VAHABOV N.G.

LOCALIZATION OF SPECTRUM AND ITS APPLICATIONS, III
NUMERICAL RANGE AND SPECTRUM OF OPERATOR-FUNCTIONS

Abstract

In the paper the classes of operator-functions are selected in a Banach space for which the analogs of theorems on behavior of numerical ranges different geometrical operations are proved. These geometrical properties of numerical ranges of operator-functions are applied to obtain the localization relations for spectrum of operator-functions by its numerical ranges.

Introduction. One of the problems of spectral theory is obtaining for operator functions analogies of classical theorems on localization of spectrum of operator by its numerical ranges. As was noted by Hadeler K. [4] it is impossible for arbitrary operator functions and he proved the analogy of Wintner-Stone's theorem on localization of spectrum of linear multiparametric operator in a real Hilbert space. The sufficient condition of localizability of spectrum of o.f. given by Hadeler suggests the selection of one-(multi) parameter operators in Banach space on which a series of facts of the classical theory of numerical ranges can be carried over. On the other hand technique of work A.Brown's and R.Duglas' [5] allows to adapt and use the scheme of Hadeler's proof for holomorphic o.f. in a complex Banach space.

The basic aim of the present paper is to obtain for o.f. analogies of the theorems on behavior of numerical ranges at different geometric operators and apply them to the questions on localization of spectrum of operator functions. The notice of these results is given in [14b].

Attraction of geometric properties of numerical ranges of o.f. and Banach spaces brings to light the main role of Teoplitz's theorem on localization of point spectrum and Wintner-Lumer's theorem on localization of approximative points spectrum in the questions on localizability of spectrum (and its parts) by numerical ranges. It turned out that all the localization theorems can be derived from these two theorems and if we use the adaptation of Berberian's construction for the spaces with semiinner product-just from Teoplitz's theorem.

We'll describe contents of the paper which is a continuation of the first two parts [14c]. In §5 the three types of domains of regularity of o.f. in Banach space are introduced and influence of geometric properties of the space and numerical ranges on its hierarchy is considered. The three natural classes of holomorphic o.f. in Banach space for which the analogs of geometric and spectral properties of numerical ranges of operators are selected.

In §6 the one-parameter analogs of geometric properties of numerical ranges of operators: G. Lumer's (K.Mc. Gregor) theorem on closed convex hull of Lumerian (Bauerian) numerical range; B. Bollobas's theorem on behavior of Bauerian numerical range relative to conjugation and S.Berberian's and G.Orland's theorem on extension (by Berberian) of Hausdorff numerical range are proved. Then using these geometric properties of numerical congruences and also the localization correlation for the compression spectrum [14c, §1, proposition 1] the one-parameter analog of theorems: Winter-Lumer's theorem on localization of approximative point spectrum by Lumerian numerical range; Lumer's theorem on localization of spectrum by algebraic numerical range; William's theorem on localization of spectrum by Bauerian *numerical range and*

Winter-Stone's theorem on localization of spectrum by Hausdorfian numerical range are derived.

The notions, terms and notations used here are reflected in previous parts of the paper [14c].

§5. Numerical ranges and regular operator functions.

5.1. We'll consider the following objects whose defining is motivated by the notion of numerical range of linear o.f. in Hilbert space introduced by Hadeler [4].

Availability of three types of numerical range of operators in Banach space [14c] generates three kinds of numerical ranges of o.f. of general form which are the exact analogs of Lumerian (Bauerian) (or spatial) and algebraic numerical ranges of operator.

Let $A: G \to B(x)$ be arbitrary operator valued function (briefly o.f.) defined in domain G of complex plane C with the values in algebra $B(x)$ of bounded linear operators acting in Banach space X with the norm ν and let s be semiscalar product (s.i.p.) in X, coordinated with the norm γ.

$$W_s[A] = \{\lambda \in G : s[A(\lambda)x, x],\ \ x \in S(x)\} \tag{5.1}$$

is called Lumerian numerical range $W_s[A]$ of the o.f. A respondent to s.i.p. s

$$V_\nu[A] = \{\lambda \in G : f(A(\lambda)x) = 0,\ f \in D(x, X),\ x \in S(X)\} \tag{5.2}$$

is called Bauerian numerical range $V_\nu[A]$ of the o.f. A.

Here $S(X)$ is a unit sphere in X and $D(x, X) = \{f \in X^* : f(x) = \|f\| = \|x\| = 1\}$.

To the previous two typer of numerical ranges of o.f. we'll add one more denote it by $B(X)$, briefly by B and consider the set $D(I, B) = \{f \in S(B^*) : f(I) = 1\}$, where I is a unit in B and $S(B^*)$ is a unit sphere in the conjugate Banach space B^*. The set

$$\mathcal{V}[A] = \{\lambda \in G : f(A(\lambda)) = 0,\ f \in D(I, B)\} \tag{5.3}$$

is called algebraic numerical range $\mathcal{V}[A]$ of the o.f. A.

Right away note that for the operator $T \in B(X)$ the "classical" numerical range $W_s(T), V_\nu(T)$ and $\mathcal{V}(T)$ are particular cases of $W_s[A], V_\nu[A]$ and $\mathcal{V}[A]$ respectively if we'll take o.f. of the form

$$A(\lambda) = T - \lambda,\ \lambda \in G = \mathbf{C}, \tag{5.4}$$

which later on we'll call "classical" o.f., generated by the operator T.

Some of initial properties of numerical ranges of operators remain valid unchanged for arbitrary or continuous o.f. in Banach spaces.

Not dwelling on this in detail just note two facts that we'll need later on. At first, the correlation between three types of numerical ranges of o.f. in Banach space is the same as in the case of operators: at any s.i.p. s, generated the norm ν in X

$$W_s[A] \subset V_\nu[A] \subset \mathcal{V}[A] \tag{5.5}$$

holds and secondly for continuous o.f. $A: G \to B(X)$ the algebraic numerical range $\mathcal{V}[A]$ is closed in domain G.

However, the way of exact copy of classical case rapidly terminates. For example, $\mathcal{V}[A]$ and $V_\nu[A]$ may be disconnected sets, whereas for the operators $V(T)$ is connected one and $\mathcal{V}(T)$ is even convex. For continuous o.f. many other important properties of numerical ranges of operators are not valid. For example, the following relation $\overline{W}(T) = \mathcal{V}(T)$ between Hausdorff and algebraic numerical ranges is broken and as was first noted by Hadeler [4] in a Hilbert space for o.f. the Winter-Stone's localization relation $\overline{W}(T)=\mathcal{V}(T)$ between spectrum and numerical range of operator is broken.

This situation induces to find condition on o.f. at which the containing facts from the theory of numerical ranges of o.f. can be obtained. The approach to this problem is suggested by the ideology of Hadeler's paper [4] confirmed by Brown's and Duglas [5] technique.

5.2. We'll associate with each o.f. $A:G\to B(X)$ in Banach space X with the norm v three types of subsets from domain G

$$\begin{aligned}\Omega_s[A]&=\{\lambda\in G:0\notin\overline{W}_s(A(\lambda))\},\\ \Omega_v[A]&=\{\lambda\in G:0\notin\overline{V}_v(A(\lambda))\},\\ \Omega[A]&=\{\lambda\in G:0\notin\overline{co}\,W_s(A(\lambda))\},\end{aligned}\tag{5.6}$$

which are called domains of s-weak regularity, regularity and strong regularity of the o.f. A respectively.

The following proposition describing the domains of regularity of "classical" o.f. (5.4) generated by the operator $T\in B(X)$ establishes relation with the theory of numerical ranges of operators.

Proposition 5.1: *For the o.f. (5.4) at any s.i.p. generated the norm v in X the following equalities are valid:* $\Omega_s[A]=\mathbf{C}\setminus\overline{W}_s(T);\ \Omega_v[A]=\mathbf{C}\setminus\overline{V}(T);\Omega[A]=\mathbf{C}\setminus\mathcal{V}(T)$.

Proof immediately follows from translation property of numerical ranges of operators, compactness of their closures and behavior of arithmetical sum of sets at closing.

This proposition justifies the naturalness and non-triviality of domains of regularity and also motivates the choice of terminology. Indeed if take into account the correlation of these three types of numerical ranges of operators, closeness and boundedness of algebraic numerical range, Wintner-Lumer's and William's localization theorems (see I part of the present paper) and non-emptiness of approximative point spectrum of operator, then the following statement is evident: for the o.f. (5.4) the set $\Omega_s[A]$ lies in field of regularity of the operator T at any s.i.p. s, generated the norm v in X, and $\Omega_v[A]$ and $\Omega[A]$ consist only of regular points of this operator where any of these three domains of regularity doesn't coincide with the domain of definition of the o.f. (5.4) and isn't empty. Remaining that for the operator $T\in B(H)$ in Hilbert space H; is called field of regularity $\mathbf{C}\setminus\sigma_\pi(T)$, or set of regular type; $\mathbf{C}\setminus\overline{W}(T)$ is called an external field of regularity; and the resolvent set $\mathbf{C}\setminus\sigma(T)$ is called set of regular points. It's important to note that at the same time there exists o.f. with empty domain of regularity (see example in the proof of theorem 6.4 from §6).

We'll dwell briefly on the question of comparison of different types of regularity domains of o.f. and influence of geometry of space on their hierarchy. First of all, for any o.f. A in arbitrary Banach space its domain of regularity are related with each other by the following chain of inclusions

$$\Omega[A]\subset\Omega_v[A]\subset\Omega_s[A],\tag{5.7}$$

where s is any of s.i.p. generating the norm v. If Banach space is smooth then any o.f. has only one domain of weak regularity which coincides with the domain of regularity. In particular in Hilbert space of the space all three types of domains of regularity coincide between each other. In a general Banach space the chain of inclusions (5.7) will be strict. If while the right inclusion from (5.7) may be strict only in non-smooth spaces then the strictness of the left inclusion is possible even in smooth uniformly rotund finite-dimensional Banach space.

All that was said can be confirmed by examples from geometry of numerical ranges in a finite-dimensional Banach spaces. Some of them will be considered in the next part of the paper in connection with estimation of Gershgorian domain of matrices by their numerical ranges. Note that smoothness of Banach space which is sufficient isn't necessary for coincidence of the sets $\Omega_v[A]$ and $\Omega_s[A]$. It follows from the following geometrical effect.

Proposition 5.2. *In two-dimensional complex space l_1^2 with the octahedral norm $v(x)=|x_1|+|x_2|, x=(x_1,x_2)\in l_1^2$ there exists s.i.p. $\underline{s}$ generating v and such that for any linear operator T in l_1^2 the equality $\overline{W}_s(T)=V(T)$ holds.*

5.3. For obtaining the contensive statements it's necessary to narrow the general notion of o.f. We'll use the following class of o.f. [2, p.95]: $A:G\to B(X)$ is called holomorphic o.f. in the domain $G\subset \mathbf{C}$, if one of the following equivalent conditions holds: a) for any functional $f\in B(X)^*$ function $f\circ A:G\to \mathbf{C}$ is holomorphic in the domain G; b) o.f. A is differentiable by norm of the space $B(X)$ in each point of the domain G.

As we see in contrast to the case of operators domain of regularity of o.f. may be empty one. Therefore in the theory of numerical ranges natural one-parametric analogies of operators containing the classical case are o.f. with non-empty domains of regularity. Here each of these three types of domains of regularity assigns its class of o.f.

Definition. *Let X be Banach space with the norm v. Holomorphic o.f. $A:G\to B(X)$ is called strongly regular, s-weakly regular if $\Omega[A],\Omega_v[A],\Omega_s[A]$ are non-empty respectively, where s is s.i.p. generating v.*

From consideration of item 5.2 follows the chain of inclusions of these three classes of o.f. and their non-triviality. Also note that conditions of weak regularity of o.f. respective to different s.i.p. s_1 and s_2 generating the norm v in X generally speaking aren't connected between each other from simultaneous weak regularity of o.f. at all s.i.p., generating v, in general case regularity of this o.f doesn't follow.

§6. Geometrical behavior of numerical ranges and localization of spectrum of operator functions.

6.1. In this item we'll give the series of statements on geometrical properties of numerical ranges of regular operator functions whose applications to the question of localization of spectrum are posed in the next item.

The theorems on geometrical behavior of numerical ranges of o.f. simplifying proofs of localization relations for spectrum give new proofs of localization theorems even for the case of operators.

The first suggested theorem gives one-parametric analogies of Bollobas' theorem [8] on behavior of Bauerian numerical range of the operator at Banach conjugation operator and as a corollary invariance relative to conjugation of closure of numerical range (numerical range itself) of o.f. in Banach (reflexive) space.

It's worth to note that two facts from geometry of Banach spaces underlie the proof of these properties of numerical range: improved by Bollobas *[8] Bishop's and* Phelps's theorem on density by norm in $S(X^*)$ of support functional set of Banach space

X and Goldstine's theorem [1, p.460] on *-weak density in the bidual space X^{**} of the ranges of unit ball $U(X)$ at canonical embedding X into X^{**}.

In the next theorem for o.f. $A: G \to B(X)$ in Banach space X with the norm v the conjugate o.f. $A^*: G \to B(X^*)$ in the conjugate space X^* with dual norm v^* is defined by the relation $A^*(\lambda) = A(\lambda)^*$ for any $\lambda \in G$.

Theorem 6.1. *(on behavior of numerical ranges of o.f. at conjugation). For the regular o.f. $A: G \to B(X)$ and conjugate to it $A^*: G \to B(X^*)$ the following chain of inclusions takes place*

$$V_v[A] \subset V_{v^*}[A^*] \subset \overline{V}_v[A] \tag{6.1}$$

here left inclusion in (6.1) is valid for any o.f.

Proof of the left inclusion follows from analogous relation for operators and the right inclusion for regular o.f. follows from above mentioned Bollobas' theorem and reasoning in the proof of theorem 1 from [14a, p.137] about domain of regularity of o.f. We'll enumerate some corollaries of this theorem. First of them gives invariance of closure of Bauerian numerical range of o.f. relative to conjugation.

Corollary 6.1. *For the regular o.f. $A: G \to B(X)$ the equality*

$$\overline{V}_v[A] = \overline{V}_{v^*}[A^*] \tag{6.2}$$

is true.

Like the previous corollary the next proposition is about invariance of algebraic numerical range of o.f. at conjugation in proof of localization theorem for spectrum of o.f.

Corollary 6.2. *For arbitrary o.f. $A: G \to B(X)$ the following symmetry is true.*

$$\mathcal{V}[A] = \mathcal{V}[A^*]. \tag{6.3}$$

The some symmetry of Bauerian numerical range of o.f. takes place at restriction on the space geometry.

Corollary 6.3. *In reflexive Banach space for any o.f. the following equality is true*

$$V_v[A] = V_{v^*}[A^*]. \tag{6.4}$$

The second suggested theorem combines at the same time the analogs of the next two known facts on behavior of numerical ranges of operator in Banach space relative to convex closure operation. This is Lumer's theorem [6] on independence of closed convex hull of Lumerian numerical range of operator on the choice of s.i.p., conformed with the norm of the space and consisting of the equality

$$\mathcal{V}(T) = \overline{co}W_s(T), \tag{6.5}$$

where $T \in B(X)$, s is an arbitrary s.i.p., assigning the norm v in X and $\overline{co}$ is a convex closure. While Mc Gregor's result [7] asserts realizability of the relation

$$\mathcal{V}(T) = \overline{co}V_v(T) \tag{6.6}$$

which, however immediately follows from (6.5)

Theorem 6.2. *(on convex hull of numerical ranges of o.f.). For strongly regular o.f. $A: G \to B(X)$ in Banach space X at any s.i.p. s, generated the norm v in X the following relation is valid*

$$\overline{co}W_s[A] = \overline{co}V_v[A] = co\mathcal{V}[A]. \tag{6.7}$$

Proof of the inclusions to the right of equality (6.7) corrected for arbitrary o.f. immediately follows from the chain of inclusions (5.5) in force of isotone property of the operator $\overline{co}$ relative to inclusion of sets. Check up of reverse inclusions in the sequence of sets (6.7) already suppose the strong regularity of the o.f. A and consist of proof's fragment of theorem 2 from [14a, p.138] regarded to the domain of strong regularity of o.f.

We'll finish the present item by the one-parameter analog of Berberian's and Orland's theorem [10] on behavior of Hausdorff numerical range of operator in Hilbert space at extension (on Berberian). Here we'll include the relation between algebraic $\mathcal{V}[A]$ and Hausdorff $W[A]$ numerical ranges of o.f. $A:G\to B(H)$ in Hilbert space H.

Berberian's representation for o.f. looks as follows. The o.f. $A^0:G\to B(K)$: is called extended for the o.f. $A:G\to B(H)$ defined by the relation $A^0(\lambda)=A(\lambda)^0$ at each $\lambda\in G$, where Hilbert space K is Berberain extension of the space H, and $\circ:B(H)\to B(K)$ is Berberian representation of the algebra $B(H)$ [9, 13].

Theorem 6.3. *(on behavior of Hausdorff set of o.f. at extension). Let $A:G\to B(H)$ be regular o.f. in Hirbert space H and $A^0:G\to B(K)$ be its extended o.f. Then the following equalities are satisfied*

$$W\left[A^0\right]=\mathcal{V}[A]=\overline{W}[A]. \tag{6.8}$$

Proof of the first equality in (6.8) follows from coincidence of the algebraic numerical range in Hilbert space with the closure of its Hausdorff numerical range with consequent application of Berberian-Orland's theorem [10]. The inclusion $\overline{W}[A]\subset\mathcal{V}[A]$ in the second inequality from (6.8) follows from the following fact: algebraic numerical range of continuous o.f. is closed. The reverse inclusion subject to Toeplitz-Hausdorff's theorem on convexity of Hausdorff numerical range [3, chapter 17] is a particular case of the following more general statement: if Bauerian numerical range $V(A(\lambda))$ of the operator $A(\lambda)$ of regular o.f. $A\to B(X)$ in Banach space X is convex at any $\lambda\in G$, then $\overline{V}[A]$ coincides with $\mathcal{V}[A]$.

Note that Berberian construction may be adapted for the spaces with s.i.p. and consider behavior of numerical ranges of o.f. in Banach space at extension operation. Then the localization theorems for spectrum, which we'll consider in the next item can be derived from Teoplitz theorem on localization of a point spectrum using geometric properties of numerical ranges of o.f.

6.2. Now using obtained above results on geometric properties of numerical ranges we'll obtain one-parameter analogs of the classical theorems on localization of operator spectrum by numerical ranges. For that consider the notions of spectrum and its parts for the o.f. $A:G\to B(X)$ in Banach space X and note some of its simplest properties. These objects are defined by the following way: spectrum $\sigma[A]=\{\lambda\in G:0\in\sigma(A(\lambda))\}$, point spectrum $\sigma_p[A]=\{\lambda\in G:0\in\sigma_p(A)\}$, approximately point spectrum $\sigma_\pi[A]=\{\lambda\in G:0\in\sigma_\pi(A(\lambda))$, defect spectrum $\sigma_\delta[A]=\{\lambda\in G:0\in\sigma_\delta(A(\lambda))\}$ and compression spectrum $\sigma_\gamma[A]=\{\lambda\in G:0\in\sigma_\gamma(A(\lambda))\}$. It's obvious that for "classical" o.f. generated by the operator $T\in B(X)$ (see (5.4) from i.5.1) these definitions give the corresponding concepts of $\sigma_\pi(T),\sigma_\delta(T)$ and $\sigma_\gamma(T)$ for the operator T.

Among the properties of operator spectrum (and its parts) remaining valid for o.f. note those of which we'll use by proving the theorem on localization of spectrum. This is a correlation between parts of spectrum of o.f. and its behavior at passage to the conjugate o.f.:

$\sigma_\rho[A]\subset\sigma_\pi[A]$; $\sigma_\gamma[A]\subset\sigma_\delta[A]$; $\sigma[A]=\sigma_\rho[A]\cup\sigma_\delta[A]=\sigma_\pi[A]\cup\sigma_\gamma[A]$;

$\sigma_\delta[A]=\sigma_\pi[A^*]$; $\sigma_\gamma[A]=\sigma_\rho[A^*]$; and at reflexivity of the space X $\sigma_\rho[A]=\sigma_\gamma[A^*]$.

In the following theorem for o.f. analogs of Wintner-Lumer theorem on localization of approximately point spectrum of operator by closure of Bauerian numerical range and Lumer's theorem on exclusion of a spectrum by algebraic numerical range of operator are given.

Theorem 6.4. *Let $A:G\to B(X)$ be o.f. in Banach space X. Then the following statement are valid.*

1. If A is s-weakly regular at some s.i.p. s assigned with the norm in X, then the following inclusion takes place
$$\sigma_\pi[A]\subset\overline{W}_s[A]. \tag{6.9}$$
2. For the regular o.f. A in arbitrary Banach space X or for s-weakly regular o.f. A in reflexive X the relation
$$\sigma_\pi[A]\subset\overline{W}_s[A] \tag{6.10}$$
is valid.
3. At some s-weakly regularity of A with bounded spectrum or at strong regularity A the inclusion
$$co\sigma[A]\subset\overline{co}W_s[A] \tag{6.11}$$
holds.

Conditions of regularity in all these statements may not be omitted.

Brief proof. 1) Localizability of $\sigma_\pi[A]$ by $\overline{W}_s[A]$ for s-weakly regular o.f. A is justified by reasoning from proof of theorem 3 in [14a, p.139] and therefore we omit them. The statement on independence of relation (6.9) on choice of s.i.p. s for regular o.f. is obvious.

2) Localization correlation (6.10) for s-weakly regular o.f. in a reflexive X follows from the first part of the theorem and proportion 1 [14c., §1] on exclusion of compression spectrum of operator by its Bauerian numerical range. In the case of regularity of the o.f. A in arbitrary Banach space X at first one-parameter Toeplitz theorem on localization of point spectrum is applied to prove inclusion (6.10): at any s.i.p. s, generated the norm in X for arbitrary o.f. A the inclusion $\sigma_\rho[A]\subset W_s[A]$ is valid. To complete the proof it remains just to check the correctness of the following proposition.

Lemma. *The defect spectrum $\sigma_\delta[A]$ of the regular o.f. A is contained in closure $\overline{V}[A]$ of its Bauerian numerical range.*

3) In order to justify the localization relation (6.11) for s-weakly regular o.f. A at boundedness of spectrum $\sigma[A]$ at first note that using continuity of A it's easy to show that the boundary $\partial\sigma[A]$ of the spectrum is contained in $\sigma_\pi[A]$. This according to the first part of the theorem implies the inclusion $\partial\sigma[A]$ to $\overline{W}_s[A]$. Then the variant of Krein-Milman's theorem [2, p.86] by virtue of compactness of $\sigma[A]$ subject to previous inclusion leads to the relation (6.11) follows from the second part of the theorem and theorem on convex hull of numerical ranges of o.f. (see i.6.1).

In order to be convinced in importance of the conditions of regularity of o.f. in the statement of theorem we'll use the following adaptation from example [4, §4].

We'll take in Banach space X the o.f. of type $A(\lambda)=\lambda T$, where $\lambda \in C$ and $T \in B(X)$ is compact operator whose numerical range lies on a real positive semi-axis. Then it's easy to see that the spectrum $\sigma[A]$ coincides with the set C of all complex numbers. At the same time according to [14c. §4] approximately point spectrum $\sigma_\pi[A]$ contains whole spectrum. On the other hand, it's easy to check that $W_s[A]=\{0\}$ at any s.i.p. s, generated the norm in X and therefore $V[A]=\{0\}$. Thus, all statements of the theorem are checked.

6.3. In conclusion we'll make some remarks on the previous theorem. William's classical theorem [11] on localization of operator spectrum in a Banach space by closure of Bauerian numerical range is obtained as a particular case of relation (6.10), moreover with the proof which differs from the original one.

In addition the same relation in particular for s-weakly regular o.f. A in a smooth Banach space gives localizability of its spectrum by the closure $\overline{W_s}[A]$ of Lumerian numerical range, proved in [12] for the operator at additional assumption of uniform rotund of the space.

The second statement of theorem 6.4 at regularity of the o.f. A is exact one-parameter analog of Williams theorem and at s-weakly regularity-its weak variant, but both of them in a Hilbert space turn into one-parameter Wintner-Stone theorem [13]. The last theorem follows also from Toeplitz theorem on localization of point spectrum by Hausdorff numerical range of o.f. at Berberian extension (see [13]). The analogous way of proving is right also for o.f. in a Banach space, if to adapt Berberian construction for the spaces with s.i.p.

If we'll based on the classical Lumer's theorem on localizability of spectrum $\sigma(T)$ of the operator $T \in B(X)$ by algebraic numerical range $V(T)$, then for any o.f. A in a Banach space its following one-parameter analog is immediately obtained

$$co\sigma[A] \subset coV[A]. \qquad (6.12)$$

However, the third part of theorem 6.4 gives other two variants of such analog without using this classical Lumer's theorem. At that the last theorem in both variants, in particular, obtains proofs which differ from the original one. In the case of strong regularity of the o.f. A the relations (6.11) and (6.12) coincide.

We'll complete by the remark that all stated above are correct for multiparametric operators (m.p.o) $A:G \to B(X)$ in Banach space X, where G is a subset of C^n and even for the systems $\tilde{A}(A_1,...,A_n)$ of m.p.o $A_j:G \to B(X)$, $j=1,...,n$ under conditions of definiteness of the system by Atkinson.

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Nazim G. Vahabov
Institute of Mathematics & Mechanics of NAS Azerbaijan.
9, F.Agayev str., 370141, Baku, Azerbaijan.
Tel.:39-47-20.

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MECHANICS

ABBASOVA L.A.

PROPAGATION OF THE FAILURE FRONT IN THE DAMAGED ROUND THICK PIPE SUBJECTED TO TEMPERATURE

Abstract

The investigation of the thermoelastic failure of the thick pipe which occurs due to the temperature difference of pumpable product and environment is an important engineering problem. In the present paper this process is investigated in the aggregate with the failure process of the material of the pipe. Taking into account the resistance of the material of the pipe behind the failure front is important. The influence of this factor on the character of distribution of the failure front defining the process of failure of the pipe is clarified.

In spite of numerous articles on the thermoplastic failure of solids, this tendency: remains actual. It is associated with the fact, that majority of machine components, mechanisms and constructions work in extremal thermal conditions. However, we have to take into account the factor of formation and accumulation of defects in the bulk. The attendant process of damaging can make important contribution and at times is determining one in the process of thermoelastic failure. In the paper [1] the process of thermoelastic failure of the thick cylindrical pipe was investigated under conditions of the plane deformation, when the temperatures on the interior and exterior surfaces of the pipe different by their value are given. Then criterion of failure of the damaging theory [2] on the greatest stress, which is tangential one was used. The time of failure of the interior surface the incubating time was found. The equation of motion of the failure front provided that the material of the pipe behind the front of failure completely loses its load-carrying capacity, was obtained and analyzed. In the present paper this investigation was carried out taking into account the resistance to loading of the material of the pipe behind the failure front. It's supposed that the material of the pipe behind the failure front preserves its load-carrying capacity to a less extent. That is in each moment of time which is greater than the incubating one is the pipe of two-layer construction, whose a part before the failure front of the source material and the other part behind the failure front is the material with sharply decreased rigid characteristics.

Analysis of the formulas of hoop thermoelastic stress, which is maximal one shows that in the case when the temperature of interior surface exceeds the temperature of exterior surface which is accepted in [1], failure first begins on the interior surface, where the hoop stress achieves its maximum and later on the failure front representing on expanding circular zone moves to the exterior surface of the pipe. In t he present paper it is also assumed that the interior temperature exceeds the exterior one. In this case the source material before the failure represents the domain S_2, adjoining to the exterior boundary of the radius R_2 of the pipe (fig.1). The domain S_1 adjoining to the interior surface of the radiuses R_0 of the pipe represents the domain behind the failure front. The failure front is a cylindrical surface of the alternate increasing radius R_1.

We'll denote all the parameters related to the domains S_1 and S_2 by the corresponding index numbers. Previously we'll give the solution of the thermoelastic