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THE INVERSE SCATTERING PROBLEM ON A SEMI-AXIS FOR FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS SYSTEM

Abstract

In the paper the direct and inverse scattering problems are studied for a system of ordinary differential equations of the first order on a semi-axis under joint consideration of $n-1$ *problems.*

The algorithm for restoration of coefficients of equations are given when singular numbers are absent.

In the paper the inverse scattering problem is studied on a semi-axis for the system of first order $n \geq 3$ equations system of the form

$$-i\frac{dy_k}{dx}+\sum_{j=1}^{n}c_{k_j}(x)y_j(x)=\lambda\xi_k y_k(x), \quad x \geq 0 \qquad (1)$$
$$(\xi_1>\xi_2>\ldots>\xi_{n-1}0>\xi_n),$$

where

$$c_{j_k}(x)\in L_1(0,+\infty),\, c_{jj}(x)=0, \quad j,k=1,\ldots,n. \qquad (2)$$

The inverse scattering problem on a semi-axis for the Dirac system of equations with self-adjoint potential has been solved in papers [1,2] and in [3] with non-self-adjoint potential. The direct scattering problem for the system (1) of equations on a semi-axis has been considered in [4,5]. The scattering problem for the system (1) of equations on the whole axis, when $\xi_i \neq \xi_k \, (i,k=1,2,\ldots,n)\, \mathrm{Im}\,\xi_i=0$ has been studied in [6-10] and a complete solution of the general problem ($\mathrm{Im}\,\xi_i \neq 0$) have been studied in [11].

1. The scattering problem. Let λ be a fixed and real number. If the coefficients of the equations system (1) satisfy the condition (2), then there exists such a solution that

$$y_k(x)=A_k e^{i\lambda\xi_k x}+0(1), \quad k=1,\ldots,n-1,$$
$$y_n(x)=Be^{i\lambda\xi_n x}+0(1), \quad x\to+\infty. \qquad (3)$$

These statements follow from [4,5,13].

Consider the $n-1$ problem:

the k-th ($k=1,2,\ldots,n-1$) problem is in the finding the solution of the system (1) of equations of the system (1) with a boundary condition

$$y_n^k(0)=y_k^k(0), \quad k=1,2,\ldots,n-1 \qquad (4)$$

under given asymptotics

$$y_j^k(x)=A_j e^{i\lambda\xi_j x}+0(1), \quad x\to+\infty. \qquad (5)$$

The joint consideration of these $n-1$ problems will be called the scattering problem for the system (1) on a semi-axis.

The scattering problem for the k-th problem is equivalent to the following system of integral equations

$$y_j^k(x)=A_j e^{i\lambda\xi_j x}+i\int_x^{+\infty}\sum_{p=1}^{n}c_{jp}(x')y_p^k(x')e^{i\lambda\xi_j(x-x')}dx',$$
$$y_n^k(x)=B_k e^{i\lambda\xi_n x}+i\int_x^{+\infty}\sum_{p=1}^{n}c_{np}(x')y_p^k(x')e^{i\lambda\xi_n(x-x')}dx', \qquad (6)$$

where

$$B_k = A_k + i\int_x^{+\infty}\sum_{p=1}^{n}\left[c_{kp}(x')\exp(-i\lambda\xi_k x') - c_{np}(x')\exp(-i\lambda\xi_n x')\right]\times$$
$$\times\, y_p^k(x')dx', \quad k = 1,2,\dots,n-1 .$$

The existence and uniqueness of solutions of the system (6) follow from the Volterra property of these integral equations of the system [5].

From (6) and (2) we get

$$y_n^k(x) = B_k e^{i\lambda\xi_n x} + 0(1), \quad x\to+\infty, \quad k = 1,\dots,n-1 . \tag{7}$$

Thus, there exists such a quadratic matrix $S(\lambda) = \|S_{ij}\|_{i,j=1}^{n-1}$, that

$$S(\lambda)(A_1\exp(i\lambda\xi_1 x),\dots,A_{n-1}\exp(i\lambda\xi_{n-1}x))^T = (B_1\exp(i\lambda\xi_n x),\dots,B_{n-1}\exp(i\lambda\xi_n x))^T . \tag{8}$$

For $x = 0$ the matrix $S(\lambda)$ transfers $(A_1,\dots,A_{n-1})^T$ to $(B_1,\dots,B_{n-1})^T$:

$$S(\lambda)(A_1,\dots,A_{n-1})^T = (B_1,\dots,B_{n-1})^T . \tag{9}$$

The matrix $S(\lambda)$ we shall call the scattering matrix for the system (1).

2. Transformation operators. The transformation operators allow to study the structure of the scattering operator and to solve the inverse problem. We consider $2n$ vector functions

$h^1(x,\lambda) = \{y_1(0)\exp(i\xi_1\lambda x),\dots,y_n(0)\exp(i\xi_n\lambda x)\}$,

$h^k(x,\lambda) = \{A_1\exp(i\xi_1\lambda x),\dots,A_{k-1}\exp(i\xi_{k-1}\lambda x),\ y_k(0)\exp(i\xi_k\lambda x),\dots,$
$y_n(0)\exp(i\xi_n\lambda x)\}$, $2\le k\le n$,

$h^{n+1}(x,\lambda) = \{A_1\exp(i\xi_1\lambda x),\dots,A_{n-1}\exp(i\xi_{n-1}\lambda x), B\exp(i\xi_n\lambda x)\}$,

$h^{n+k}(x,\lambda) = \{y_1(0)\exp(i\xi_1\lambda x),\dots,y_{k-1}(0)\exp(i\xi_{k-1}\lambda x), A_k\exp(i\xi_k\lambda x),\dots,$
$A_{n-1}\exp(i\xi_{n-1}\lambda x), B\exp(i\xi_n\lambda x)\}$, $2\le k\le n-1$,

$h^{2n}(x,\lambda) = \{y_1(0)\exp(i\xi_1\lambda x),\dots,y_{n-1}(0)\exp(i\xi_{n-1}\lambda x), B\exp(i\xi_n\lambda x)\}$.

Lemma 1 [5]. *Let the coefficients of the system (1) satisfy the condition (2). Then each bounded solution has integral representation*

$$y_1(x) = h_i^1(x) + \sum_{j=1}^{n}\int_{\xi_n x}^{\xi_1 x} R_{ij}^1(x,\tau)\exp(i\lambda\tau)h_j^1(0), \tag{10}$$

$$y_i(x) = h_i^2(x) + \int_{-\infty}^{\xi_1 x} R_{i1}^2(x,\tau)\exp(i\lambda\tau)d\tau\cdot h_1^1(0) + \sum_{j=2}^{n}\int_{-\infty}^{\xi_2 x} R_{ij}^2(x,\tau)\exp(i\lambda\tau)d\tau\cdot h_j^2(0), \tag{11}$$

$$y_i(x) = h_i^k(x) + \sum_{j=1}^{k-2}\int_{-\infty}^{\infty} R_{ij}^k(x,\tau)\exp(i\lambda\tau)d\tau h_j^k(0) + \int_{-\infty}^{\xi_{k-1}x} R_{i,k-1}(x,\tau)\exp(i\lambda\tau)h_{k-1}^k(0) +$$
$$+ \sum_{j=k}^{n}\int_{-\infty}^{\xi_k x} R_{ij}^k(x,\tau)\exp(i\lambda\tau)d\tau\cdot h_j^k(0), \ (3\le k\le n), \tag{12}$$

$$y_i(x) = h_i^{n+1}(x) + \int_{\xi_1 x}^{\infty} R_{i1}^{n+1}(x,\tau)\exp(i\lambda\tau)h_1^{n-1}(0) + \sum_{j=2}^{n-1}\int_{-\infty}^{\infty} R_{ij}^{n+1}(x,\tau)\exp(i\lambda\tau)h_j^{n+1}(0) +$$
$$+ \int_{-\infty}^{\xi_n x} R_{in}^{n+1}(x,\tau)\exp(i\lambda\tau)d\tau\, h_n^{n+1}(0), \tag{13}$$

$$y_i(x)=h_i^{n+k}(x)+\sum_{j=1}^{k-1}\int_{\xi_{k-1}x}^{+\infty}R_{ij}^{n+k}(x,\tau)\exp(i\lambda\tau)d\tau\, h_j^{n+k}(0)+\int_{\xi_n x}^{+\infty}R_{ik}^{n+k}(x,\tau)\exp(i\lambda\tau)d\tau\, h_k^{n+k}(0)+$$

$$+\sum_{j=k+1}^{n}\int_{-\infty}^{\infty}R_{ij}^{n+k}(x,\tau)\exp(i\lambda\tau)d\tau\, h_j^{n+k}(0),\quad (2\le k\le n-1), \tag{14}$$

$$y_i(x)=h_i^{2n}(x)+\sum_{j=1}^{n-1}\int_{\xi_{n-1}x}^{+\infty}R_{ij}^{2n}(x,\tau)\exp(i\lambda\tau)d\tau\, h_j^{2n}(0)+\int_{\xi_n x}^{+\infty}R_{in}^{2n}(x,\tau)\exp(i\lambda\tau)d\tau\, h_n^{2n}(0). \tag{15}$$

The kernels of these transformations are uniquely determined by the coefficients $c_{kj}(x)\,(k,j=1,...,n)$.

3. The properties of scattering matrices. The integral transformations (10)-(15) allow us to establish important relations between the components of $h^i(t)$. This relation is realized under special assumptions by means of analytic functions.

Lemma 2. *For any* $B_i\ (i=1,...,n)$

$$y_p^p(0,\lambda)-y_q^p(0,\lambda)=\left(1+a_{q_-}(\lambda)\right)\left(B_p-B_q\right),\quad p,q\in\{1,...,n-1\}\ , \tag{16}$$

where

$$a_{q_-}(\lambda)=\int_{-\infty}^{0}\left[R_{nn}^{n+1}(0,\tau)-R_{qn}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau\ .$$

They are analytic functions at the lower half-plane $\operatorname{Im}\lambda\le 0$.

Proof. Assuming in the representation (13) $h^{n+1}(0,\lambda)=\{A_1,...,A_{n-1},B_p\}$ and $h^{n+1}(0,\lambda)=\{A_1,...,A_{n-1},B_q\}$, that correspond to the solution of the p-th and q-th problems, by the substitution we get

$$y_i^p(0,\lambda)-y_i^q(0,\lambda)=\int_{-\infty}^{0}R_{in}^{n+1}(0,\tau)\exp(i\lambda\tau)d\tau\left(B_p-B_q\right),$$

$$y_n^p(0,\lambda)-y_n^q(0,\lambda)=y_p^p(0,\lambda)-y_q^q(0,\lambda)=\left(1+\int_{-\infty}^{0}R_{nn}^{n+1}(0,\tau)\exp(i\lambda\tau)d\tau\right)\left(B_p-B_q\right).$$

Hence for $i=q$ the equality (16) follows.

Lemma 3. *If* $A_{k+1}=...=A_{n-1}=B_1=...=B_{k-1}=0\ (2\le k\le n-2)$, *then*

$$y_1^i(0,\lambda)=...=y_{k-1}^i(0,\lambda)=B_{1k+}(\lambda)A_k,\ i=1,...,n-1, \tag{17}$$

$$y_k^i(0,\lambda)=\left(1+B_{2k+}(\lambda)\right)A_k, \tag{18}$$

$$A_k=\left(1+b_{k+}(\lambda)\right)^{-1}\left(1+a_{k-}(\lambda)\right)B_k, \tag{19}$$

where

$$B_{1k+}(\lambda)=\left(1-\sum_{j=1}^{k-1}\int_0^{\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{nk}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\ ,$$

$$B_{2k+}(\lambda)=\int_0^{+\infty}R_k^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau+\sum_{j=1}^{k-1}\int_0^{+\infty}R_{kj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\cdot B_{1k+}(\lambda),$$

$$b_{k+}(\lambda)=B_{2k+}(\lambda)-B_{1k+}(\lambda).$$

Proof. Taking into account the condition of the lemma for the first problem when $x=0$ from (14) we have

$$y_k^1(0,\lambda)=\left(1+\int_0^{+\infty}R_{kk}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)A_k+\sum_{j=1}^{k-1}\int_0^{+\infty}R_{kj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\cdot y_j^1(0,\lambda),$$

$$y_n^1(0,\lambda)=\int_0^{+\infty}R_{nk}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\cdot A_k+\sum_{j=1}^{k-1}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\cdot y_j^1(0,\lambda), \qquad (20)$$

$$(2\le k\le n-2).$$

For $B_1=...=B_{k-1}$ from lemma 2, it follows

$$y_1^i(0,\lambda)=y_2^i(0,\lambda)=...=y_{k-1}^i(0,\lambda),\ \ i=1,...,k-1.$$

Taking this and $y_n^1(0,\lambda)=y_1^1(0,\lambda)$ into account from the system (20) we get the equalities (17), (18) and (19).

Lemma 4. *Let* $B\equiv B_1=...=B_{n-1}$. *Then the solutions of these* $n-1$ *problems coincide. If* $\alpha\equiv y_k^1(0)=...=y_k^{n-1}(0)$, *then*

$$\alpha=(1+N_+(\lambda))B, \qquad (21)$$

$$\alpha=(1+N_-(\lambda))A_1, \qquad (22)$$

where

$$N_+(\lambda)=\left(1-\sum_{j=1}^{n-1}\int_0^{+\infty}R_{nj}^{2n}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\left(1+\int_0^{+\infty}R_{nn}^{2n}(0,\tau)\exp(i\lambda\tau)d\tau\right)-1,$$

$$N_-(\lambda)=\left(1-\sum_{j=2}^{n}\int_{-\infty}^{0}R_{1j}^{2}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\left(1+\int_{-\infty}^{0}R_{11}^{2}(0,\tau)\exp(i\lambda\tau)d\tau\right)-1.$$

Proof. Let $B_1=...=B_{n-1}$. It follows from lemma 2 that

$$y_1^i(0,\lambda)=...=y_{n-1}^i(0,\lambda)=y_n^i(0,\lambda)\ (i=1,...,n-1).$$

From the uniqueness of the Cauchy problem for the system (1) we also get that the solutions of these $n-1$ problems coincide. The remaining parts of the proof follow from (11) and (15).

Lemma 5. *If* $B_k\neq 0$ *($k\neq n-1$ is fixed),* $B_i=0,\ i\neq k$, *then*

$$y_k^i(0,\lambda)=(1+c_{k+}(\lambda))(1+a_{k-}(\lambda))B_k, \qquad (23)$$

$$y_n^i(0,\lambda)=y_{n-1}^i(0,\lambda)=y_i^i(0,\lambda)=c_{k+}(\lambda)(1+a_{k-}(\lambda))B_k, \qquad (24)$$

$$A_{n-1}=r_{k+}(\lambda)(1+a_{k-}(\lambda))B_k,\ \ k=1,...,n-2, \qquad (25)$$

and for $k=n-1$, *i.e.* $B_{n-1}\neq 0,\ B_1=...=B_{n-2}=0$,

$$y_n^i(0,\lambda)=y_i^i(0,\lambda)=c_{n-1+}(\lambda)(1+a_{n-1-}(\lambda))B_{n-1},\ \ i=1,...,n-2, \qquad (26)$$

$$y_{n-1}^i=(1+c_{n-1+}(\lambda))(1+a_{n-1-}(\lambda))B_{n-1}, \qquad (27)$$

$$A_{n-1}=(1+b_{n-1+}(\lambda))(1+a_{n-1-}(\lambda)), \qquad (28)$$

where

$$c_{k+}(\lambda)=(1-\hat{c}_{k+}(\lambda))^{-1}-1,\ \hat{c}_{k+}(\lambda)=\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\times$$

$$\times\left\{\int_0^{+\infty}R_{n-1,k}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau+\left(1+\int_0^{+\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)\left[\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\times\right.\right.\right.$$

$$\times\exp(i\tau\lambda)d\tau\Big)^{-1}\left(1+\int_0^{\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)\times$$

$$\left.\times\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{n,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right]^{-1}\times$$

$$\times\left[\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{nk}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau-\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\lambda)\times\right.\right.$$

$$\left.\left.\left.\times\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{n-1,k}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right]\right\},\ (k=1,...,n-2),$$

$$c_{n-1+}(\lambda)=(1-\hat{c}_{n-1+}(\lambda))^{-1}-1,\ \ \hat{c}_{n-1+}(\lambda)=\left(1-\sum_{j=1}^{n-2}\int_0^{+\infty}R_{nj}^{2n+1}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\times$$

$$\times\int_0^{+\infty}R_{n,n-1}^{2n-1}(0,\tau)\exp(i\lambda\tau)d\tau\left[1+\int_0^{+\infty}R_{n-1,n-1}^{2n-1}(0,\tau)\exp(i\lambda\tau)d\tau+\sum_{j=1}^{n-2}\int_0^{+\infty}R_{n-1,j}^{2n-1}(0,\tau)\exp(i\lambda\tau)d\tau\times\right.$$

$$\left.\times\left(1-\sum_{j=1}^{n-2}\int_0^{+\infty}R_{nj}^{2n-1}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\cdot\int_0^{+\infty}R_{n,n-1}^{2n-1}(0,\tau)\exp(i\lambda\tau)d\tau\right];$$

$$r_{k+}(\lambda)=\left(1+\int_0^{+\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\left[\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)\hat{c}_{k-}(\lambda)-\right.$$

$$\left.-\int_0^{+\infty}R_{n-1,k}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right](1-\hat{c}_{k-})^{-1},\ \ k=1,...,n-2,$$

$$b_{n-1+}(\lambda)=\left[1+\int_0^{+\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau+\sum_{j=1}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\times\right.$$

$$\left.\times\left(1-\sum_{j=1}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{n,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right]^{-1}(1-\hat{c}_{n-1+}(\lambda))^{-1}-1.$$

Proof. In the first case from (14) we obtain

$$y_{n-1}^{\prime}(0,\lambda)=\left(1+\int_0^{+\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)A_{n-1}+\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\times$$

$$\times y_{n-1}^i(0,\lambda)+\int_0^{+\infty}R_{n-1,k}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\cdot y_k^i(0,\lambda),$$

$$y_n^i(0,\lambda)=y_{n-1}^i(0,\lambda)=\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\, y_k^i(0,\lambda)+\int_0^{+\infty}R_{n,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\times \tag{29}$$

$$\times A_{n-1}.$$

By solving of the system we have $y_{n-1}^i(0,\lambda)=\hat{c}_{k+}(\lambda)y_k^i(0,\lambda)$ or

$$y_k^i(0,\lambda)-y_{n-1}^i(0,\lambda)=(1-\hat{c}_{k+}(\lambda))y_k^i(0,\lambda). \tag{30}$$

Since $y_{n-1}^i(0,\lambda)=y_n^i(0,\lambda)=y_i^i(0,\lambda)$, then

$$y_k^i(0,\lambda)-y_i^i(0,\lambda)=(1-\hat{c}_{k+}(\lambda))y_k^i(0,\lambda). \tag{31}$$

Comparing (16), (29) and (31) we get (23) and (24). The equality (25) also follows from (16), (29) and (31). In the second case from (14) analogously we'll have:

$$y_n^i(0,\lambda)=\left(1-\sum_{j=1}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{n,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\cdot A_{n-1},$$

$$y_{n-1}^i(0,\lambda)=\left[1+\int_0^{+\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau+\sum_{j=1}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\times\right.$$

$$\left.\times\left(1-\sum_{j=1}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{n,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau\right]\cdot A_{n-1}.$$

Hence $y_n^i(0,\lambda)=\hat{c}_{n-1+}(\lambda)y_{n-1}^i(0,\lambda)$ or

$$y_{n-1}^i(0,\lambda)-y_n^i(0,\lambda)=(1-\hat{c}_{n-1+}(\lambda))y_{n-1}^i(0,\lambda). \tag{32}$$

Comparing (16) and (32) we get (26), (27). The equality (28) is proved similarly.

Lemma 6. *If* $A_1=...=A_{n-2}=0$, *then*

$$y_i^i(0,\lambda)=\left(1-\int_0^{+\infty}R_{in}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\int_0^{+\infty}R_{i,n-1}^n(0,\tau)\exp(i\lambda\tau)d\tau\cdot A_{n-1},\ i=1,...,n-2, \tag{33}$$

$$y_{n-1}^{n-1}(0,\lambda)=\left(1-\int_0^{\infty}R_{n-1,n}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\left(1+\int_0^{+\infty}R_{n-1,n-1}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)A_{n-1}. \tag{34}$$

Proof. From the representation (12) $(k=n)$ for $A_1=...=A_{n-2}=0$ we get for each problem the following

$$y_i^i(0,\lambda)=\int_0^{+\infty}R_{i,n-1}^n(0,\tau)\exp(i\lambda\tau)d\tau\cdot A_{n-1}+\int_0^{+\infty}R_{in}^n(0,\tau)\exp(i\lambda\tau)d\tau\cdot y_i^i(0,\lambda),\ i=1,...,n-2,$$

$$y_{n-1}^{n-1}(0,\lambda)=\left(1+\int_0^{+\infty}R_{n-1,n-1}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)\cdot A_{n-1}+\int_0^{+\infty}R_{n-1,n}^n(0,\tau)\exp(i\lambda\tau)d\tau\cdot y_{n-1}^{n-1}(0,\lambda).$$

The solution of the system has the form (33) and (34).

Note that in these lemmas are assumed that the functions

$$1+b_{k+}(\lambda);\ 1-\sum_{j=1}^{k-1}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau,\ (k=1,...,n-1),$$

$$1-\sum_{j=1}^{n-1}\int_0^{+\infty}R_{nj}^{2n}(0,\tau)\exp(i\lambda\tau)d\tau;\ 1-\sum_{j=2}^{n}\int_{-\infty}^{0}R_{1j}^{2}(0,\tau)\exp(i\lambda\tau)d\tau;$$

$$1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{n-1,j}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau;\ 1-\sum_{\substack{j=1\\ j\neq k}}^{n-2}\int_0^{+\infty}R_{nj}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau;\ (k=1,...,n-2),$$

$$1-\hat{c}_{k+}(\lambda);\ 1+\int_0^{+\infty}R_{n-1,n-1}^{n+k}(0,\tau)\exp(i\lambda\tau)d\tau;\ 1-\int_0^{\infty}R_{kn}^{n}(0,\tau)\exp(i\lambda\tau)d\tau;$$

$$(k=1,...,n-1);\ 1+b_{n-1+}(\lambda);\ 1-\sum_{j=1}^{n-2}\int_0^{+\infty}R_{nj}^{2n-1}(0,\tau)\exp(i\lambda\tau)d\tau$$

have no zeros. The set of zeros of these functions denote by $O_1(\lambda)$.

By $O_2(\lambda)$ we denote a set of zeros of the functions

$$\det\left\|S_{ij}(\lambda)\right\|_{i,j=1}^{k}\ (1\le k\le n-1);\ \gamma_{11}(\lambda)+...+\gamma_{1,n-1}(\lambda);\ 1+N_-(\lambda),$$

$$1+\int_{-\infty}^{0}R_{11}^{2}(0,\tau)\exp(i\lambda\tau)d\tau\ ,\ 1+\int_0^{+\infty}R_{nn}^{n+1}(0,\tau)\exp(i\lambda\tau)\,d\tau\ .$$

Let $O(\lambda)=O_1(\lambda)\cup O_2(\lambda)$. These values $\lambda\in O(\lambda)$ we shall call singular numbers of the system (1) of equations on a semi-axis. For the simplicity of the statement we restrict ourselves with the case when singular numbers are absent.

The property of the scattering matrix is studied in the following theorem.

Theorem 1. *Let the coefficients of the system (1) satisfy the conditions (2), and singular numbers are absent. Then* $S(\lambda)$ *has the inverse* $S^{-1}(\lambda)=\left\|\gamma_{ij}(\lambda)\right\|_{i,j=1}^{n-1}$. *Moreover* $S(\lambda)$ *and* $S^{-1}(\lambda)$ *have the form:*

$$S(\lambda)=I+\int_{-\infty}^{+\infty}F(\tau)e^{i\lambda\tau}d\tau\ ,\ F\in L_1(R),\tag{35}$$

$$S^{-1}(\lambda)=I+\int_{-\infty}^{+\infty}J(\tau)e^{i\lambda\tau}d\tau\ ,\ J\in L_1(R).\tag{36}$$

The matrix functions $\Delta_k(\lambda)=\left\|S_{ij}(\lambda)\right\|_{i,j=1}^{k}(k=1,...,n-1)$ *are also invertible. Moreover, the functions*

$$\left(\gamma_{11}(\lambda)+...+\gamma_{1,n-1}(\lambda)\right)^{-1},\ \gamma_{n-1,n-1}(\lambda),\ \left(\Delta_k^{-1}(\lambda)\right)_{kk}\ (1\le k\le n-1)$$

admit the factorization

$$\left(\gamma_{11}(\lambda)+...+\gamma_{1,n-1}(\lambda)\right)^{-1}=\left(1+N_+(\lambda)\right)^{-1}\left(1+N_-(\lambda)\right),\tag{37}$$

$$\left(\Delta_k^{-1}(\lambda)\right)_{kk}=\left(1+b_{k+}(\lambda)\right)^{-1}\left(1+a_{k-}(\lambda)\right),\ 1\le k\le n-1\,,\tag{38}$$

and the functions $S_{n-1,n-1}(\lambda)-S_{k,n-1}(\lambda)$ $(k=1,...,n-2)$; $S_{k1}(\lambda)\,(k=2,...,n-1)$; $\gamma_{1k}(\lambda)$ $(k=1,...,n-1)$; $\gamma_{n-1,k}(\lambda)\,(k=1,...,n-2)$ *have the form*

$$S_{n-1,n-1}(\lambda)-S_{k,n-1}(\lambda)=1+G_{k-}(\lambda),\tag{39}$$

$$S_{k1}(\lambda)=\left(1+a_{k-}(\lambda)\right)^{-1}q_{k_+}(\lambda),\ q_{k+}(\lambda)=\int_0^{+\infty}\left[R_{k1}^{n+1}(0,\tau)-R_{n1}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau\,,\tag{40}$$

$$\gamma_{1k}(\lambda)=\left(1+N_-(\lambda)\right)^{-1}c_{k_+}(\lambda)\left(1+a_{k\_}(\lambda)\right)+\delta_{1k}+t_{k-}(\lambda)\,,\tag{41}$$

$$\delta_{1k} = \begin{cases} 1 & k=1 \\ 0 & k \neq 1 \end{cases}$$

$$\gamma_{n-1,k}(\lambda) = r_{k+}(\lambda)(1 + a_{k\_}(\lambda)). \qquad (42)$$

Proof. From the representation (13) taking into account the boundary condition (4) $(k = 1,...,n-1)$ we get equalities

$$B_1 = (1 + a_{1-}(\lambda))^{-1}\left\{1 + \int_0^{+\infty}\left[R_{11}^{n+1}(0,\tau) - R_{n1}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau \cdot A_1 + \right.$$

$$\left. + \sum_{j=2}^{n-1}\int_{-\infty}^{+\infty}\left[R_{1j}^{n+1}(0,\tau) - R_{nj}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau \cdot A_j\right\},$$

$$B_k = (1 + a_{k-}(\lambda))^{-1}\left\{\int_0^{+\infty}\left[R_{k1}^{n+1}(0,\tau) - R_{n1}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau \cdot A_1 + \right.$$

$$+ 1 + \int_{-\infty}^{+\infty}\left[R_{kk}^{n+1}(0,\tau) - R_{nk}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau \cdot A_k +$$

$$\left. + \sum_{\substack{j=2 \\ j\neq k}}^{n-1}\int_{-\infty}^{+\infty}\left[R_{kj}^{n+1}(0,\tau) - R_{nj}^{n+1}(0,\tau)\right]\exp(i\lambda\tau) \cdot A_j\right\}, \quad (2 \le k \le n-1) \ .$$

From here and the definition of the matrix $S(\lambda)$ we get

$$S_{11}(\lambda) = (1 + a_{1-}(\lambda))^{-1}(1 + b_{1+}(\lambda)),$$

$$S_{1p}(\lambda) = (1 + a_{1-}(\lambda))^{-1} \cdot \int_{-\infty}^{\infty}\left[R_{1p}^{n+1}(0,\tau) - R_{np}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau \ , \ (p = 2,...,n-1),$$

$$S_{k1}(\lambda) = (1 + a_{1-}(\lambda))^{-1} \cdot \int_0^{\infty}\left[R_{k1}^{n+1}(0,\tau) - R_{n1}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau,$$

$$S_{kk}(\lambda) = (1 + a_{k-}(\lambda))^{-1}\left(1 + \int_{-\infty}^{\infty}\left[R_{kk}^{n+1}(0,\tau) - R_{nk}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau\right),$$

$$S_{kp}(\lambda) = (1 + a_{k-}(\lambda))^{-1}\left(\int_{-\infty}^{\infty}\left[R_{kp}^{n+1}(0,\tau) - R_{np}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau\right), p \neq k \ , \ k = 2,...,n-1 \ ,$$

where $b_{1+}(\lambda) = \int_0^{+\infty}\left[R_{11}^{n+1}(0,\tau) - R_{n1}^{n+1}(0,\tau)\right]\exp(i\lambda\tau)d\tau$.

Thus, the factorization (38) $(k = 1)$ and the property (40) are established. Moreover, it is shown that (35) and (36) are valid.

From the lemma 5 we get (by definition of $S^{-1}(\lambda)$) (38) $(k = n-1)$ and (42) respectively.

On the other hand, by the definition $(\Delta_k^{-1})_{kk}$ lemma 3, implies (38) $(2 \le k \le n-2)$.

Let the conditions of lemma 5 be fulfilled. Then from the representation (11) for $B_1 \neq 0, B_2 = ... = B_{n-1} = 0$ we have

$$y_1(0,\lambda)=\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)A_1+\sum_{j=2}^{n}\int_{-\infty}^{0}R_{1j}^2(0,\tau)e^{i\lambda\tau}d\tau\cdot y_2(0,\lambda),$$

or

$$A_1=\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}\left[y_1(0,\lambda)-\sum_{j=2}^{n}\int_{-\infty}^{0}R_{1j}^2(0,\tau)e^{i\tau\lambda}d\tau\cdot y_2(0,\lambda)\right]=$$

$$=\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}\left(1+c_{1+}(\lambda)-\sum_{j=2}^{n}\int_{-\infty}^{0}R_{1j}^2(0,\tau)e^{i\tau\lambda}d\tau\cdot c_{1+}(\lambda)\right)(1+a_{1-}(\lambda))B_1 .$$

Hence taking into account (22), by definition of $S^{-1}(\lambda)$ we have

$$\gamma_{11}(\lambda)=\left[\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}(1+a_{1-}(\lambda))+\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}\times\right.$$
$$\left.\times\left(1-\sum_{j=2}^{n}\int_{-\infty}^{0}R_{1j}^2(0,\tau)e^{i\lambda\tau}d\tau\right)c_{1+}(\lambda)(1+a_{1-}(\lambda))\right]. \tag{43}$$

Similarly, for $b_k\neq 0$, $b_i=0$, $i\neq k$, $k\geq 2$ it follows from (11) that

$$\gamma_{1k}(\lambda)=(1+N_-(\lambda))^{-1}c_{k+}(\lambda)(1+a_{k-}(\lambda))-\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}\times$$
$$\times\int_{-\infty}^{0}R_{1k}^2(0,\tau)e^{i\lambda\tau}d\tau(1+a_{1-}(\lambda)). \tag{44}$$

Denoting

$$t_{1-}(\lambda)=\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}(1+a_{1-}(\lambda))-1,$$

$$t_{k+}(\lambda)=-\left(1+\int_{-\infty}^{0}R_{11}^2(0,\tau)e^{i\lambda\tau}d\tau\right)^{-1}\int_{-\infty}^{0}R_{1k}^2(0,\tau)e^{i\lambda\tau}d\tau(1+a_{1-}(\lambda)),\ k=2,...,n-1$$

from (43) and (44) we get (41).

Prove the equalities (37), and (39). Let $B_1=B_2=...=B_{n-1}\equiv B$. In this case $a_1=(\gamma_{11}+\gamma_{12}+...+\gamma_{1,n-1})B$ and by lemma 4 $a_1=(1+N_-(\lambda))^{-1}(1+N_+(\lambda))B$, i.e. the equality (37) is valid.

Let $A_1=...=A_{n-2}=0$. From lemma 6 we have

$$y_n^{n-1}(0,\lambda)-y_n^k(0,\lambda)=\left[\left(1-\int_{0}^{\infty}R_{n-1,n}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\times\right.$$
$$\times\left(1+\int_{0}^{+\infty}R_{n-1,n-1}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)-\left(1-\int_{0}^{+\infty}R_{kn}^n(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\times$$
$$\left.\times\int_{0}^{\infty}R_{k,n-1}^n(0,\tau)\exp(i\lambda\tau)d\tau\right]A_{n-1}.$$

On the other hand it follows from (13) that

$$y_n^{n-1}(0,\lambda)-y_n^k(0,\lambda)=\left(1+\int_0^{+\infty}R_{n,n}^{n+1}(0,\tau)\exp(i\lambda\tau)d\tau\right)(B_{n-1}-B_k).$$

Comparing these last two equalities we have

$$B_{n+1}-B_k=(1+G_{k-}(\lambda))a_{n-1}$$

or

$$S_{n-1,n-1}-S_{k,n-1}=1+G_{k-}(\lambda),$$

where

$$G_{k-}(\lambda)=\left(1+\int_0^{+\infty}R_{n,n}^{n+1}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\left[\left(1-\int_0^{+\infty}R_{n-1,n}^{n}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\times\right.$$

$$\times\left(1+\int_0^{+\infty}R_{n-1,n-1}^{n}(0,\tau)\exp(i\lambda\tau)d\tau\right)-\left(1-\int_0^{+\infty}R_{kn}^{n}(0,\tau)\exp(i\lambda\tau)d\tau\right)^{-1}\times$$

$$\left.\times\int_0^{\infty}R_{k,n-1}^{n}(0,\tau)\exp(i\lambda\tau)d\tau\right].$$

4. The inverse scattering problem. The inverse scattering problem for the system (1) is in the restoration of coefficients of equations on the known functions $S(\lambda)$. The inverse scattering problem on a semi-axis with boundary conditions (4) lead to the inverse problem for the system (11) on the whole axis with additional condition of equaling to zero of coefficients of the system for $x<0$.

Consider the passage matrix $\Pi(\lambda)$ connecting $(A_1,\dots,A_{n-1},B)$ with boundary values of the solution for $x=0$, i.e. $(y_1(0,\lambda),\dots,y_n(0,\lambda))$

$$\Pi(\lambda)=\begin{pmatrix}A_1\\ \vdots\\ A_{n-1}\\ B\end{pmatrix}=\begin{pmatrix}y_1(0,\lambda)\\ \vdots\\ y_{n-1}(0,\lambda)\\ y_n(0,\lambda)\end{pmatrix}. \tag{45}$$

Theorem 2. *Let the coefficients of the equation system (1) satisfy the condition (2), and singular numbers be absent. Then the passage matrix* $\Pi(\lambda)$ *is expressed by the elements* $S(\lambda)$, $S(^{-1}\lambda)$, *and analytic functions* $N_+(\lambda)$, $N_-(\lambda)$, $c_{k+}(\lambda)$ $(k=1,\dots,n-1)$ *from the factorizational equalities (37) and (38) by the following way:*

$$\Pi(\lambda)=\begin{pmatrix}1+c_{1+}(\lambda) & \cdots & c_{n-1+}(\lambda), & 1+N_+(\lambda)\\ c_{1+}(\lambda) & \cdots & c_{n-1+}(\lambda), & 1+N_+(\lambda)\\ \cdots & \cdots & \cdots & \cdots\\ c_{1+}(\lambda) & \cdots & 1+c_{n-1+}(\lambda), & 1+N_+(\lambda)\\ c_{1+}(\lambda) & \cdots & c_{n-1+}(\lambda), & 1+N_+(\lambda)\end{pmatrix}\times \tag{46}$$

$$\times\, diag(1+a_{1-}(\lambda),\dots,1+a_{n-1}(\lambda))\begin{pmatrix}S & -I\\ 0 & I\end{pmatrix},$$

where the functions $c_{k+}(\lambda)$ $(k=1,\dots,n-1)$ *are found from the formula (41) by means of solution of Riemann's problem [12].*

Proof. Let $y(x,\lambda)$ be a solution of the i-th problem $(i=2,\dots,n-1)$ with $B_2=\dots=B_{n-1}=0$, $B_2=B_{n-1}=0$. Then $A_1=\gamma_{11}(\lambda)B_1,\dots,A_{n-1}=\gamma_{n-1,1}(\lambda)B_1$ by virtue of lemma 5 $y_1(0,\lambda)$ is expressed by B_1 according the formula (23) $(k=1)$, and $y_2(0,\lambda)=...=y_n(0,\lambda)$ and it is expressed by B_1 according to formula (24). Substituting these values in (45), we have

$$\Pi(\lambda)\begin{pmatrix}\gamma_{11}(\lambda)\\ \vdots\\ \gamma_{n-1,1}(\lambda)\\ 0\end{pmatrix}=\begin{pmatrix}(1+c_{1+}(\lambda))(1+a_{1-}(\lambda))\\ \vdots\\ c_{1+}(\lambda)(1+a_{1-}(\lambda))\\ c_{1-}(\lambda)(1+a_{1-}(\lambda))\end{pmatrix}. \quad (47)$$

Similarly, assuming $B_1=B_{k-1}=B_{k+1}=0 \quad (k=2,...,n-1)$, $A_1=\gamma_{1k}B_k,...,A_{n-2}=\gamma_{n-2,k}B_k$, $A_{n-1}=\gamma_{n-1,k}B_k$ we get from lemma 5 that

$$\Pi(\lambda)\begin{pmatrix}\gamma_{1k}(\lambda)\\ \vdots\\ \gamma_{kk}(\lambda)\\ \vdots\\ \gamma_{n-1,k}(\lambda)\\ 0\end{pmatrix}=\begin{pmatrix}c_{k+}(\lambda)(1+a_{k-}(\lambda))\\ \vdots\\ (1+c_{k+}(\lambda))(1+a_{k-}(\lambda))\\ \vdots\\ c_{k+}(\lambda)(1+a_{k-}(\lambda))\end{pmatrix}, k=2,...,n-1. \quad (48)$$

Assuming $B_1=...=B_{n-1}=B$, i.e. $A_1=(\gamma_{11}+...+\gamma_{1n-1})b,...,A_{n-1}=(\gamma_{n-1,1}+...+\gamma_{n-1,n-1})B$ we get from lemma 4 that

$$\Pi(\lambda)\begin{pmatrix}\gamma_{11}(\lambda)+...+\gamma_{1,n-1}(\lambda)\\ \vdots\\ \gamma_{n-1,1}(\lambda)+...+\gamma_{n-1,n-1}(\lambda)\\ 1\end{pmatrix}=\begin{pmatrix}1+N_+(\lambda)\\ \vdots\\ 1+N_+(\lambda)\\ 1+N_+(\lambda)\end{pmatrix}. \quad (49)$$

Joining the equalities (47)-(49) in a matrix equality, we get

$$\Pi(\lambda)\begin{pmatrix}\gamma_{1,1}(\lambda) & \cdots & \gamma_{1,n-1}(\lambda), & \gamma_{11}(\lambda)+...+\gamma_{1,n-1}(\lambda)\\ \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots\\ \gamma_{n-1,1}(\lambda) & \cdots & \gamma_{n-1,n-1}(\lambda), & \gamma_{n-1,1}(\lambda)+...+\gamma_{n-1,n-1}(\lambda)\\ 0 & \cdots & 0, & 1\end{pmatrix}=$$

$$=\begin{pmatrix}1+c_{1+}(\lambda) & \cdots & c_{n-1+}(\lambda), & 1+N_+(\lambda)\\ \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots\\ c_{1+}(\lambda) & \cdots & 1+c_{n-1+}(\lambda), & 1+N_+(\lambda)\\ c_{1+}(\lambda) & \cdots & c_{n-1+}(\lambda), & 1+N_+(\lambda)\end{pmatrix}. \quad (50)$$

Since

$$\begin{pmatrix}\gamma_{11}(\lambda) & \cdots & \gamma_{1,n-1}(\lambda), & \gamma_{11}(\lambda)+...+\gamma_{1,n-1}(\lambda)\\ \cdots & \cdots & \cdots & \cdots\\ \gamma_{n-1,1}(\lambda) & \cdots & \gamma_{n-1,n-1}(\lambda) & \gamma_{n-1,1}(\lambda)+...+\gamma_{n-1,n-1}(\lambda)\\ 0 & \cdots & 0, & 1\end{pmatrix}^{-1}=\begin{pmatrix}S_{11}(\lambda) & \cdots & S_{1,n-1}(\lambda), & -1\\ \cdots & \cdots & \cdots & \cdots\\ S_{n-1,1}(\lambda) & \cdots & S_{n-1,n-1}(\lambda), & -1\\ 0 & \cdots & 0, & 1\end{pmatrix}$$

The equality (50) leads to (46).

The passage matrix $\Pi(\lambda)$ introduced by the equality (45) is closely connected with a distance matrix on the whole axis. Indeed, continuing the coefficients in the system (1) identically by zero for $x<0$, we get that the passage matrix for the initial system coincides with a distance matrix for transformed system. Since, for linear systems of the first order on the axis, the inverse problem has been studied in [6-10], then using the equality (46) we arrive at the following result in the inverse scattering problem for the system (1) on semi-axis.

Theorem 3. *Let scattering matrix* $S(\lambda)$ *be given for the system (1) with coefficients satisfying the conditions of (2), and let singular numbers be absent. Then the coefficients of the system (1) are uniquely determined by* $S(\lambda)$. *And the formulas (37), (38), (41) and (46) together with solution procedure of the inverse scattering problem on the whole axis compose a solution algorithm of the inverse scattering problem for the system (1) on a semi-axis.*

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