MAMEDOV I.T., SALMANOVA Sh.Yu.

THE A.D. ALEKSANDROV TYPE INEQUALITY FOR A CLASS OF SECOND ORDER EQUATIONS WITH NON-NEGATIVE CHARACTERISTIC FORM

Abstract

The analogue of the classical A.D. Aleksandrov inequality is proved for a class degenerating on boundary of domain of second order elliptic-parabolic equations of non-divergent structure with generally speaking discontinuous coefficients.

Let $\mathbf{R}_{n+1}$ be an $(n+1)$ dimensional Euclidean space of the points $(x,t)=(x_1,...,x_n,t)$, $Q_T=\Omega\times(0,T)$ be a cylindrical domain in $\mathbf{R}_{n+1}$, where Ω is a bounded n-dimensional domain with the boundary $\partial\Omega$, and $T\in(0,\infty)$. Let further $Q_0=\{(x,t): x\in\Omega,\ t=0\}$, and $\Gamma(Q_T)=Q_0\cup(\partial\Omega\times[0,T])$ be a parabolic boundary of Q_T. Consider the following second order degenerate elliptic-parabolic operator in Q_T

$$L=\sum_{i,j=1}^{n}a_{ij}(x,t)\frac{\partial^2 u}{\partial x_i\partial x_j}+\sum_{i=1}^{n}b_i(x,t)\frac{\partial u}{\partial x_i}+c(x,t)+w(x,t)\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}$$

in assumption that $\|a_{ij}(x,t)\|$ is a real symmetric matrix where for all $(x,t)\in Q_T$ and any n-dimensional vector ξ

$$\gamma|\xi|\le\sum_{i,j=1}^{n}a_{ij}(x,t)\xi_i\xi_j\le\gamma^{-1}|\xi|^2,\ \gamma\in(0,1]=const. \qquad (1)$$

We determine the function $w(x,t)$ by the equality $w(x,t)=\psi_1(\rho)\psi_2(t)\varphi(T-t)$, where $\rho=dist(x,\partial\Omega)$, ψ_1,ψ_2 and φ are continuous, non-negative and non-decreasing functions of themselves arguments, where

$$\int_0^T\left(\frac{\varphi(v)}{v^2}\right)^{n+1}dv<\infty. \qquad (2)$$

Besides we'll assume that all coefficients of the operator L are measurable in Q_T functions.

Denote by $W_w^{2,2}(Q_T)$ a Banach space of the functions $u(x,t)$ given on Q_T with the finite norm

$$\|u\|_{W_w^{2,2}(Q_T)}=\|u\|_{C(Q_T)}+\sum_{i=1}^{n}\left\|\frac{\partial u}{\partial x_i}\right\|_{L_{n+1}(Q_T)}+$$
$$+\sum_{i,j=1}^{n}\left\|\frac{\partial^2 u}{\partial x_i\partial x_j}\right\|_{L_{n+1}(Q_T)}+\left\|\frac{\partial u}{\partial t_i}\right\|_{L_{n+1}(Q_T)}+\left\|w\frac{\partial^2 u}{\partial t^2}\right\|_{L_{n+1}(Q_T)},$$

and let $\dot{W}_w^{2,2}(Q_T)$ be a subspace of $W_w^{2,2}(Q_T)$, dense set in which is a set of all functions from $C^\infty(\overline{Q}_T)$ vanishing on $\Gamma(Q_T)$.

The aim of the present note is determination of conditions on the coefficients $b_1(x,t),...,b_n(x,t)$ and $c(x,t)$, for fulfillment of which for arbitrary functions $u\in\dot{W}_w^{2,2}(Q_T)$ the estimation

$$\|u\|_{C(Q_T)}\le C_1\|Lu\|_{L_{n+1}(Q_T)} \qquad (3)$$

is correct with constant C_1 independent of u.

Note that for the second order elliptic operators the analogous estimations were established in classical works of A.D. Aleksandrov [1-3] and I.Ya. Bakel'man [4]. For the second order parabolic operators the estimations in the form of (3) were proved in works of N.V. Krylov [5-6] and Kaising Tso [7]. For the second order elliptic-parabolic operators without minor coefficients at power degeneration of the function $w(x,t)$ on boundary of the domain Q_T the estimation (3) was obtained by I.G. Gasanov [8]. We show also in [9] in which the estimation (3) was established for operators as L in the case $b_i(x,t) \equiv c(x,t) \equiv 0;\ i=1,\dots,n.$

Relative to the function $c(x,t)$ we'll assume that

$$c(x,t) \le 0,\ \ c(x,t) \in L_{n+1}(Q_T). \qquad (4)$$

Relating to the vector $\overline{B}(x,t) = (b_1(x,t),\dots,b_n(x,t))$ then for obtaining the local estimation as (3) we impose on it the following condition

$$\left|\overline{B}(x,t)\right| \in L_{2(n+1)}(Q_T), \qquad (5)$$

where $\left|\overline{B}(x,t)\right| = \left(\sum_{i=1}^{n} b_i^2(x,t)\right)^{1/2}$.

Theorem 1. *Let relative to coefficients of the operator L in the domain $Q_T \subset \mathbf{R}_{n+1}$ the conditions (1)-(2) and (4)-(5) be satisfied. Then there exists $T_0 = T_0(\gamma, n, d, \overline{B})$ such that if $T \le T_0$ then for any function $u(x,t) \in \dot{W}_w^{2,2}(Q_T)$ the estimation (3) is correct. In addition $C_1 = C_1(\gamma, n, d)$.*

Here and further the note $C(\dots)$ means that the positive constant C depends only on the contents of parenthesis.

Proof. It's obvious that it's sufficient to prove the estimation (3) for the function $u(x,t) \in C^\infty(\overline{Q}_T)$ vanishing on $\Gamma(Q_T)$. Besides without loss of generality we'll assume that $a_{ij}(x,t) \in C^\infty(\overline{Q}_T)$ $(i,j=1,\dots,n)$ and $\sup_{Q_T}|u| = \sup_{Q_T} u = M > 0$. Denote by (x^0,t^0) the point $\overline{Q}_T$, in which $u(x^0,t^0) = M$. Two cases are possible:
i) $(x^0,t^0) \in Q_T$, ii) $(x^0,t^0) = (x^0,T)$, $x^0 \in \Omega$.

At first we assume that the alternative i) holds. Let $v = u^2$ and for $i,j=1,\dots,n$ $v_i = \frac{\partial v}{\partial x_i}$, $v_{ij} = \frac{\partial^2 v}{\partial x_i \partial x_j}$. Denote by A_v the set $\{(x,t) : (x,t) \in Q_T, u(x,t) \ge 0,$ $v_t(x,t) \ge 0, v_{tt}(x,t) \le 0$, the matrix $\left\|v_{ij}(x,t)\right\|$ is non-positive defined$\}$. Since (x^0,t^0) is a interior point of Q_T, then it belongs to one of connected components of the set A_v. For simplicity we'll denote this connected component again by A_v.

To every point $(x,t) \in A_v$ we associate the vector

$$F(x,t) = \left(v_1(x,t),\dots,v_n(x,t), v(x,t) - \sum_{i=1}^{n}(x_i - x_i^0)v_i(x,t)\right).$$

Consider the following set in $\mathbf{R}_{n+1}$

$$G=\left\{(\xi,h):|\xi|<\frac{M^2}{d},d|\xi|<h<M^2\right\}.$$

Prove that $G\subset F(A_v)$. To this end we fix an arbitrary point $(\xi,h)\in G$ and $t'\in(0,T)$ and consider the graph of the function $\tilde{v}_{t'}=v(x,t')$. In the space (x,Y) we draw the hyperplane $P:Y=<\xi,x-x^0>+h$.

It's easy to see that $Y|_{\partial\Omega}\geq h-|\xi||x-x^0|\geq h-|\xi|d>0$. Besides $Y|_{x=x^0}=h<M^2$. Therefore moving the graph of the function $\tilde{v}_{t'}(x)$ along the axis t we find such t that hyperplane P will be tangent to the graph of $\tilde{v}_t(x)$ at the point x. But at this point it's obvious $F(x,t)=(\xi,h)$. Thus $G\subset F(A_v)$. We calculate now the Jacobian J of the transformation F. We have

$$J=\begin{vmatrix} v_{11} & v_{21} & \dots & v_{n1} & -\sum_{i=1}^n(x_i-x_i^0)v_{i1}\\ v_{12} & v_{22} & \dots & v_{n2} & -\sum_{i=1}^n(x_i-x_i^0)v_{i2}\\ - & - & \dots & - & -\\ v_{1n} & v_{2n} & \dots & v_{nn} & -\sum_{i=1}^n(x_i-x_i^0)v_{in}\\ v_{1t} & v_{2t} & \dots & v_{nt} & v_t-\sum_{i=1}^n(x_i-x_i^0)v_{it}\end{vmatrix}.$$

We multiply the first column by $x_1-x_1^0$, the second - by $x_2-x_2^0$, the $n-th$ - by $x_n-x_n^0$ and their sum to the last column. We obtain

$$J=\begin{vmatrix} v_{11} & v_{21} & \dots & v_{n1} & 0\\ v_{12} & v_{22} & \dots & v_{n2} & 0\\ - & - & \dots & - & 0\\ v_{1n} & v_{2n} & \dots & v_{nn} & 0\\ v_{1t} & v_{2t} & \dots & v_{nt} & v_t\end{vmatrix}=v_t\det\|v_{ij}\|.$$

Thus

$$mesG\leq\int_{A_v}|v_t\det\|v_{ij}\||dxdt. \tag{6}$$

On the other hand

$$mesG=\int_{|\xi|<\frac{M^2}{d}}d\xi\int_{d|\xi|}^{M^2}dh=\int_{|\xi|<\frac{M^2}{d}}(M^2-d|\xi|)d\xi=w_n\int_0^{M^2/d}(M^2-\rho d)\rho^{n-1}d\rho=$$

$$=w_nM^2\int_0^{M^2/d}\rho^{n-1}d\rho-w_nd\int_0^{M^2/d}\rho^nd\rho=\frac{w_nM^{2(n+1)}}{d^n}\cdot\frac{1}{n(n+1)}, \tag{7}$$

where w_n is measure of unique n-dimensional ball. From (6)-(7) we obtain

$$M^{2(n+1)}\leq C_2\int_{A_v}|v_t\det\|v_{ij}\||dxdt=C_2\int_{A_v}\frac{|v_t\det\|a_{ij}v_{ij}\||}{\det\|a_{ij}\|}dxdt\leq$$

$$\leq \frac{C_2}{\gamma^n}\int_{A_v}\left|v_t \det\left\|a_{ij}v_{ij}\right\|\right|dxdt, \tag{8}$$

where $C_2=\dfrac{d^n n(n+1}{w_n}$.

From the condition (1) and the determination of the set A_v it follows that the matrix $\left\|a_{ij}(x,t)v_{ij}(x,t)\right\|$ is non-positive valued for $(x,t)\in A_v$ and its trace is equal to

$$\sum_{i,j=1}^{n}a_{ij}(x,t)v_{ij}(x,t).$$

Let $\lambda_1,\ldots,\lambda_n$ be eigen values of the matrix $\left\|a_{ij}v_{ij}\right\|$. According to abovementioned $\lambda_i\leq 0,\ i=1,\ldots,n$. From (8) we conclude

$$M^{2(n+1)}\leq\frac{C_2}{\gamma^n}\int_{A_v}\left|v_t\lambda_1\ldots\lambda_n\right|dxdt=\frac{C_2}{\gamma^n}\int_{A_v}v_t(-\lambda_1)\ldots(-\lambda_n)dxdt\leq$$

$$\leq\frac{C_2}{\gamma^n(n+1)^{n+1}}\int_{A_v}(v_t-\lambda_1\ldots-\lambda_n)^{n+1}dxdt=C_3\int_{A_v}\left(v_t-\sum_{i,j=1}^{n}a_{ij}v_{ij}\right)^{n+1}dxdt,$$

where $C_3=\dfrac{C_2}{\gamma^n(n+1)^{n+1}}$. We have further

$$M^{2(n+1)}\leq C_3\int_{A_v}\left(v_t-\sum_{i,j=1}^{n}a_{ij}v_{ij}-2cv-w(x,t)v_{tt}\right)^{n+1}dxdt=$$

$$=2^{n+1}C_3\int_{A_v}\left[u\left(u_t-\sum_{i,j=1}^{n}a_{ij}u_{ij}-cu-w(x,t)u_{tt}\right)-\sum_{i,j=1}^{n}a_{ij}u_iu_j-\right.$$

$$\left.-w(x,t)u_t^2\right]^{n+1}dxdt\leq 2^{n+1}C_3\int_{A_v}\left[u(-Lu)+u\sum_{i=1}^{n}|b_i||u_i|-\gamma\left|\nabla_x u\right|^2\right]^{n+1}dxdt\leq$$

$$\leq 2^{n+1}C_3\int_{A_v}\left[u(-Lu)+u\left|\overline{B}\right|\left|\nabla_x u\right|-\gamma\left|\nabla_x u\right|^2\right]^{n+1}dxdt, \tag{9}$$

where $\nabla_x u=(u_1,\ldots,u_n)$. If now the point $(x,t)\in A_v$ is such that $\left|\nabla_x u(x,t)\right|\geq\dfrac{\left|\overline{B}(x,t)\right|}{\gamma}u(x,t)$ then

$$u\left|\overline{B}\right|\left|\nabla_x u\right|-\gamma\left|\nabla_x u\right|^2\leq 0.$$

If for $(x,t)\in A_v$, $\left|\nabla_x u(x,t)\right|<\dfrac{\left|\overline{B}(x,t)\right|}{\gamma}u(x,t)$ is satisfied, then

$$u\left|\overline{B}\right|\left|\nabla_x u\right|-\gamma\left|\nabla_x u\right|^2\leq\frac{1}{\gamma}\left|\overline{B}\right|^2u^2.$$

Thus from (9) we conclude

$$M^{2(n+1)}\leq 2^{n+1}C_3\int_{A_v}\left[u\left|Lu-\frac{1}{\gamma}\left|\overline{B}\right|^2u\right|\right]^{n+1}dxdt.$$

But on the other hand

$$\iint_{A_v}\left|Lu-\frac{1}{\gamma}\left|\overline{B}\right|^2u\right|^{n+1}dxdt\le 2^n\left(\iint_{Q_T}|Lu|^{n+1}dxdt+\frac{M^{n+1}}{\gamma^{n+1}}\iint_{Q_T}\left|\overline{B}\right|^{2(n+1)}dxdt\right).$$

Therefore

$$M^{n+1}\le 2^{2n+1}C_3\left(\iint_{Q_T}|Lu|^{n+1}dxdt+\frac{M^{n+1}}{\gamma^{n+1}}\left\|\overline{B}\right\|^{2(n+1)}_{L_{2(n+1)}(Q_T)}\right). \qquad (10)$$

By virtue of the condition (5) there exists $T_0=T_0\left(\gamma,n,d,\overline{B}\right)$ such that if $T\le T_0$ then

$$\frac{2^{2n+1}C_3}{\gamma^{n+1}}\left\|\overline{B}\right\|^{2(n+1)}_{L_{2(n+1)}(Q_T)}\le\frac{1}{2}.$$

Now from (10) it follows that

$$M^{n+1}\le 2^{2n+2}C_3\iint_{Q_T}|Lu|^{n+1}dxdt,$$

i.e. for fulfillment of alternative i) the estimation (3) is proved with $C_1=4C_3^{\frac{1}{n+1}}$. Let now alternative ii) hold. We fix an arbitrary $\varepsilon\in(0,1)$ and consider the auxiliary function $v^\varepsilon(x,t)=(T-t)^\varepsilon u(x,t)$. It's easy to see that the exact upper bound of the function $v^\varepsilon(x,t)$ in Q_T can't be reached when $t=T$. We denote $\sup\limits_{Q_T}v^\varepsilon(x,t)$ by M_ε.

According to the previously proved

$$M_\varepsilon^{2(n+1)}\le C_3\int_{A_{v^\varepsilon}}\left(v_t^\varepsilon-\sum_{i,j=1}^n a_{ij}v_{ij}^\varepsilon-2cv^\varepsilon-w(x,t)v_{tt}^\varepsilon\right)^{n+1}dxdt,$$

where the set A_{v^ε} for the function $v^\varepsilon(x,t)$ is determined as the set A_{v^ε} for the function $v(x,t)$. But for $(x,t)\in A_{v^\varepsilon}$

$$v_t^\varepsilon=-\varepsilon(T-t)^{\varepsilon-1}u+(T-t)^\varepsilon u_t\le(T-t)^\varepsilon u_t,$$

$$v_{tt}^\varepsilon=-\varepsilon(1-\varepsilon)(T-t)^{\varepsilon-2}u-2\varepsilon(T-t)^{\varepsilon-1}u_t+(T-t)^\varepsilon u_{tt}.$$

Now acting as in consideration of alternative i) we obtain for $T\le T_0$

$$M_\varepsilon^{n+1}\le C_4\int_{Q_T}(T-t)^{\varepsilon(n+1)}|Lu|^{n+1}dxdt+C_4\varepsilon^{n+1}(1-\varepsilon)^{n+1}\times$$

$$\times\iint_{Q_T}[w(x,t)]^{n+1}(T-t)^{(\varepsilon-2)(n+1)}|u|^{n+1}dxdt+2^{n+1}C_4\varepsilon^{n+1}\times$$

$$\times\iint_{Q_T}[w(x,t)]^{n+1}(T-t)^{(\varepsilon-1)(n+1)}|u_t|^{n+1}dxdt, \qquad (11)$$

where $C_4=4^{n+1}3^nC_3$. We have further

$$\iint_{Q_T}[w(x,t)]^{n+1}(T-t)^{(\varepsilon-2)(n+1)}|u|^{n+1}dxdt\le M^{n+1}[\psi_1(d)\psi_2(T)]^{n+1}\times$$

$$\times T^{\varepsilon(n+1)}\int_{Q_T}(T-t)^{-2(n+1)}(\varphi(T-t))^{n+1}dxdt\le$$

$$\le M^{n+1}[\psi_1(d)\psi_2(T)]^{n+1}T^{\varepsilon(n+1)}d^nw_n\int_0^T\left(\frac{\varphi(t)}{t^2}\right)^{n+1}dt. \qquad (12)$$

Since $u(x,t)\in C^{\infty}(\overline{Q}_T)$, then $|u_t(x,t)|\le C_5(u)$ for $(x,t)\in Q_T$. Therefore

$$\iint_{Q_T}[w(x,t)]^{n+1}(T-t)^{(\varepsilon-1)(n+1)}|u_t|^{n+1}dxdt\le C_5^{n+1}[\psi_1(d)\psi_2(T)]^{n+1}\times$$

$$\times T^{\varepsilon(n+1)}d^n w_n\int_0^T\left(\frac{\varphi(t)}{t}\right)^{n+1}dt. \qquad (13)$$

Denote for $\sigma\in(0,T)$ $\sup\limits_{Q_T\cap\{(x,t):t\le T-\sigma\}} u$ by $M(\sigma)$. Then allowing for the condition (2) and the inequalities (12)-(13) in (11) and tending ε to zero we obtain

$$(M(\sigma))^{n+1}\le C_4\iint_{Q_T}|Lu|^{n+1}dxdt.$$

Now it's sufficient to tend σ to zero and we come the required estimation (3) with $C_1=C_4^{\frac{1}{n+1}}$. The theorem is proved.

For obtaining the global estimation as (3) (for any $T\in(0,\infty)$) instead of the condition (5) we'll assume that there exists a constant $K>0$ such that for $(x,t)\in Q_T$

$$|\overline{B}(x,t)|^2+Kc(x,t)\le 0,\ |\overline{B}(x,t)|\in L_{(n+2)}(Q_T) \qquad (14)$$

Theorem 2. *If relative to the coefficients of the operator L in the domain $Q_T\subset\mathbf{R}_{n+1}$ the conditions (1)-(2), (4) and (14) are satisfied, then for any function $u(x,t)\in\dot{W}_w^{2,2}(Q_T)$ the estimation (3) is valid. In addition $C_1=C_1(\gamma,n,d,K)$.*

Proof. As in proof of the previous theorem we'll assume that $\sup\limits_{Q_T}|u|=\sup\limits_{Q_T}u=u(x^0,t^0)=M>0$. At first we consider the case when $(x^0,t^0)\in Q_T$. We introduce the auxiliary function $z=u^k$, where we'll choose the natural number $k\ge 2$ later, and also the set A_z by analogue with the set A_v from the proof of theorem 1. We have

$$M^{k(n+1)}\le C_3\iint_{A_z}\left(z_t-\sum_{i,j=1}^n a_{ij}z_{ij}\right)^{n+1}dxdt\le C_3\iint_{A_z}\left(z_t-\sum_{i,j=1}^n a_{ij}z_{ij}-w(x,t)z_{tt}\right)^{n+1}dxdt=$$

$$=C_3\iint_{A_z}\Bigg[ku^{k-1}\left(u_t-\sum_{i,j=1}^n a_{ij}u_{ij}-w(x,t)u_{tt}\right)-k(k-1)u^{k-1}\sum_{i,j=1}^n a_{ij}u_iu_j-$$

$$-w(x,t)k(k-1)u^{k-2}u_t^2\Bigg]^{n+1}dxdt=C_3\iint_{A_z}\Bigg[ku^{k-1}(-Lu)+ku^{k-1}\sum_{i=1}^n b_iu_i+ku^{k-2}cu-$$

$$-k(k-1)u^{k-2}\sum_{i,j=1}^n a_{ij}u_iu_j-w(x,t)k(k-1)u^{k-2}u_t^2\Bigg]^{n+1}dxdt\le$$

$$\le C_3\iint_{A_z}\left[ku^{k-1}(-Lu)+ku^{k-2}\left(u|\overline{B}||\nabla_x u|+cu^2-(k-1)\gamma|\nabla_x u|^2\right)\right]^{n+1}dxdt. \qquad (15)$$

If the point $(x,t)\in A_z$ such that

$$|\nabla_x u(x,t)|\ge\frac{|\overline{B}(x,t)|}{(k-1)\gamma}u(x,t),$$

then $u|\overline{B}||\nabla_x u|+cu^2-(k-1)\gamma|\nabla_x u|^2\le 0$. If for $(x,t)\in A_z$

$$|\nabla_x u(x,t)| \le \frac{|\overline{B}(x,t)|}{(k-1)\gamma} u(x,t)$$

is satisfied, then $u|\overline{B}||\nabla_x u| + cu^2 - (k-1)\gamma|\nabla_x u|^2 \le \frac{u^2}{(k-1)\gamma}\left[|\overline{B}|^2 + (k-1)\gamma c\right]$. We choose and fix k as the least natural number exceeding $\max\left\{2, 1+\frac{K}{\gamma}\right\}$. Then from (15) we conclude

$$M^{k(n+1)} \le C_3 k^{n-1} M^{(k-1)(n+1)} \iint_{Q_T} |Lu|^{n+1} dxdt,$$

hence and the required value (3) follows with $C_1 = C_3^{\frac{1}{n+1}} k$. The case $(x^0, t^0) = (x^0, T)$, $x^0 \in \Omega$ is considered as in proof of the previous theorem.

We stop now on the conditions

$$c(x,t) \in L_{n+1}(Q_T),\ |\overline{B}(x,t)| \in L_{(n+2)}(Q_T). \tag{16}$$

They provide the finiteness of the integral $\iint_{Q_T} |Lu|^{n+1} dxdt$ for any function $u(x,t) \in \dot{W}_w^{2,2}(Q_T)$. It's sufficient to prove that by fulfillment of the second one from the conditions (16)

$$J_i = \iint_{Q_T} |b_i|^{n+1} |u_i|^{n+1} dxdt < \infty;\ i = 1,\ldots,n. \tag{17}$$

We'll use the following imbedding theorem proved in [10]. Let $1 < q < n+2$ and the function $u(x,t)$ given on Q_T is such that

$$D_q(u) = \sum_{i,j=1}^{n} \|u_{ij}\|_{L_q(Q_T)} + \|u_t\|_{L_q(Q_T)} + \|u\|_{L_q(Q_T)} < \infty.$$

Then if $p = \frac{q(n+2)}{n+2-q}$ then there exists a constant $C_6 = C_6(n,q,d,T)$ such that

$$\sum_{i=1}^{n} \|u_i\|_{L_p(Q_T)} \le C_6 D_q(u).$$

Assume $q = n+1$ we have for $i = 1,\ldots,n$

$$J_i \le \left(\iint_{Q_T} |u_i|^{(n+1)(n+2)} dxdt\right)^{\frac{1}{n+2}} \left(\iint_{Q_T} |b_i|^{(n+2)} dxdt\right)^{\frac{n+1}{n+2}} \le C_6^{n+1} \|u\|^{n+1}_{W_w^{2,2}(Q_T)} \left\||\overline{B}|\right\|^{n+1}_{L_{(n+2)}(Q_T)},$$

whence the correctness of the inequality (17) follows. The theorem is proved.

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Ilham T. Mamedov, Shahla Yu. Salmanova
Institute of Mathematics & Mechanics of NAS Azerbaijan.
9, F.Agayev str., 370141, Baku, Azerbaijan.
Tel.:39-39-24.
E-mail: ilham@lan.ab.az

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NAJAFOV Kh.M.

ON ONE GENERAL BOUNDARY VALUE PROBLEM WITH DISPLACEMENT FOR ONE SECOND ORDER HYPERBOLIC EQUATION

Abstract

In the present paper a general boundary value problem with shift for model hyperbolic type equation

$$U_{xx} - U_{yy} = 0 \qquad (1)$$

is considered. The boundary value problem for the equation (1) is investigated in domain D*, when on* $\bar{J} = \{(x,y): y = 0, 0 \le x \le 1\}$ *the value of the function* $U(x,y)$ *is given, but in characteristics* AC *and* BC *- the condition that pointwisely connect the value* $U(x,y)$ *and its derivative in some direction.*

As a model of second order hyperbolic partial differential equation of two independent variables x, y consider the wave equation

$$U_{xx} - U_{yy} = 0. \qquad (1)$$

Let D be a finite simply connected domain of plane of variables x, y bounded by characteristics $AC: x + y = 0$, $BC: x - y = 1$ of equation (1) and segment AB of the axis $y = 0; J \equiv AB$ is unit interval $0 < x < 1$. As regular solution of equation (1) in domain D we'll understand a function $U(x,y) \in C(\bar{D}) \cap C^2(D)$ satisfying equation (1) in D.

Consider the following boundary-value problem for equation (1) in domain D, when the value of function $U(x,y)$ is given on J, and on $AC \cup BC$ the condition pointwise connecting the values of $U(x,y)$ and its derivatives in some direction.

Problem C. Find a regular in domain D solution $U(x,y)$ of equation (1), satisfying the boundary conditions

$$U(x,0) = \tau(x), \ \forall x \in \bar{J}, \qquad (2)$$

$$\alpha(x) U\left(\frac{x}{2}, -\frac{x}{2}\right) + \alpha_1(x) \frac{d}{dx} U\left(\frac{x}{2}, -\frac{x}{2}\right) + \beta(x) U\left(\frac{x+1}{2}, \frac{x-1}{2}\right) + $$
$$+ \beta_1(x) \frac{d}{dx} U\left(\frac{x+1}{2}, \frac{x-1}{2}\right) = \delta(x), \ \forall x \in \bar{J}, \qquad (3)$$

where $\tau(x)$, $\alpha(x), \beta(x), \alpha_1(x), \beta_1(x), \delta(x)$ are given sufficiently smooth functions.

Note that the considered problem is the generalization of two problems from paper [2] and analogous problem from [3]. We'll go over to investigation of problem C.

Regular in domain D general solution $U(x,y)$ of equation (1) satisfying boundary condition (2) can be represented in the following form

$$U(x,y) = \frac{1}{2}\left[\tau(x+y) + \tau(x-y)\right] + \frac{1}{2}\int_{x-y}^{x+y} \nu(t)\,dt, \qquad (4)$$

where $\nu(x) \in C^1(J)$ is an arbitrary function.

Taking into account (4) in boundary condition (3) we'll obtain

$$\left[\alpha(x) + \beta(x)\right]\tau(x) + \left[\alpha_1(x) + \beta_1(x)\right]\tau'(x) - \left[\alpha_1(x) - \beta_1(x)\right]\nu(x) -$$