MECHANICS

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PROPAGATION OF THE FAILURE FRONT IN THE DAMAGED ROUND THICK PIPE SUBJECTED TO TEMPERATURE

Abstract

The investigation of the thermoelastic failure of the thick pipe which occurs due to the temperature difference of pumpable product and environment is an important engineering problem. In the present paper this process is investigated in the aggregate with the failure process of the material of the pipe. Taking into account the resistance of the material of the pipe behind the failure front is important. The influence of this factor on the character of distribution of the failure front defining the process of failure of the pipe is clarified.

In spite of numerous articles on the thermoplastic failure of solids, this tendency: remains actual. It is associated with the fact, that majority of machine components, mechanisms and constructions work in extremal thermal conditions. However, we have to take into account the factor of formation and accumulation of defects in the bulk. The attendant process of damaging can make important contribution and at times is determining one in the process of thermoelastic failure. In the paper [1] the process of thermoelastic failure of the thick cylindrical pipe was investigated under conditions of the plane deformation, when the temperatures on the interior and exterior surfaces of the pipe different by their value are given. Then criterion of failure of the damaging theory [2] on the greatest stress, which is tangential one was used. The time of failure of the interior surface the incubating time was found. The equation of motion of the failure front provided that the material of the pipe behind the front of failure completely loses its load-carrying capacity, was obtained and analyzed. In the present paper this investigation was carried out taking into account the resistance to loading of the material of the pipe behind the failure front. It's supposed that the material of the pipe behind the failure front preserves its load-carrying capacity to a less extent. That is in each moment of time which is greater than the incubating one is the pipe of two-layer construction, whose a part before the failure front of the source material and the other part behind the failure front is the material with sharply decreased rigid characteristics.

Analysis of the formulas of hoop thermoelastic stress, which is maximal one shows that in the case when the temperature of interior surface exceeds the temperature of exterior surface which is accepted in [1], failure first begins on the interior surface, where the hoop stress achieves its maximum and later on the failure front representing on expanding circular zone moves to the exterior surface of the pipe. In t he present paper it is also assumed that the interior temperature exceeds the exterior one. In this case the source material before the failure represents the domain S_2, adjoining to the exterior boundary of the radius R_2 of the pipe (fig.1). The domain S_1 adjoining to the interior surface of the radiuses R_0 of the pipe represents the domain behind the failure front. The failure front is a cylindrical surface of the alternate increasing radius R_1.

We'll denote all the parameters related to the domains S_1 and S_2 by the corresponding index numbers. Previously we'll give the solution of the thermoelastic problem for the thick two-layer cylindrical pipe under conditions of plane deformation when the temperatures T_0 and T_2 on the interior and exterior surfaces are given.

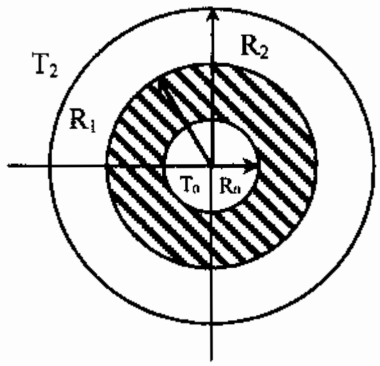

Fig.1.

For the stationary temperature field the distribution of temperature on circular section of the pipe will be the same as in [3]:

$$\begin{cases} T = P_1 \ln r + P_2 \ ; \\ P_1 = \dfrac{T_2 - T_0}{\ln R_2 / R_1} ; \ \ P_2 = \dfrac{T_0 \ln R_2 - T_2 \ln R_0}{\ln R_2 / R_0} , \end{cases} \tag{1}$$

where r is the current radius.

Then the necessary later on radial displacements in both sections and the hoop stress in the domain S_2 will have the form:

$$\begin{gathered} rU_r^{(1)} = \frac{1+\nu_1}{1-\nu_1} \cdot \frac{\chi_1}{E_1} \left[\int_{R_0}^{r} \xi T(\xi) d\xi + \frac{(1-2\nu_1) r^2 + R_0^2}{R_1^2 - R_0^2} \int_{R_0}^{R_1} \xi T(\xi) d\xi \right] + \\ + \frac{Q R_1^2 (1+\nu_1)}{E_1 \left(R_1^2 - R_0^2\right)} \left[(1-2\nu_1) r^2 + R_0^2 \right]; \\ rU_r^{(2)} = \frac{1+\nu_2}{1-\nu_2} \cdot \frac{\chi_2}{E_2} \left[\int_{R_1}^{r} \xi T(\xi) d\xi + \frac{(1-2\nu_2) r^2 + R_1^2}{R_2^2 - R_1^2} \int_{R_1}^{R_2} \xi T(\xi) d\xi \right] - \\ - \frac{Q R_1^2 (1+\nu_2)}{E_2 \left(R_2^2 - R_1^2\right)} \left[(1-2\nu_2) r^2 + R_2^2 \right]; \end{gathered} \tag{2}$$

$$\sigma_\theta^{(2)} = \frac{\chi}{1-\nu_2} \cdot \frac{1}{r^2} \left[\frac{r^2 + R_1^2}{R_2^2 - R_1^2} \int_{R_1}^{R_2} \xi T(\xi) d\xi + \int_{R_1}^{r} \xi T(\xi) d\xi - r^2 T(r) \right] - \frac{Q R_1^2}{R_2^2 - R_1^2} \left(1 + \frac{R_2^2}{r^2} \right). \tag{3}$$

There Q is the pressure on the contact surface of layers.

The incubating time will be defined by the formula, obtained in [1] based on the criterion of failure [2]:

$$\left(1 + M^*\right) \sigma_\theta = \sigma_0 , \tag{4}$$

where M^* is an operator of damaging, σ_0 is the ultimate strength of defectless material.

For obtaining the incubating time for the hoop stress it's necessary to take its value on the interior surface of the pipe, i.e. it's necessary to assign in (4) that

$\sigma_\theta = \sigma_\theta \big|_{r=R_0}$. Moreover, given value can be obtained from (3), where we have to replace R_1 by R_0 and assign $Q=0$. Then for the incubating time to

$$\int_0^{t_0} M(\tau)d\tau = \frac{\sigma_0\left(1-\beta_0^2\right)\ln\beta_0}{1-\beta_0^2+2\ln\beta_0} - 1; \;\; \beta_0 = \frac{R_0}{R_2} \tag{5}$$

takes place, here $M(t)$ is the kernel of the operator of damaging.

For the time $t > t_0$ the picture of the process of failure which is led in fig.1 takes place.

For obtaining the equation of the failure front we'll previously transform the expressions (2), (3) replacing the modulus of elasticity E_2 by the corresponding operator containing the operators of damaging which according to the theory developed in [2] has the form:

$$\frac{1}{\tilde{E}_2} = \frac{1}{E_2}\left(1 + M^*\right), \tag{6}$$

where M^* is the integral operator of hereditary type. We'll assign also the following denotations

$$\frac{R_1}{R_2} = \beta(\tau); \;\; \frac{r}{R_2} = \begin{cases} \beta(t); & for \;\; R_1 \le r \le R_2 \\ s; & for \;\; R_0 \le r \le R_1 \end{cases}; \; t \ge \tau . \tag{7}$$

For passing to dimensionless quantities we'll give the following form to the formula (1):

$$\begin{gathered} \frac{T}{T_0} = \hat{T}(x) = p\ln x + T_i , \\ x = \frac{r}{R_2}; \; T_i = \frac{T_2}{T_0}; \; p = \frac{1-T_i}{\ln\beta_0} . \end{gathered} \tag{8}$$

Besides we'll assign the following dimensionless quantities:

$$\begin{cases} \dfrac{U_r^{(k)}}{R_2} = U_r^{(k)}; \;\; k=1,2; \quad \dfrac{\chi_k T_0}{E_k} = \gamma_k ; \; \dfrac{\sigma_\theta^{(2)}(1-\nu_2)}{\chi_2 T_0} = \hat{\sigma}_\theta^{(2)} ; \\ \dfrac{Q(1-\nu_2)}{\chi_2 T_0} = q ; \;\; \dfrac{\sigma_0(1-\nu_2)}{\chi_2 T_0} = \sigma_0^* . \end{cases} \tag{9}$$

In addition assume that times t and r are dismeasured simultaneously with dismeasuring the parameters of the kernel of the operators of damaging M^*. For example, if $M(t) = m\exp(-\lambda t)$, then we have to consider $\lambda \to \lambda / m$ and $t \to mt$ dimensionless herewith that the parameter m have to be assumed as unity, i.e. the dimensionless kernel has the form $\exp(-\lambda t)$.

Taking into account denotations introduced above we'll obtain the following expressions for the dimensionless displacements and stress

$$\begin{cases} sU_r^{(1)}(s,\tau)=\gamma_1\frac{1+\nu_1}{1-\nu_1}\left\{\int_{\beta_0}^{s}\xi\hat{T}(\xi)d\xi+\frac{(1-2\nu_1)s^2+\beta_0^2}{\beta^2(\tau)-\beta_0^2}\int_{\beta_0}^{\beta(\tau)}\xi\hat{T}(\xi)d\xi\right\}+ \\ +\gamma_1\frac{1+\nu_1}{1-\nu_2}\cdot\frac{\chi_2}{\chi_1}q(\tau)\frac{\beta^2(\tau)}{\beta^2(\tau)-\beta_0^2}\left[(1-2\nu_1)s^2+\beta_0^2\right]; \\ \beta(t)U_r^{(2)}(t,\tau)=(1+M^*)\left\{\frac{1+\nu_2}{1-\nu_2}\gamma_2\left[\int_{\beta(\tau)}^{\beta(t)}\xi\hat{T}(\xi)d\xi+\frac{(1-2\nu_2)\beta^2(t)+\beta^2(\tau)}{1-\beta^2(\tau)}\int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi\right]-\right. \\ \left.-\gamma_2\frac{1+\nu_2}{1-\nu_2}q(\tau)\frac{\beta^2(\tau)}{1-\beta^2(\tau)}\left[(1-2\nu_2)\beta^2(t)+1\right]\right\} \end{cases} \tag{10}$$

$$\hat{\sigma}_\theta^{(2)}(t,\tau)=\frac{1}{\beta^2(t)}\left\{\frac{\beta^2(t)+\beta^2(\tau)}{1-\beta^2(\tau)}\int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi+\int_{\beta(\tau)}^{\beta(t)}\xi\hat{T}(\xi)d\xi-\beta^2(t)\hat{T}(\beta(t))\right\}-$$
$$-\frac{q(\tau)\beta^2(\tau)}{1-\beta^2(\tau)}\left(1+\frac{1}{\beta^2(t)}\right). \tag{11}$$

Here $U_2^{(2)}(t,\tau)$ and $\hat{\sigma}_\theta^{(2)}(t,\tau)$ are the displacement and the stress on the surface of the radius $r, R_1\le r\le R_2$ respectively where the failure front will reach at the moment of time t, when the failure front has the coordinate R_1 corresponding to time τ. In addition $\beta=\beta(t)$ is the dimensionless pressure on the failure front.

The criterion of failure defining the low of motion of the failure front according to (4) has the form:

$$\hat{\sigma}_\theta^{(2)}(t,t)+\int_0^t M(t-\tau)\hat{\sigma}_\theta^{(2)}(t,\tau)d\tau=\sigma_0^*. \tag{12}$$

Taking into account (11) in (12) we'll find the explicit form of the equation of failure front

$$\frac{2}{1-\beta^2(t)}\int_{\beta(t)}^{1}\xi\hat{T}(\xi)d\xi-\hat{T}(\beta(t))-q(t)\frac{1+\beta^2(t)}{1-\beta^2(t)}+$$
$$+\frac{1}{\beta^2(t)}\int_0^t M(t-\tau)\left\{\frac{\beta^2(t)+\beta^2(\tau)}{1-\beta^2(\tau)}\int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi+\int_{\beta(\tau)}^{\beta(t)}\xi\hat{T}(\xi)d\xi\right\}d\tau-$$
$$-\hat{T}(\beta(t))\int_0^t M(\tau)d\tau-\left(1+\frac{1}{\beta^2(t)}\right)\int_0^t M(t-\tau)\frac{q(\tau)\beta^2(\tau)}{1-\beta^2(t)}d\tau=\sigma_0^*. \tag{13}$$

This equation contains two unknown functions $\beta(t)$ and $q(t)$. We'll obtain the second equation from condition of continuity of displacement on the failure front

$$\frac{\gamma_1}{\gamma_2}\frac{1+\nu_1}{1-\nu_1}\frac{1-\nu_2}{1+\nu_2}\left\{\left[1+\frac{(1-2\nu_1)\beta^2(t)+\beta_0^2}{\beta^2(t)-\beta_0^2}\right]\int_{\beta_0}^{\beta(t)}\xi\hat{T}(\xi)d\xi+\frac{1-\nu_1}{1-\nu_2}\frac{\chi_2}{\chi_1}q(t)\times\right.$$
$$\left.\times\frac{\beta^2(t)\left[(1-2\nu_1)\beta^2(t)+\beta_0^2\right]}{\beta^2(t)-\beta_0^2}\right\}=\frac{2(1-\nu_2)\beta^2(t)}{1-\beta^2(t)}\int_{\beta}^{1}\xi\hat{T}(\xi)d\xi-$$
$$-q(t)\frac{\beta^2(t)}{1-\beta^2(t)}\left[(1-2\nu_2)\beta^2(t)+1\right]+\int_0^t M(t-\tau)\left\{\int_{\beta(t)}^{\beta(t)}\xi\hat{T}(\xi)d\xi+\right.$$
$$\left.+\frac{(1-2\nu_1)\beta^2(t)+\beta^2(\tau)}{1-\beta^2(\tau)}\int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi\right\}-\left[(1-2\nu_2)\beta^2(t)+1\right]\int_0^t M(t-\tau)\frac{q(\tau)\beta^2(\tau)}{1-\beta^2(\tau)}d\tau\,. \quad (14)$$

Thus, we have system of two nonlinear integral equations (13) and (14) with respect to two unknown functions $q(t)$ and $\beta(t)$. It must be noted that evaluating the integral term with the function $q(\tau)$ from (13) and substituting into (14) we can find the explicit analytical expression for the function $q(t)$ by means of the function $\beta(t)$. Then substituting the obtained expression in (13) we can obtain one integral equation with respect to one function $\beta(t)$ - the radial coordinate of the failure front.

For instantiation of the system of equations (13), (14) we'll take into account the expression of the functions of distribution of the temperature (8). Consider the particular case $T_i=0$. Then for the temperature integrals we'll obtain:

$$\begin{cases}\int_{\beta(t)}^{1}\xi\hat{T}(\xi)d\xi=-\frac{1}{4\ln\beta_0}\left\{1-(1-2\ln\beta(t))\beta^2(t)\right\};\\[2mm] \int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi=-\frac{1}{4\ln\beta_0}\left\{1-(1-2\ln\beta(\tau))\beta^2(\tau)\right\};\\[2mm] \int_{\beta(\tau)}^{\beta(t)}\xi\hat{T}(\xi)d\xi=\frac{1}{4\ln\beta_0}\left\{-(1-2\ln\beta(t))\beta^2(t)+(1-2\ln\beta(\tau))\beta^2(\tau)\right\};\\[2mm] \int_{\beta_0}^{\beta(t)}\xi\hat{T}(\xi)d\xi=\frac{1}{4\ln\beta_0}\left\{-(1-2\ln\beta(t))\beta^2(t)+(1-2\ln\beta_0)\beta_0^2\right\};\end{cases} \quad (15)$$

In addition we have to note that for unknown functions $q(t)$ and $\beta(t)$ appearing in the system of equations (13), (14) the following representation is true

$$\beta(t)=\begin{cases}\beta_0;\ 0\le t\le t_0;\\ \beta(t);\ t>t_0;\end{cases}\qquad q(t)=\begin{cases}0;\ 0\le t\le t_0;\\ q(t);\ t>t_0;\end{cases} \quad (16)$$

and equations (13) and (14) hold for the time $t>t_0$. We'll introduce denotations

$$
\begin{cases}
A(\beta(t),t)=\left\{\dfrac{2}{1-\beta^2(t)}\displaystyle\int_{\beta(t)}^{1}\xi\hat{T}(\xi)d\xi - T(\beta(t))\left(1+\int_0^t M(\tau)d\tau\right)\right\}\dfrac{\beta^2(t)}{1+\beta^2(t)};\\
B(\beta(t),\beta(\tau))=\left\{\dfrac{\beta^2(t)+\beta^2(\tau)}{1-\beta^2(\tau)}\displaystyle\int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi+\int_{\beta(\tau)}^{\beta(t)}\xi\hat{T}(\xi)d\xi\right\}\dfrac{1}{\beta^2(t)+1};\\
\dfrac{\gamma_1}{\gamma_2}\dfrac{(1+v_1)(1-v_2)}{(1-v_1)(1+v_2)}=a;\ \dfrac{1-v_1}{1-v_2}\dfrac{\chi_2}{\chi_1}=b;\ \dfrac{\sigma_0^*\beta^2(t)}{1+\beta^2(t)}=\sigma(\beta(t));\\
C(\beta(t))=\left\{a\left[1+\dfrac{(1-2v_1)\beta^2(t)+\beta_0^2}{\beta^2(t)-\beta_0^2}\right]\displaystyle\int_{\beta_0}^{\beta(t)}\xi\hat{T}(\xi)d\xi-\dfrac{2(1-v_2)\beta^2(t)}{1-\beta^2(t)}\int_{\beta(t)}^{1}\xi\hat{T}(\xi)d\xi\right\}\times\\
\times\dfrac{1}{(1-2v_2)\beta^2(t)+1};\\
D(\beta(t))=\left\{ab\dfrac{\beta^2(t)\left[(1-2v_1)\beta^2(t)+\beta_0^2\right]}{\beta^2(t)-\beta_0^2}+\dfrac{\beta^2(t)}{1-\beta^2(t)}\left[(1-2v_2)\beta^2(t)+1\right]\right\}\times\\
\times\dfrac{1}{(1-2v_2)\beta^2(t)+1};\\
E(\beta(t),\beta(\tau))=\left\{\displaystyle\int_{\beta(\tau)}^{\beta(t)}\xi\hat{T}(\xi)d\xi+\dfrac{(1-2v_2)\beta^2(t)+\beta^2(\tau)}{1-\beta^2(\tau)}\int_{\beta(\tau)}^{1}\xi\hat{T}(\xi)d\xi\right\}\times\\
\times\dfrac{1}{(1-2v_2)\beta^2(t)+1}.
\end{cases}
\tag{17}
$$

Then we'll give the form to the equations (13) and (14)

$$
A(\beta(t),t)+\int_0^t M(t-\tau)B(\beta(t),\beta(\tau))d\tau-\frac{\beta^2(t)}{1-\beta^2(t)}q(t)-
$$

$$
-\int_0^t M(t-\tau)\frac{q(\tau)\beta^2(\tau)}{1-\beta^2(\tau)}d\tau=\sigma(\beta(t)); \tag{18}
$$

$$
C(\beta(t))-\int_0^t M(t-\tau)E(\beta(t),\beta(\tau))d\tau+
$$

$$
+D(\beta(t))q(t)+\int_0^t M(t-\tau)\frac{q(\tau)\beta^2(\tau)}{1-\beta^2(\tau)}d\tau=0. \tag{19}
$$

Adding (18) and (19) we'll find the explicit expression for $q(t)$ the function of pressure on the failure front

$$
q(t)=\left\{\sigma(\beta(t))-A(\beta(t),t)-C(\beta(t))+\right.
$$

$$
\left.+\int_0^t M(t-\tau)\left[E(\beta(t),\beta(\tau))-B(\beta(t),\beta(\tau))\right]d\tau\right\}\left[D(\beta(t))-\frac{\beta^2(t)}{1-\beta^2(t)}\right]^{-1}. \tag{20}
$$

Taking into account the obtained expression in (18) or (19) we'll obtain one integral equation with respect to the function of radial coordinate of the failure front $\beta(t)$. We'll substitute (20) in (18)

$$A(\beta(t),t)+\int_0^t M(t-\tau)B(\beta(t),\beta(\tau))d\tau-\frac{\beta^2(t)}{(1-\beta^2(t))D(\beta(t)-\beta^2(t))}\times$$

$$\times\left\{\sigma(\beta(t))-A(\beta(t),t)-C(\beta(t))+\int_0^t M(t-\tau)[E(\beta(t),\beta(\tau))-B(\beta(t),\beta(\tau))]d\tau\right\}-$$

$$-\int_0^t M(t-\tau)\frac{\beta^2(\tau)}{(1-\beta^2(\tau))D(\beta(\tau)-\beta^2(\tau))}\left\{\sigma(\beta(\tau))-A(\beta(\tau),\tau)-C(\beta(\tau))\ +\right.$$

$$\left.+\int_0^\tau M(\tau-\omega)[E(\beta(\tau),\beta(\omega))-B(\beta(\tau),\beta(\omega))]d\omega\right\}d\tau=\sigma(\beta(t)). \qquad (21)$$

The equation (21) is nonlinear concerned with none of known types of integral equations. Its analytical solving is not possible. We'll use approximate numerical method for solving it based on replacing integral by integral sums. On each step of calculation this leads to necessity of solving the nonlinear algebraic equation.

The calculation was realized for the particular case of dimensionless kernel

$$M(t)=1,\ \text{for}\ \chi_2\neq\chi_1=1;\ \nu_1=\nu_2;\ \sigma_0^*=4;$$

for dimensional relation of modulus of elasticity behind the failure front and before it

$$\frac{\gamma_2}{\gamma_1}=\frac{E_1}{E_2}=\alpha=0;\ 0{,}001;\ 0{,}005;\ 0{,}01;\ 902;\ 903.$$

The curves characterizing the moving of the failure front constructed on the basis of numerical calculation is in the fig.2.

From them it follows that the failure front is spread with the accelerated velocity and the weakening of the material behind the failure front leads to noticeable acceleration of the failure process.

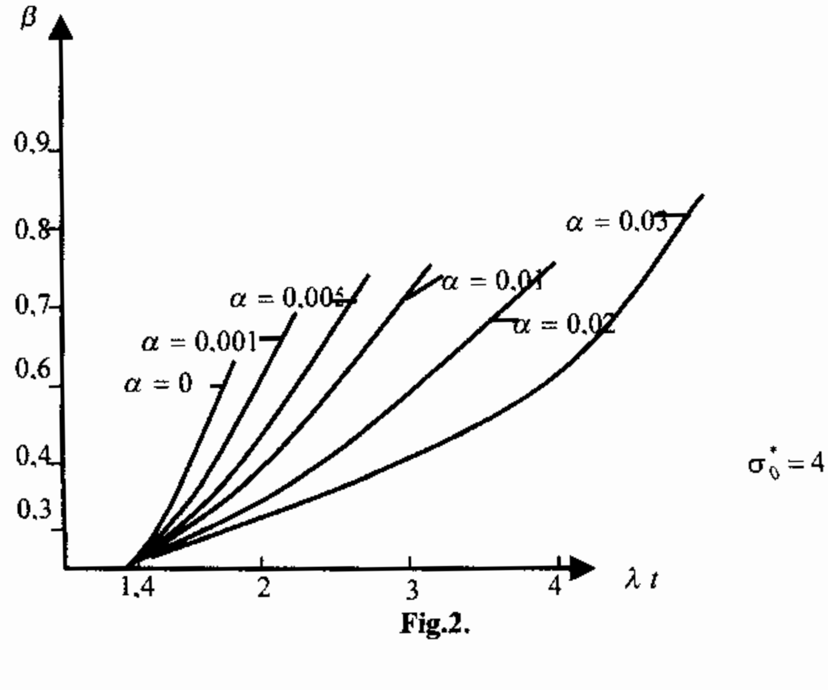

Fig.2.

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DEVELOPMENT OF CANAL IN A PACKED LAYER

Abstract

The problem on the development of surface canal with increase of expenditure of flow is solved, the domains of physical parameters of problem responding to homogeneous pseudo-liquefaction and condition with the formation of open canal in a packed layer are determined.

One of the basic problems frequently originating on pseudo-liquefacted layer is formation of open canals in layer. Sufficiently small size initial canals are always present in dense layer of rigid particles in consequence of non-uniformity of packing. In pseudo-liquefacted layer we can consider such initial canals as some unavoidable fluctuations in random mutual disposition of particles. In this addition the question on the laws of development of long and narrow canals in dense and pseudo-liquefacted layers of rigid particles arises.

In paper [2] effective solution of hydrodynamic problem of flow in the given canal of finite length in packed layer of rigid particles is constructed and the sewing parameter is determined.

Now we consider the question on development of initial canal in packed layer. It's obvious that on the bottom of canal the greatest gradient of pressure is valid, therefore with increase of ϑ_0 the local zone of pseudo-liquefacted state, and also separation and wear of particles by flow of liquid arise first on the bottom of canal. Namely the local separation of particles from the bottom of canal is responsible for its development along its axis in packed layer of particles.

Later on we'll apply the general approach of collapse mechanics [5].

The force moving the canal is given by the next invariant of the Γ-integral [3,4,6]

$$\Gamma = \gamma \int_{\Sigma} \left(-\vartheta^2 n_z + 2\vartheta_n \vartheta_z \right) d\Sigma ,$$

$$\left(\gamma = \rho / 2\varepsilon^2 ,\ \vartheta^2 = \vartheta_r^2 + \vartheta_z^2 \right). \tag{1.1}$$

Here Σ is an arbitrary non-closed surface covering the bottom of cylindric cavity in packed layer, the contour of Σ coincides with some closed curve on the cylinder $r = r_0$, n_z is a constituent of the vector by unit external normal to Σ on the axis z.

We remined that the right hand side of the equality (1.1) is equal to zero for any closed surface of integration in packed layer. Using the invariance property, it's convenient to choose Σ that $\Sigma = \Sigma_0 + \Sigma_+ + \Sigma_-$, where Σ_0 is a lateral surface of a cylinder $r = r_*$, Σ_+ is an end-wall of a cylinder $z = -l - L$, Σ_- is an end wall of a cylinder with the hole $z = -l + L$, $r_0 < r < r_*$.

We choose the distance L as minimal so that one could ignore the influence of three-dimensional effects of flow near the bottom of the canal, not subjected to the description in frames of suggested approach. We remind that the bottom of canal corresponds to $z < -l$, $r < r_0$. The integral (1.1) by the surface Σ_0 is equal to zero, since $n_z = 0, \vartheta_r = 0$ on Σ_0. The integral (1.1) by the end-wall Σ_+ is equal to $-\pi \gamma r_n^2 \vartheta_0^2$,