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TORSION OF VISCO-ELASTIC PRISMATIC PIVOTS IN GEOMETRIC AND PHYSICAL NON-LINEAR STATEMENT

Abstract

Geometrical non-linear torsion of physically non-linear visco-elastic prismatic pivot is stated in a general form. As a special case, the problem on the torsion of circular cross section of a solved is solved by successive approximations method.

Consider the torsion of a visco-elastic prismatic pivot by action of torque. We'll accept the coordinates of initial state. We choose origin of coordinates from one of the end-walls of pivot. We direct the axis Ox_3 along the axis of pivot. We introduce the following designations: R is length of pivot (L is assumed as sufficiently large), S is a lateral surface, S_1 is area of torsion.

Since we consider the problem in geometric and physical non-linear statement, then we calculate the components of the deformation tensor e_{ij} by the components of the displacement vector u_i by Green's formula

$$2e_{i,j} = u_{i,j} + u_{j,i} + u_i u_j . \tag{1}$$

We accept the physical relations between the components of stress tensor and deformation tensor in the form of [2]:

$$\frac{S_y}{2G_0} = э_y\left(1-\omega(\varepsilon_u)\right) - \int_0^t R(t-\tau)\left(1-\omega(\varepsilon_u)\right)э_{ij}(\tau)d\tau , \tag{2}$$

$$\frac{\tau}{K} = \theta . \tag{3}$$

Here $S_y = \sigma_{ij} - \sigma\delta_{ij}$ is a deviator of the stress tensor σ_{ij}; $э_{ij} = e_{ij} - e\delta_{ij}$ is a deviator of the deformation tensor e_{ij}, $\theta = e_{kk} = 3e$ is relative variation of volume; $\sigma = \sigma_{kk}/3$ is mean stress, G_0- is instantly-elastic shear modulus; K is modulus of deformation tensor. The kernel $R(t-\tau)$ characterizes rheological properties of material; $\varepsilon_{ij} = \left(\frac{2}{3}э_{ij}\cdot э_{ij}\right)^{1/2}$ is deformation intensity.

When $\omega(\varepsilon_u)=0$ the equations (2) describe physical linear visco-elasticity characteristics.

We write an equilibrium differential equation and the boundary conditions [3]

$$\left[\sigma_{ij}\left(\delta_{ki} + u_{k,j}\right)\right]_{,j} = F_k ,$$

$$\left[\sigma_{ij}\left(\delta_{ki} + u_{k,i}\right)\right]l_j = R_{vk} .$$

We solve the problem in displacements. To this end we put the components of the stress (2) in equilibrium equations and boundary conditions and subject to relation between the components of deformation and displacements (1).

As a result we obtain

$$L_k(u) = F_k + F_k' + F_{\omega k} , \tag{4}$$

$$M_k(u) = R_{vk} + R_{vk}' + R_{v\omega k} . \tag{5}$$

Here the following designations are introduced.

$$L_k(u)=G_0\nabla^2u_k+a_1\Delta_{,k}+u_{k,j}\left(G_0\nabla^2u_k+a_1\Delta_k\right)+G_0\left(u_{s,kj}u_{s,j}+\right.$$
$$\left.+u_{s,jj}u_{s,k}+u_{j,ij}\left(u_{i,j}+u_{j,i}\right)\right)+a_3\left(u_{s,s}u_{k,jj}+u_{r,s}u_{r,sk}\right),$$
$$M_k(u)=G_0\left(u_{k,i}+u_{j,i}\right)l_j+a_3u_{s,s}l_j+$$
$$+G_0\left(u_{s,k}u_{s,j}+u_{k,i}\left(u_{i,j}+u_{j,i}\right)\right)l_j+a_3\left(u_{r,s}^2+u_{s,s}u_{i,i}\right)l_i\ ,$$
$$F_k'=2G_0\int_0^t R(t-\tau)\vartheta_{kj,j}(\tau)d\tau+2G_0u_{k,j}\int_0^t R(t-\tau)\vartheta_{kj,j}(\tau)d\tau+$$
$$+2G_0u_{k,jj}\int_0^t R(t-\tau)\vartheta_{kj,j}(\tau)d\tau\ ,$$
$$F_{k\omega}=-2G_0\int_0^t R(t-\tau)\left(\omega(\varepsilon_u)\vartheta_{kj}\right)_{,j}d\tau+2G_0\left(\omega(\varepsilon_u)\vartheta_{kj}\right)_{,j}+2G_0\left(\omega(\varepsilon_u)\vartheta_{kj}\right)_{,j}u_{k,j}+$$
$$+2G_0\omega(\varepsilon_u)\vartheta_{kj}u_{k,jj}-2G_0u_{k,jj}\int_0^t R(t-\tau)\omega(\varepsilon_u)\vartheta_{kj}d\tau-2G_0u_{k,j}\int_0^t R(t-\tau)\left(\vartheta_{kj}\omega(\varepsilon_u)\right)_{,j}d\tau \quad (6)$$
$$R_{vk}'=2G_0\int_0^t R(t-\tau)\vartheta_{kj}^{(\tau)}d\tau\cdot l_j+2G_0u_{k,i}\int_0^t R(t-\tau)\vartheta_{kj}(\tau)d\tau\cdot l_j\ ,$$
$$R_{\omega vk}=2G_0\omega(\varepsilon_u)\vartheta_{kj}l_j+2G_0\omega(\varepsilon_u)u_{k,i}\vartheta_{ij}l_j-2G_0\int_0^t R(t-\tau)\omega(\varepsilon_u)\vartheta_{ij}(\tau)d\tau\cdot l_j-$$
$$-2G_0u_{k,i}\int_0^t R(t-\tau)\omega(\varepsilon_u)\vartheta_{ij}(\tau)d\tau\cdot l_j\ ,$$
$$\nabla^2v_i=u_{i,11}+u_{i,22},\quad \Delta=v_{1,1}+v_{2,2},\quad a_1=K+\frac{1}{3}G_0,$$
$$a_2=K+\frac{4}{3}G_0,\quad a_3=K-\frac{2}{3}G_0.$$

We associate to them two integral conditions on end-walls of pivot

$$\int_{S_1}\sigma_{33}dx_1dx_2=0\ , \quad (7)$$

$$\int_{S_1}\left(x_1\sigma_{13}-x_2\sigma_{23}\right)dx_1dx_2=M\ . \quad (8)$$

The obtained equilibrium equation (4), the boundary conditions (5) and the integral conditions (7),(8) completely describe a torsion problem of prismatic visco-elastic pivot of arbitrary section in geometrical and physical non-linear statement and are essential non-linear relative to the unknown displacements u_k. They differ from corresponding equations of geometrical and physical non-linear torsion problem of elastic pivot by that in them the fictitious forces F_k, R_{vk} are added to the given external forces $F_k', F_{vk}, R_{vk}', R_{v\omega k}$.

If we follow the Sent-Venant's semi-inverse method, we represent the displacement components analogously to [4] in the form of

$$u_1(x_1,x_2,x_3)=-\alpha(t)x_2x_3+\alpha^2(t)\left[-\frac{1}{2}x_1x_3+V_1(x_1,x_2,t)\right],$$

$$u_2(x_1,x_2,x_3)=\alpha(t)x_1x_3+\alpha^2(t)\left[-\frac{1}{2}x_2x_3^2+V_2(x_1,x_2,t)\right],$$

$$u_3(x_1,x_2,x_3)=\alpha(t)\varphi(x_1,x_2)+\alpha^2(t)c(t)x_3, \tag{9}$$

where $\alpha(t)$ is a torsion angle, $\varphi(x_1,x_2)$, $V_1(x_1,x_2,t)$, $V_2(x_1,x_2,t)$, $c(t)$ are desired functions.

If we substitute (9) in (4), (5), (7), (8) we are convinced that the problem fall into two problems. One of them is a classical torsion problem of elastic prismatic bodies which is described by the following system of equations

$$\Delta\varphi=0,$$
$$\left[(-x_2+\varphi_{,1})l_1+(x_1+\varphi_{,2})\right]_s=0. \tag{10}$$

The second problem corresponds to secondary effect by torsion of visco-elastic pivot arising due to geometrical and physical non-linearity and consists of the followings:

The equilibrium equations are

$$G_0\nabla^2V_1+a_1\Delta_{,1}=F_1+F_1'+F_{\omega 1},$$
$$G_0\nabla^2V_2+a_1\Delta_{,2}=F_2+F_2'+F_{\omega 2} \tag{11}$$

and the boundary conditions on lateral surface are

$$\left[(a_2V_{1,1}+a_3V_{2,2})\cdot l_1+G_0(V_{1,2}+V_{2,1})\cdot l_2\right]_S=$$
$$=\left[(R_{\nu 1}+R_{\nu 1}'+R_{\nu\omega 1})l_1+(R_{\nu 2}+R_{\nu 2}'+R_{\nu\omega 1})l_2\right]_S,$$
$$\left[(a_2V_{2,2}+a_3V_{1,1})\cdot l_2+G_0(V_{1,2}+V_{2,1})\cdot l_1\right]_S=$$
$$=\left[(R_{\nu 3}+R_{\nu 3}'+R_{\nu\omega 3})l_2+(R_{\nu 4}+R_{\nu 4}'+R_{\nu\omega 4})l_1\right]_S, \tag{12}$$

where $F_k', F_{\omega k}, R_{\nu k}', R_{\nu\omega k}$ are determined from (6) allowing for the permutation (9).

The obtained system (7), (8), (10), (11), (12) is a general statement of the torsion problem of visco-elastic pivot of arbitrary cross-section subject to geometrical physical non-linearity from which we can obtain a statement for the following cases:

1. $R(t-\tau)=0$, $\omega(\varepsilon_u)=0$ is geometrical non-linear torsion of elastic pivot.
2. $R(t-\tau)=0$, $\omega(\varepsilon_u)\neq 0$ is geometrical and physical non-linear torsion of elastic pivot.
3. $R(t-\tau)\neq 0$, $\omega(\varepsilon_u)=0$ is geometrical non-linear torsion of physical linear visco-elastic pivot.
4. $R(t-\tau)\neq 0$, $\omega(\varepsilon_u)\neq 0$ is geometrical non-linear torsion of physical non-linear visco-elastic pivot.

As a partial case we consider the torsion of visco-elastic pivot with circular cross-section of the radius R. The classical solution of this problem is given by the first sum of (9), in addition $\varphi(x_1,x_2)=0$. Consider the case when $R(t-\tau)\neq 0$, $\omega(\varepsilon_u)\neq 0$. In addition the formulas (11), (12), (7), (8) relatively get the form.

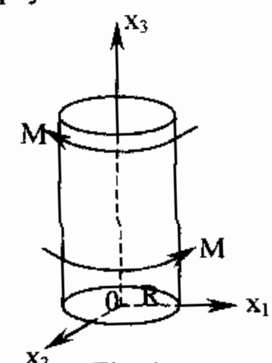

Fig. 1.

The equilibrium equation is

$$G_0\nabla^2V_1+a_1\Delta_{,1}=F_{11}+F_{11}'+F_{\omega 11},$$
$$G_0\nabla^2V_2+a_1\Delta_{,2}=F_{22}+F_{22}'+F_{\omega 22} \tag{13}$$

the boundary on lateral surface are

$$
\begin{gathered}
\left[\left(a_2 V_{1,1}+a_3 V_{2,2}\right)\cdot x_1+G_0\left(V_{1,2}+V_{2,1}\right)\cdot x_2\right]_S= \\
=\left[\left(R_{v11}+R'_{v11}+R_{v\omega 11}\right)x_1+\left(R_{v22}+R'_{v22}+R_{v\omega 22}\right)x_2\right]_S, \\
\left[\left(a_2 V_{2,2}+a_3 V_{1,1}\right)\cdot x_2+G_0\left(V_{1,2}+V_{2,1}\right)\cdot x_1\right]_S= \\
=\left[\left(R_{v33}+R'_{v33}+R_{v\omega 33}\right)x_2+\left(R_{v22}+R'_{v22}+R_{v\omega 22}\right)x_1\right]_S
\end{gathered} \tag{14}
$$

and the integral conditions on end-walls are

$$
\iint_{S_1}\left[c+a_3\left(V_{1,1}+V_{2,2}\right)\right]dx_1 dx_2=L_1+L'_1+L_{\omega 1}, \tag{15}
$$

$$
\alpha(t)\iint_{S_1}\left[\left(x_1^2+x_2^2\right)dx_1 dx_2+\alpha^3(t)\left(L_2+L'_2+L_{\omega 2}\right)\right]=M, \tag{16}
$$

where by F_{ii} and R_{vii} the known forces F_i, R_{vi} are denoted for simplicity and $F'_{ii}, F_{\omega ii}, R_{vii}, R_{v\omega\, ii}$ are calculated from (6), and $L_i, L'_i, L_{\omega i}$ from integral conditions for $\varphi(x_1,x_2)=0$. We solve the problem by the successive approximations method. For zero approximation we take $R^{(0)}(t-\tau)=0, \omega^{(0)}(\varepsilon_u=0)$ and we have geometrical non-linear torsion of elastic pivot with circular cross-section. This problem is solved by the author [5] and it has the form

$$
\begin{gathered}
V_1^{(0)}(x_1,x_2)=b_1^{(0)}R^2 x_1+3b_2^{(0)}x_1 x_2^2+b_3^{(0)}x_1^3, \\
V_2^{(0)}(x_1,x_2)=b_1^{(0)}R^2 x_2+3b_2^{(0)}x_2 x_1^2+b_3^{(0)}x_2^3,
\end{gathered}
$$

where $b_i^{(0)}$ are determined by the modulus K, G and the radius R [5]

$$
b_i^{(0)}=b_i^{(0)}(k,G_0,R).
$$

From the integral conditions the constant C is determined [5]:

$$
C^{(0)}=C^{(0)}(k,G,R).
$$

From the second integral conditions the connection between the torque M and the angle of torsion $\alpha^{(0)}$ [5]

$$
M=\frac{1}{2}\pi G R^4\alpha\left[1+(\alpha R)^2\gamma^{(0)}\right]
$$

is found, where $\gamma^{(0)}=\gamma^{(0)}\left(b_i^{(0)},C^{(0)}\right)$.

Substituting $V_i^{(0)}$ in (9) for zero approximation we determine $u_i^{(0)}$. For the problem of the first approximation we accept $R^{(0)}(t-\tau)\neq 0,\ \omega^{(0)}(\varepsilon_u)=0$. By the found $u_i^{(0)}$ we calculate $F_{ii}^{\prime(0)}, R_{vii}^{(0)}, L_i^{(0)}, L_i^{(0)}$ which enter the relations (13)-(16). Substituting them in the equilibrium equation (13), the boundary conditions (14) and the integral condition (15), (16) we have a geometrical non-linear problem, but in the existence of the fictitious forces $F_{ii}^{\prime(0)}$, $R_{vii}^{(0)}$, $L_i^{\prime(0)}$ the obtained problem is solved analogously to zero approximation. If we determine $V_i^{(1)}(t), C^{(1)}(t)$ we find $u_i^{(1)}(x_1,x_2,x_3,t)$.

For the second approximation we have $R^{(1)}(t-\tau)\neq 0,\ \omega^{(1)}(\varepsilon_u)\neq 0$, from the found $u_i^{(1)}$ we determine $F_{ii}^{\prime(1)}, F_{\omega ii}^{(1)}, R_{vii}^{(1)}, R_{v\omega\, ii}^{(1)}, L_i^{(1)}, L_i^{\prime(1)}, L_{\omega i}^{(1)}$ and we substitute them in (13)-(16). And at the second approximation we have a problem analogously to zero approximation for the known and additional fictitious forces. The problem is solved analogously to geometrical non-linear elastic torsion problem. Beginning with the first approximation the components of the displacement vector are functions of coordinate and time.

For arbitrary k -th approximation we have the problem:

The equilibrium equation is

$$G_0\nabla^2 V_1^{(k)} + a_1\Delta_{,1} = F_{11}^{(k-1)} + F_{11}'^{(k-1)} + F_{\omega 11}^{(k-1)},$$
$$G_0\nabla^2 V_2^{(k)} + a_1\Delta_{,2} = F_{22}^{(k-1)} + F_{22}'^{(k-1)} + F_{\omega 22}^{(k-1)}$$

the boundary conditions are

$$\left[\left(a_2 V_{1,1}^{(k)} + a_3 V_{2,2}^{(k)}\right)\cdot x_1 + G_0\left(V_{1,2}^{(k)} + V_{2,1}^{(k)}\right)\cdot x_2\right]_S =$$
$$= \left[\left(R_{v11}^{(k-1)} + R_{v11}'^{(k-1)} + R_{v\omega 11}^{(k-1)}\right)x_1 + \left(R_{v22}^{(k-1)} + R_{v22}'^{(k-1)} + R_{v\omega 22}\right)x_2\right]_S ,$$
$$\left[\left(a_2 V_{2,2}^{(k)} + a_3 V_{1,1}^{(k)}\right)\cdot x_2 + G\left(V_{1,2}^{(k)} + V_{2,1}^{(k)}\right)\cdot x_1\right]_S =$$
$$= \left[\left(R_{v33}^{(k-1)} + R_{v33}'^{k-1} + R_{v\omega 33}^{(k-1)}\right)x_2 + \left(R_{v22}^{(k-1)} + R_{v22}'^{(k-1)} + R_{v\omega 22}\right)x_1\right]_S .$$

The integral conditions on end-walls are

$$\iint_{S_1}\left[a_2 c^{(k)} + a_3\left(V_{1,1}^{(k)} + V_{2,2}^{(k)}\right)\right]dx_1 dx_2 = L_1^{(k-1)} + L_1'^{(k-1)} + L_{\omega 1}^{(k-1)},$$

$$\alpha^k(t)\iint\left(x_1^2 + x_2^2\right)dx_1 dx_2 + \left(\alpha^{(k)}(t)\right)^3\left(L_2^{(k-1)} + L'^{(k-1)}_2 + L_{\omega 2}^{(k-1)}\right) = M .$$

Thus for determination of k -th approximation we have a problem on geometrical non-linear torsion of elastic pivot with additional fictitious forces determined by the previous $(k-1)$-th approximation. The problem of every next approximation is solved analogously to zero approximation, $V_i^{(k)}(x_1, x_2, t), C^{(k)}(t), \alpha^{(k)}(t)$ are found. Then the displacements $u_i^{(k)}$ with the help of these quantities following [5] are determined in the form of

$$u_1^{(k)}(x_1, x_2, x_3, t) = -\alpha^{(k)}(t)x_2 x_3 + \left(\alpha^{(k)}(t)\right)^3\left[-\frac{1}{2}x_1 x_3^2 + V_1^{(k)}(x_1, x_2, t)\right],$$

$$u_2^{(k)}(x_1, x_2, x_3, t) = \alpha^{(k)}(t)x_1 x_3 + \left(\alpha^{(k)}(t)\right)^3\left[-\frac{1}{2}x_1 x_3^2 + V_2^{(k)}(x_1, x_2, t)\right],$$

$$u_3^{(k)}(x_1, x_2, x_3, t) = \left(\alpha^{(k)}(t)\right)^2 c(t)x_3 .$$

Thus the considered successive approximation method allows to construct the solution of torsion problem of visco-elastic circular pivot in geometrical and physical non-linear statement.

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ON NORMAL IMPACT ON FLEXIBLE ELASTIC FILAMENT

Abstract

In the paper the construction of solving of the problem on normal impact with constant velocity by obtuse rigid wedge on elastic filament is given. It is assumed that the velocity of the breakpoint is less than the velocity of elastic wave in filament.

In works [1-4] the behaviour of flexible filament at transverse impact by rigid wedge is investigated when the flexure part of filament covers to check of wedge. In the present paper the solution of the problem on normal impact by rigid symmetric wedge having plane fore-part on flexible elastic filament is investigated. It's accepted that domain beyond breakpoints A and A_1 covers the surface of bombarding body, and velocity of the breakpoint A (and A_1) is less that velocity of elastic wave in the filament $b = Vctg\gamma < a_0$.

§1. Let the normal impact by symmetric wedge with plane fore-part with the constant velocity V be performed by infinite long flexible linear-elastic, rectilinear non-strained filament. After impact in filament four elastic waves whose fronts are N_1, C_1, C, N and two waves of strong break (break point) A and A_1 arise (pic.1). Denote the width BB_2 by $2L$. The behaviour of the filament in the domains $NABCO$ and $N_1A_1B_1C_1O$ are the same. The velocity of particles of filament in these domains are directed along the filament respectively. In the domains OC and OC_1 the filament is at rest to the zero time $t = \frac{L}{a_0}$ $\left(0 \le t \le \frac{L}{\alpha_0}\right)$ relative to "wedge". Since the impact is performed with constant velocity, then in originating domains the filaments determining the parameters are constant. It's assumed that the friction is absent in covering domain between the filament and bombarding body.

In fig.1 B_1 and B are stationary break points and the motion of filaments relative to these points are taken as motion via fixed block [5]. The following designations are accepted: ε is deformation, σ is stress, ϑ is velocity of particles of filament, $a_0 = \sqrt{E\rho^{-1}}$ is velocity of elastic wave; E is Young's modulus, ρ is density, γ is an angle between the initial position of filament and the check of wedge BA (and B_1A_1) (pic.1), t is time, x is Lagrangian coordinate.

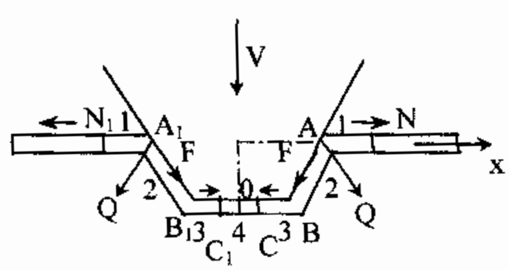

Fig. 1.