_1$ and upper of this line $(V \geq a_0 tg\gamma)$ corresponds to supersonic regime, and lower of this linear $(V < a_0 tg\gamma)$ - to subsonic. In domain (fig.3) of left hand-side of the line $\overline{C}O_1O_2$ and upper of the linear $OO_2\,(\gamma < 2\gamma_* \;\; \mu_* = tg\gamma_*)$ the next inequality [2] is valid

$$F < \mu_* Q, \tag{1.5}$$

and right hand side of the line $\overline{C}O_1O_2$ and upper of the line O_2C_2 condition [2] is valid.

$$F = \mu_* Q; \; (\gamma > 2\gamma_*) \tag{1.6}$$

Here μ_* is a coefficient of coulomb friction at breakpoint; F,Q are tangent and normal to check of wedge component of the point force at the point A (fig.1). The lines OO_2C_2 correspond to the solution of the problem at impact on filament for which $F = Q = 0$ and to deformation continuous at the break point [1,2]

$$\varepsilon_1 = \varepsilon_2 \tag{1.7}$$

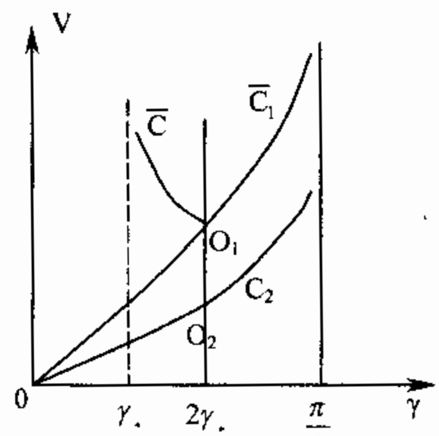

Fig.3.

§2. Let the inequality (1.5) be valid on wave of strong break (at the point A) – in the domain OO_2O_1O (pic.3). The condition (1.5) implies the condition [2,3,4]

$$\vartheta_2 = 0 \tag{2.1}$$

at the breakpoint A.

At the point B (pic.1) we have the kinematics condition in the form of

$$\vartheta_3 = \vartheta_2 \cos\gamma \ . \tag{2.2}$$

On the front C (pic.1) or on the front BK (pic.2) we have

$$\vartheta_3 - \vartheta_4 = -a_0(\varepsilon_4 - \varepsilon_3) \tag{2.3}$$

since

$$\vartheta_4 = 0; \ \varepsilon_4 = 0 \ , \tag{2.4}$$

then from (2.1)-(2.4) it follows that

$$\varepsilon_3 = 0; \vartheta_3 = 0; \ (\vartheta_2 = 0). \tag{2.5}$$

On front of the elastic wave $N(BK_0)$ (pic.1, pic.2) we have

$$\vartheta_1 - \vartheta_{01} = a_0(\varepsilon_{01} - \varepsilon_1). \tag{2.6}$$

Here $\varepsilon_{01}, \vartheta_{01}$ are deformation and velocity of particles in front of the elastic wave N (or BK_0). At $\varepsilon_{01} = 0; \vartheta_{01} = 0$ from (2.6) we obtain the relation

$$\vartheta_1 = -a_0 \varepsilon_1 \,. \tag{2.7}$$

Allowing for the condition (1.7) form (1.2), (2.1), (2.7) we determine $\varepsilon_1, \varepsilon_2, z$ in the form of [2]

$$\begin{gathered} \varepsilon_1 = \varepsilon_2 = b a_0^{-1}(\sec\gamma - 1); \ \vartheta_2 = 0; \vartheta_1 = -a_0\varepsilon_1 \neq 0; \\ z = a_0 V ctg\gamma \left[V ctg\gamma (1 - \cos\gamma) + a_0 \cos\gamma \right]^{-1} ; \end{gathered} \tag{2.8}$$

$$\sigma_1 = \sigma_2 = \rho \alpha_0^2 \varepsilon_1 \,.$$

Thus from the solutions (2.4)-(2.8) it follows that at the condition (1.5) after impact $\left(0 \le t < \dfrac{L}{a_o}\right)$ the flexure part of the filament ABB_1A (pic.1) adheres to the surface of bombarding wedge. In the domain BB_1 (fig.1) the filament is in non-stress state (and in rest) and in domains 1 and 2 the parameters are determined by the formulas (3.8) bombarding wedge. In the domain BB_1 (fig.1) the filament is in non-stress state (and in rest) and in domains 1 and 2 the parameters are determined by the formulas (2.8).

Note that the conditions (2.1), (1.5) in the domain OO_2O_1O (pic.3) will be observed only for some set of values of parameters of problem, the whose boundary is determined by the condition (1.6). At the right hand of the line O_2O_1 in the domain $\overline{C}_1O_1O_2C_2$ the condition (1.6) is taken as closing condition, in addition $\vartheta_2 \neq 0$. Now let on wave of strong break (at the point A) the condition (1.6) be satisfied, it is required to determine the parameters of problems for the domains 1,2,3,4 for period $0 \le t < \frac{L}{a_0}$. In this case the creeping of the particles of filament happens beyond the breakpoint A by the cheek of wedge. At the point B the continuing condition of deformation is taken, i.e. the condition

$$\varepsilon_1 = \varepsilon_2 . \tag{2.9}$$

Thus the unknown parameters of problem in domains 1,2,3,4 are determined from the system (1.1)-(1.4), (1.6), (2.2)-(2.4), (2.7), (2.9) and have the form

$$\varepsilon_1 = \frac{z}{a_0 - z} b_1 \left[\beta_2 (1 - \cos\gamma) + \frac{z}{a_0} \beta_3 \sin\gamma \cos\gamma \right]; \ \vartheta_1 = -a_0 \varepsilon_1 ; \tag{2.10}$$

$$\varepsilon_3 = \varepsilon_2 = \frac{z \cos\gamma}{z + a_0 \sin\gamma} b_1 \left[\beta_1 (1 - \cos\gamma) - \frac{z}{a_0} \beta_3 \sin\gamma \right]; \tag{2.11}$$

$$\vartheta_2 = a_0 \varepsilon_2 \sec\gamma \ ; \ \ \vartheta_3 == a_0 \varepsilon_3 ; \tag{2.12}$$

$$b = z \frac{\beta_2 + \beta_1 \cos\gamma}{\beta_1 \cos\gamma + \beta_2 \sec\gamma + \frac{z}{a_0} \beta_3 \sin\gamma} ; \tag{2.13}$$

$$b = V ctg\gamma . \tag{2.14}$$

here

$$\beta_1 = 1 + tg\gamma \cdot tg\gamma_* ; \ \beta_2 = \frac{a_0 + z \sec\gamma}{z + a_0 \sec\gamma} ,$$

$$\beta_3 = tg\gamma_* - tg\gamma ; b_1 = \left[\beta_2 + \beta_1 \cos^2\gamma + \frac{z}{a_0} \beta_3 \sin\gamma \cos\gamma \right]^{-1} . \tag{2.15}$$

From the formula (2.13) z is determined for given $b = V ctg\gamma$, but here it's convenient to solve this equation relative to b for given z. The solution (2.10)-(2.15) is correct for period $0 \le t < \frac{L}{a_0}$.

§3. At the time $t = \frac{L}{a_0}$ after impact the elastic waves C and C_1 met at the point 0 (fig.1), then two reflections of the elastic waves $KK_2, K\overline{K}_2$ (pic.2) arise. It's known that two identical waves moving to meet each other in place of meeting their velocities vanich. This cross-section of filament (the point 0) remains stationary with respect to wedge and we consider it as closed end of filament. In this place for $\frac{L}{a_0} \le t < \frac{2L}{a_0}$ the stress (deformation) is doubled. Thus in domain 5 (pic.2) the parameters of the problem have the form

$$\vartheta_5 = 0; \sigma_5 = 2\sigma_3 = 2\rho a_0^2 \varepsilon_3, \tag{3.1}$$

where the deformation ε_3 is determined by the formula (2.11) (or by the formula (2.5) depending on regime of motion at the breakpoint A). At the time $t = \frac{2L}{a_0}$ the reflected elastic waves KK_2 and $K\widetilde{K}_2$ interact with stationary breaks B and B_1. For $t = \frac{2L}{a_0}$ at the left from the point B the reflected elastic wave is propagated with the front K_2K_3, and from the right from the point B by BA the elastic wave is propagated with the front K_2K_1 (fig.2). In the domain $OBAN$ (pic.1) in filament five domains (domains 5,31,21,2,1) (pic.2) arise. The solutions of the problem in domains 1,2,5 are known from (2.10)-(2.15), (3.1). The parameters of the problem in domains 31,21 are determined from the relations on the fronts K_2K_3, K_2K_1

$$\vartheta_{31} - \vartheta_5 = -a_0(\varepsilon_5 - \varepsilon_{31}); \tag{3.2}$$
$$\vartheta_{21} - \vartheta_2 = a_0(\varepsilon_2 - \varepsilon_{21});$$

and from the conditions at the points B

$$\vartheta_{31} = \vartheta_{21}\cos\gamma; \tag{3.3}$$

$$\varepsilon_{31} = \varepsilon_{21}. \tag{3.4}$$

If at the breakpoint the condition (1.5) holds then from (3.2), (3.3), (3.4) subject to (2.1), (2.5), (3.1) the parameters $\vartheta_{21}, \varepsilon_{21}, \vartheta_{31}, \varepsilon_{31}$ are determined in the form of

$$\vartheta_{21} = \frac{a_0}{1+\cos\gamma}\varepsilon_2; \; \varepsilon_{21} = \frac{\cos\gamma}{1+\cos\gamma}\varepsilon_2; \tag{3.5}$$
$$\vartheta_{31} = \vartheta_{21}\cos\gamma; \; \varepsilon_{21} = \varepsilon_{31}$$

where ε_2 is expressed by the formula (2.8).

If at the breakpoint the condition (1.6) is correct then from (3.2), (3.3), (3.4) subject to (2.11), (2.12), (3.1) the parameters $\vartheta_{21}, \varepsilon_{21}, \vartheta_{31}, \varepsilon_{31}$ are determined in the form of

$$\vartheta_{21} = \varepsilon_2 \sec\gamma tg^2\frac{\gamma}{2}; \; \vartheta_{31} = \vartheta_{21}\cos\gamma,$$

$$\varepsilon_{21} = \varepsilon_{31} = \frac{3+\cos\gamma}{1+\cos\gamma}\varepsilon_2, \tag{3.6}$$

where ε_2 is expressed by the formula (2.11).

Note that depending on the velocity of impact V, the angle γ, on the width $|BB_1| = 2L$ the different wave schemes of motion arise. One of them is considered below. Since subsonic regime of motion $(z < a_0)$ is investigated i.e. the case when the velocity of elastic wave a_0 more than the velocity of wave of strong break z, is investigated, then after some time the front of elastic wave K_2K_1 overtakes the front of wave of strong break BK_6 and at the point K_5 interaction of fronts of these waves happens. Interaction time of the fronts of waves K_2K_1 and BK_6 is determined from the relations

$$zt + L = a_0 t - L. \tag{3.7}$$

and

$$t = t_1 = \frac{2L}{a_0 - z}. \qquad (3.8)$$

And at the time $t = t_2 = \frac{3L}{a_0}$ the front of elastic wave K_2K_3 is reflected from the point 0 in the form of K_3K_4 (pic.2). It follows to note that depending on the values t_1, t_2 the different wave schemes of motion arise in elastic filament at transverse impact on it by wedge having plane front. In addition the following cases exist. In the first case if $t_1 < t_2$ then subject to (3.7), (3.8), the inequality $t_1 < t_2$ is satisfied for $\frac{z}{a_0} < \frac{1}{3}$. Consequently from the inequality $t_1 < t_2 \left(\frac{z}{a_0} < \frac{1}{3}\right)$ it follows that only other contact of the fronts of waves K_2K_1 and BK_6 at the point K_5, the front of the elastic waves K_2K_3 is reflected from the point 0 in the form of K_3K_4 (pic.2). From the left from the point K_5 the elastic wave K_5K_7 is rejected in covering domain of filament, to the right from the point K_5 by longitudinal part along the front of elastic wave BK_0, the front of elastic wave K_5K_8 is propagated and beginning with the point K_5 the wave of strong break by the front K_5K_9, the velocity of which is unknown, is propagated.

In the second case if $t = t_2$ then subject to (3.7), (3.8) the equality $t_1 = t_2$ is satisfied for $\frac{z}{a_0} = \frac{1}{3}$. Consequently, for this case the reflection from the point 0 of elastic wave K_2K_3 and interaction of the waves K_2K_1 and BK_6 happen at the same time. Further the same wave situation of motion mentioned in the first case happens. Finally, the case $t_2 < t_1$ is possible. Subject to (3.7), (3.8) the inequality $t_2 < t_1$ is satisfied for $\frac{1}{3} < \frac{z}{a_0} < 1$. In addition at first the front of elastic wave K_2K_3 is reflected from the point 0 (pic.1) (from the point K_3 (pic.2)) in the form of K_3K_4, and later the fronts of elastic waves K_2K_1 and BK_6 interact at the point K_5.

Note that the solution of the problem for all above mentioned three cases subject to repeated reflections and integration of waves, is not difficult, but allowing for their awkwardness they don't cited here. However in below considered case when $t_2 < t_1$ in addition it's accepted that the front of elastic wave K_2K_3 is reflected from the point K_3 in the form of K_3K_4, but the reflected wave K_2K_3 doesn't reach the point K_4 (pic.2) (to the point B (fig.1))m and the fronts of waves K_2K_1 and BK_6 aren't met at the point K_5 get. Consequently, the problem is investigated for period $0 < t \le t_*$. Here t_* varies in period $\frac{3L}{a_0} \le t_* < t_1$. For $t_2 < t_1$ the wave scheme of motion is mentioned in pic.2. The solutions of the problem in domains $1, 2, 21, 31$ are unknown, and the solution in new domain 51 is determined from condition on front $51 - 31$.

$$\vartheta_{51} - \vartheta_{31} = a_0(\varepsilon_{31} - \varepsilon_{51}). \qquad (3.9)$$

$$\sigma_{51} = \rho a_0^2 \varepsilon_{51} . \qquad (3.10)$$

Here $\varepsilon_{31}, \vartheta_{31}$ are determined by the formulas (3.5) (for $F < \mu_* Q$) and (3.6) (for $F = \mu_* Q$). Comparing the formulas for $\varepsilon_5, \varepsilon_{31}, \varepsilon_{51}$ we obtain the inequalities in the form of

$$\varepsilon_5 < \varepsilon_{31} < \varepsilon_{51} \text{ or } \sigma_5 < \sigma_{31} < \sigma_{51} . \qquad (3.11)$$

Let $\mu_* = tg\gamma_* = 0{,}2679; \gamma_* = 15^0; \gamma = 35^0, za_0^{-1} = 0{,}023$, then by formulas (2.10)-(2.15), (3.1) the parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3, \vartheta_2, \vartheta_3, b, \varepsilon_5$ get the following values

$$\varepsilon_1 = 0{,}00211; \varepsilon_2 = \varepsilon_3 = 0{,}00206 ;$$

$$a_0^{-1}\vartheta_2 = 0{,}00251; a_0^{-1}\vartheta_3 = 0{,}00206 ;$$

$$a_0^{-1}b = 0{,}02094; \varepsilon_5 = 0{,}00416 .$$

For the same data, but for $za_0^{-1} = 0{,}33495$ the parameters get the following values relatively

$$\varepsilon_1 = 0{,}00287; \varepsilon_2 = \varepsilon_3 = 0{,}03214 ;$$

$$a_0^{-1}\vartheta_2 = 0{,}3824; a_0^{-1}\vartheta_3 = 0{,}03214 ;$$

$$a_0^{-1}b = 0{,}3153; \varepsilon_5 = 0{,}06428 .$$

It the impact on elastic filament is performed by acute wedge $(L = 0)$ then domains $3, 4, 5$ (pic.2) disappear and the parameters in domains $1, 2$ for above mentioned data by solution [2] get the next values

$$\varepsilon_1 = 0{,}00416; \varepsilon_2 = 0{,}00531 ;$$

$$\vartheta_2 = 0; a_0^{-1}b = 0{,}01894$$

for $a_0^{-1}z = 0{,}023$;

$$\varepsilon_1 = 0{,}064; \varepsilon_2 = 0{,}06568 ;$$

$$\vartheta_2 = 0; a_0^{-1}b = 0{,}29239$$

for $a_0^{-1}z = 0{,}33495$.

From above mentioned calculations it follows that independent of geometry of wedge at increase of velocity of impact, the deformation of filament in every domain increases, but at impact by wedge having a plane front, the filament is deformed smaller than at impact to it by acute wedge. Consequently for input data at impact by acute wedge the break of filament may occur at the impact point for $t = 0$ and for those input data at impact by obtused wedge, the break of filament nay occur after some moment $t = t_p \left(t_p > 0\right)$ (t_p is destruction time).

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Received November 15, 2000; Revised June 18, 2001.
Translated by Mirzoyeva K.S.

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BUCKLING AROUND THE TWO COLLINEAR MACRO-CRACKS IN THE CLAMPED COMPOSITE PLATE-STRIP

Abstract

Buckling (delamination) around the two collinear cracks in the composite plate-strip is investigated. For this purpose the approach based on the exact geometrically non-linear equations for anisiotropic body is used. It is assumed that the edges of the cracks have an initial insignificant imperfection and by employing boundary-form perturbation method the solution of the considered non-linear problem is reduced to the solution of the series linearized problems. Numerical results are obtained by employing FEM. According to these results, it is established that as a result of the interaction between the cracks the critical values of compressive force change insignificantly, but these values increase monotonically with decreasing a distance between the cracks.

1. Introduction. One of the major mechanisms of failure of unidirectional composites in compression along cracks is a stability loss of an equilibrium form around the cracks [5,6]. In the related investigations, which have been carried out in the framework of the Three-Dimensional Linearized Theory of Stability (TDLTS) of deformable solid body mechanics it has been assumed that a considered material occupies the infinite or semi-infinite region. The review of these investigations is given in [6, 7] and others. It follows from [6, 7] that up to now corresponding investigations, i.e. investigations carried out in the framework of TDLTS, for elements of constructions containing cracks are almost absent. It should be noted that in this field there are investigations [3, 8] in which the stability loss of the strip containing a macrocrack is studied. In [3] the stability loss of the simply supported strip containing a crack is investigated and it is assumed that this strip is fabricated from the isotropic, homogeneous linear elastic material. Moreover in [3] it is assumed that the crack is on the middle plane of the strip and corresponding eigen-value problem is studied by employing the finite-difference method. In this case the singularity order of the stresses and strains does not taken into account. However in [8] the stability loss of a clamped strip fabricated from the composite material and containing a macro-crack in a plane, which is parallel to the middle plane of the strip, is studied. In this study for solution of the corresponding mathematical problems the FEM is employed and by use of a special technique the order of the singularity the stresses and strains in the vicinity of the crack tips is taken into account.

In the present paper the investigations carried out in [8] is developed and the buckling around the two collinear macro-cracks in the clamped composite plate-strip is studied. The boundary- form perturbation technique and FEM are employed.

2. Formulation of the problem and the method of solution. Consider the strip fabricated from the composite material that is modelled as orthotropic material with normalized mechanical properties and contain two collinear cracks as shown in Fig.1a. Assume that the crack edges have initial insignificant imperfection in the form shown also in Fig.1a. We associate Lagrangian coordinate system $Ox_1x_2x_3$ and suppose that the principal elastic symmetry axes of the strip material are the Ox_1, Ox_2 and Ox_3 axes (Fig.1). Note that Ox_3 axis is perpendicular to the plane Ox_1x_2 and has not been shown in Fig.1. It is assumed that the plate-strip occupies the region $-\infty < x_3 < +\infty$.