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ON APPROXIMATION OF MULTIVARIABLE PERIODIC FUNCTIONS IN THE $H^l_{p,m}[\omega]$ SPACES

Abstract

Upper estimates of deviations of 2π periodic multivariable functions from their trigonometric polynomials being the best in the metric $L_p(T^m)$ approximation and particular cubic sums of Fourier-Lebesque multiseries in a norm of the generalized Hölder spaces $H^l_{p,m}[\omega]$ are obtained.

Let R^m be m-dimensional Euclidean space of the points $x=(x_1,...,x_m)$ with the norm $|x|=(x_1^2+...+x_m^2)^{1/2}$, $m\in N$, $T^m=[-\pi,\pi]^m=\{x\in R^m;-\pi\le x_i\le\pi,\ i=\overline{1,m}\}$; $L_p(T^m)$, $1\le p<\infty$, be a space of all measurable 2π periodic for any variable x_i $(i=\overline{1,m})$ functions $f(x)=f(x_1,...,x_m)$, for which $\|f\|_{p,m}\equiv\|f;L_p(T^m)\|=\{(2\pi)^{-m}\int_{T^m}|f(x)|^p dx\}^{1/p}<$ $<+\infty$, $L_\infty(T^m)\equiv C(T^m)$ be a corresponding space of continuous functions, $\|f\|_{\infty,m}\equiv\|f;C(T^m)\|=\max\{|f(x)|;\ x\in T^m\}$;

$E_{n,...,n}(f)_{p,m}$ be a complete best in the metric $L_p(T^m)$ approximation of the function f by trigonometrical polynomials of order $\le n\in Z_+$ on the variable x_i $(i=\overline{1,m})$;

$T^{(m)}_{n;p}(f;x)\equiv T_{n,...,n;p}(f;x_1,...,x_m)$ be polynomial of a complete best in $L_p(T^m)$ approximation of the function f of order $n\in Z_+$ on the variable x_i $(i=\overline{1,m})$: $\|f(\cdot)-T^{(m)}_{n;p}(f;\cdot)\|_{p,m}\equiv\|f(\cdot)-T_{n,...,n;p}(f;\cdot)\|_{p,m}=E_{n,...,n}(f)_{p,m}$;

$S^{(m)}_n(f;x)\equiv S_{n,...,n}(f;x_1,...,x_m)$ be particular cubic sum of order n on the variable x_i $(i=\overline{1,m})$ of Fourier-Lebesque multiple trigonometric series of the function $f\in L_p(T^m)$;

$\omega_l(f;\delta)_{p,m}$ be a complete modulus of smoothness of l-th order of the function $f\in L_p(T^m)$, $l\in N$, $0<\delta\in R:\omega_l(f;\delta)_{p,m}=\sup\{\|\Delta^l_h f(\cdot)\|;\ h\in R^m,|h|\le\delta\}$, where $\Delta^l_h f(x)=$ $=\sum_{\nu=0}^{l}(-1)^{l-\nu}\binom{l}{\nu}f(x+\nu h)$, $x\in R^m$, $\binom{l}{\nu}=\frac{l!}{\nu!(l-\nu)!}$;

$\Omega_l(0,d]$ be a class of functions $\omega(\delta)$, defined on $(0,d]$, $d=\pi m^{1/2}$, and satisfying the conditions: $0<\omega(\delta)\downarrow 0$ $(\delta\downarrow 0)$ and $\delta^{-l}\omega(\delta)\downarrow(\delta\uparrow)$.

Denote $(1\le p\le\infty,\ m\ge 1,\ l\in N,\ \omega\in\Omega_l(0,d])$ by $H^l_{p,m}[\omega]$ class of functions $f\in L_p(T^m)$ for each of which the norm

$$\left\| f; H^{l}_{p,m}[\omega] \right\| = \|f\|_{p;m} + \sup_{\delta\in(0,d]} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} < +\infty .$$

The estimation of deviation of the function f from $T^{(m)}_{n;p}(f)$ in the metric of the space $L_p(T^m)$ by $\omega_l(f;\delta)_{p,m}$ is given by the known inequality (for $m=1$ by so-called direct theorem of constructive theory of functions; see, f.e., [1], p.226; [2], p.274, p.338, p.288; [3], p.67)

$$\left\| f - T^{(m)}_{n;p}(f) \right\|_{p,m} \equiv E_{n,\dots,n}(f)_{p,m} \le C_1(l,m)\,\omega_l\left(f; d/(n+1)\right)_{p,m}. \tag{1}$$

In the present paper the upper estimates of deviations of the functions f from $T^{(m)}_{n;p}(f)$ (theorem 1) and $S^{(m)}_{n}(f;x)$ (theorem 2) are obtained in the norm of the space $H^{l}_{p,m}[\omega]$. Mentioned corollaries 2 and 3 verify a natural expected loss the speed of convergence of corresponding aggregates in the scale of Hölder spaces with power decrease degree of the mojorant function. In connection with theorem 2 we note, that in [4; p.442] the following estimation $(n\in N)$ is proved

$$\begin{aligned} \|A_n(f)-f\|_{\varphi} &\le \|A_n(f)-f\|_{\infty,1}\left[1+2/\varphi(1/n)\right]+ \\ &+\left(1+\|A_n\|\right)\sup\left\{2\omega_1(f;\delta)_{\infty,1}/\varphi(\delta);\ 0<\delta\le 1/n\right\}, \end{aligned} \tag{2}$$

where $\{A_n\}$ is a sequence of linear convolution operators from $C(T)$ to $C(T)$ with the operator norm $\|A_n\|$, $\varphi(\delta)$ is a some non-decreasing positive function on $(0,+\infty)$, and φ-norm $\|g\|_{\varphi}$ of the function $g\in C(T)$ is determined as following: $\|g\|_{\varphi} = \|g\|_{\infty,1} + \sup\left\{\|f(\cdot)-f(\cdot+\delta)\|_{\infty,1}/\varphi(\delta);\ \delta\in(0,+\infty)\right\}$. Assuming in (2) $A_n(f;x)=S^{(1)}_{n}(f;x)$, $\varphi(\delta)=\delta^{\beta}$, $f\in Lip(\alpha;M)=\left\{f\in C(T);\omega_1(f;\delta)_{\infty,1}\le M\delta^{\alpha},\delta\in(0,\pi]\right\}$, $0<\beta<\alpha\le 1$, we obtain the statement of mentioned below corollary 3 in the case $p=\infty$, $l=m=1$.

Theorem 1. *Let* $1\le p\le\infty$, $m\ge 1$, $l\in N$, $\omega\in\Omega_l(0,d]$; *then for any function* $f\in H^{l}_{p,m}[\omega]$ *the estimation* $(n\in N)$

$$\left\| f - T^{(m)}_{n;p}(f); H^{l}_{p,m}[\omega] \right\| \le C_2(l,m) \sup_{\delta\in(0,d/n]} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)}. \tag{3}$$

is valid.

Proof. We beforehand obtain the upper estimate of the quantity $\sup\left\{\omega_l\left(f-T^{(m)}_{n;p}(f);\delta\right)_{p;m}/\omega(\delta);\delta\in(0,d]\right\}$. For fixed $n\in N$ and arbitrary $\delta\in(0,d]$ two cases are possible: $\delta<d/n$ or $\delta\ge d/n$. In the case $\delta\ge d/n$ by virtue of obvious inequality $\omega_l(g;\delta)_{p,m}\le 2^l\|g\|_{p,m}\left(g\in L_p(T^m)\right)$ we have $\omega_l\left(f-T^{(m)}_{n;p}(f);\delta\right)_{p,m}\le$ $\le 2^l\left\|f-T^{(m)}_{n;p}(f)\right\|_{p,m} = 2^l E_{n,\dots,n}(f)_{p,m}$, whence taking into account $\omega(\delta)\uparrow(\delta\uparrow)$ we obtain

$$\frac{\omega_l\left(f-T^{(m)}_{n;p}(f);\delta\right)_{p,m}}{\omega(\delta)} \le 2^l \frac{E_{n,\dots,n}(f)_{p,m}}{\omega(d/n)}. \tag{4}$$

Now we consider the case $\delta < d/n$. By virtue of the right inequality from 1) of lemma 3 [3; p.68] (assume $p=q, r=l$), the inequality (29) [2; p.232], the relation (23) from theorem 7 [5; p.674] and the inequality (1) we have

$$\omega_l\left(T_{n;p}^{(m)}(f);\delta\right)_{p,m} \le m^l\delta^l \max_{|\alpha|=l}\left\|D^\alpha T_{n;p}^{(m)}(f;\cdot)\right\|_{p,m} \le$$
$$\le m^l\delta^l 2^{-l}n^l \max_{|\alpha|=l}\omega_{\alpha_1,\dots,\alpha_m}\left(T_{n,p}^{(m)}(f);\frac{\pi}{n},\dots,\frac{\pi}{n}\right)_{p,m} \le$$
$$\le m^l\delta^l 2^{-l}n^l C_3(l,m)\,\omega_l\left(T_{n;p}^{(m)}(f);d/n\right)_{p,m} \le$$
$$\le m^l\delta^l 2^{-l}n^l C_3(l,m)\left\{\omega_l\left(T_{n;p}^{(m)}(f)-f;d/n\right)_{p,m} + \omega_l(f;d/n)_{p,m}\right\} \le$$
$$\le m^l\delta^l n^l C_3(l,m)\left\{E_{n,\dots,n}(f)_{p,m} + 2^{-l}\omega_l(f;d/n)_{p,m}\right\} \le$$
$$\le m^l C_3(l,m)\delta^l n^l\left\{C_1(l,m)\omega_l(f;d/(n+1))_{p,m} + 2^{-l}\omega_l(f;d/n)_{p,m}\right\} \le$$
$$\le m^l C_3(l,m)\left[2^{-l}+C_1(l,m)\right]\delta^l n^l\omega_l(f;d/n)_{p,m},$$

where $D^\alpha g(x) \equiv \partial^{|\alpha|}g(x)/\partial x^\alpha$, $x \in R^m$, $\alpha=(\alpha_1,\dots,\alpha_m)$, $\alpha_i \in Z_+$ $\left(i=\overline{1,m}\right)$, $|\alpha|=\alpha_1+\dots+\alpha_m$, $\partial x^\alpha = \partial x_1^{\alpha_1}\dots\partial x_m^{\alpha_m}$; $\omega_{\alpha_1,\dots,\alpha_m}(g;\delta_1,\dots,\delta_m)_{p,m}$ is a mixed moduli of smoothness of the function $g \in L_p(T^m)$ of the order α_i on the variable x_i $\left(i=\overline{1,m}\right)$ (see, f.e., [5; p.673], [2; p.126]), whence

$$\omega_l\left(T_{n;p}^{(m)}(f);\delta\right)_{p,m} \le C_4(l,m)\,\delta^l n^l\omega_l(f;d/n)_{p,m}. \tag{5}$$

Note, that the inequality (5) is m-dimensional L_p-analogue of the inequality (4.3) from the paper of S.B.Stechkin [1; p.229]. Further, applying in (5) the known inequality (see, f.e., [2], inequality (6) on p.116): $\delta_2^{-l}\omega_l(f;\delta_2)_{p,m} \le 2^l\delta_1^{-l}\omega_l(f;\delta_1)_{p,m}(\delta_1<\delta_2)$, we obtain

$$\omega_l\left(T_{n;p}^{(m)}(f);\delta\right)_{p,m} \le C_4(l,m)2^l d^l\omega_l(f;\delta)_{p,m},$$

whence $\left(C_5(l,m)=2^l d^l C_4(l,m)+1\right)$ $\omega_l\left(f-T_{n;p}^{(m)}(f);\delta\right)_{p,m} \le C_5(l,m)\,\omega_l(f;\delta)_{p,m}$, and consequently for $\delta < d/n$ we have

$$\frac{\omega_l\left(f-T_{n;p}^{(m)}(f);\delta\right)_{p,m}}{\omega(\delta)} \le C_5(l,m)\frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)}. \tag{6}$$

By virtue of the obtained estimations (4), (6) and the inequality (1) we have

$$\left\|f-T_{n;p}^{(m)}(f);H_{p,m}^l[\omega]\right\| = \left\|f-T_{n;p}^{(m)}(f)\right\|_{p,m} + \sup_{\delta\in(0,d]}\frac{\omega_l\left(f-T_{n;p}^{(m)}(f);\delta\right)_{p,m}}{\omega(\delta)} \le$$
$$\le E_{n,\dots,n}(f)_{p,m} + C_5(l,m)\sup_{0<\delta<d/n}\frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} + 2^l\frac{E_{n,\dots,n}(f)_{p,m}}{\omega(d/n)} =$$
$$=\left(1+\frac{2^l}{\omega(d/n)}\right)E_{n,\dots,n}(f)_{p,m} + C_5(l,m)\sup_{0<\delta<d/n}\frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le$$
$$\le\left[\omega(d)+2^l\right]\frac{E_{n,\dots,n}(f)_{p,m}}{\omega(d/n)} + C_5(l,m)\sup_{0<\delta<d/n}\frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le$$

$$\le \left[2^l + \omega(d)\right] C_1(l,m) \frac{\omega_l(f;d/n)_{p,m}}{\omega(d/n)} + C_5(l,m) \sup_{0<\delta<d/n} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le$$

$$\le \left\{\left[2^l + \omega(d)\right] C_1(l,m) + C_5(l,m)\right\} \sup_{0<\delta\le d/n} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)}.$$

Theorem 1 is proved.

Remark. By proving theorem 1 only the property $\omega(\delta)\uparrow(\delta\uparrow)$ of the function $\omega \in \Omega_l(0,d]$ is practically used.

Corollary 1. *Let* $1 \le p \le \infty$, $m \ge 1$, $l,k \in N, l \ge k$, $\varphi \in \Omega_k$, $\omega \in \Omega_l$ *and* $\sup\{\varphi(\delta)/\omega(\delta); \delta \in (0,d]\} < +\infty$; *then for any function* $f \in H^k_{p,m}[\varphi]$ *the estimation* $(n \in N)$

$$\left\| f - T^{(m)}_{n,p}(f);\, H^l_{p,m}[\omega] \right\| \le C_6(l,k,m) \left\| f; H^k_{p,m}[\varphi] \right\| \sup_{\delta\in(0,d/n]} \frac{\varphi(\delta)}{\omega(\delta)}$$

is valid.

Proof. From the condition $\sup\{\varphi(\delta)/\omega(\delta); \delta \in (0,d]\} < +\infty$ it follows the existence of such a constant $M > 0$ that $\varphi(\delta) \le M\omega(\delta)$ for all $\delta \in (0,d]$; consequently for any $\delta \in (0,d]$ we have

$$\frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le \frac{2^{l-k}\omega_k(f;\delta)_{p,m}}{\omega(\delta)} \le 2^{l-k} M \frac{\omega_k(f;\delta)_{p,m}}{\varphi(\delta)},$$

hence $\left\| f; H^l_{p,m}[\omega] \right\| \le \left(1 + 2^{l-k}M\right)\left\| f; H^k_{p,m}[\varphi] \right\|$, i.e. $f \in H^l_{p,m}[\omega]$. Further taking into account the obvious inequality $\omega_k(f;\delta)_{p,m} \le \left\| f; H^k_{p,m}[\varphi] \right\| \varphi(\delta)$, $\delta \in (0,d]$, by virtue of the theorem 1 we obtain

$$\left\| f - T^{(m)}_{n;p}(f); H^l_{p,m}[\omega] \right\| \le C_2(l,m) \sup_{0<\delta\le d/n} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le$$

$$\le C_2(l,m) 2^{l-k} \sup_{\delta\in(0,d/n]} \frac{\omega_k(f;\delta)_{p,m}}{\varphi(\delta)} \frac{\varphi(\delta)}{\omega(\delta)} \le$$

$$\le C_2(l,m) 2^{l-k} \left\| f; H^k_{p,m}[\varphi] \right\| \sup\{\varphi(\delta)/\omega(\delta); \delta \in (0,d/n]\}.$$

Corollary 2. *Let* $1 \le p \le \infty$, $m \ge 1$, $f \in L_p(T^m)$, $l \in N$, $0 < \beta < \alpha \le l$ *and* $\omega_l(f;\delta)_{p,m} \le M_1\delta^\alpha$, $\delta \in (0,d]$, $M_1 = M_1(f)$ *be a positive constant; then* $(n \in N)$

$$\left\| f - T^{(m)}_{n;p}(f); H^l_{p,m}\left[\delta^\beta\right] \right\| \le C_7(l,m,\alpha,\beta,M_1) n^{-(\alpha-\beta)}.$$

Really, assuming in the estimation (3) $\omega(\delta) = \delta^\beta$ we get

$$\left\| f - T^{(m)}_{n;p}(f); H^l_{p,m}\left[\delta^\beta\right] \right\| \le C_2(l,m) \sup_{\delta\in(0,d/n]} M_1\delta^{\alpha-\beta} =$$

$$= C_2(l,m) M_1 (d/n)^{\alpha-\beta} = C_2(l,m) d^{\alpha-\beta} M_1 n^{-(\alpha-\beta)}.$$

Theorem 2. *Let* $1 \le p \le \infty$, $m \ge 1$, $l \in N$, $\omega \in \Omega_l(0,d]$; *then for any function* $f \in H^l_{p,m}[\omega]$ *the estimation* $(n \in N)$ *is valid*

$$\left\| f - S^{(m)}_n(f); H^l_{p,m}[\omega] \right\| \le C_8(l,m)\left(1 + \left\| S^{(m)}_n \right\|_p\right) \sup_{\delta\in(0,d/n]} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)}, \qquad (7)$$

where $\left\|S_n^{(m)}\right\|_p = \sup\left\{\left\|S_n^{(m)}(f;\cdot)\right\|_{p,m}; f \in L_p(T^m),\ \|f\|_{p,m} \le 1\right\}$.

Proof. We have (in the case $m=1$, see, f.e., [6], p.279, p.594; [7], c.116; [2], c.339)

$$\left\|f - S_n^{(m)}(f)\right\|_{p,m} = \left\|f - T_{n;p}^{(m)}(f) + T_{n;p}^{(m)}(f) - S_n^{(m)}(f)\right\|_{p,m} \le$$
$$\le \left\|f - T_{n;p}^{(m)}(f)\right\|_{p,m} + \left\|S_n^{(m)}\left[T_{n;p}^{(m)}(f) - f\right]\right\|_{p,m} \le$$
$$\le E_{n,\dots,n}(f)_{p,m} + \left\|S_n^{(m)}\right\|_p E_{n,\dots,n}(f)_{p,m} = \left(1 + \left\|S_n^{(m)}\right\|_p\right) E_{n,\dots,n}(f)_{p,m},$$

whence by virtue of the inequality (1) we obtain

$$\left\|f - S_n^{(m)}(f)\right\|_{p,m} \le C_1(l,m)\left(1 + \left\|S_n^{(m)}\right\|_p\right)\omega_l(f; d/n)_{p,m}. \tag{8}$$

Further, since $\Delta_h^l\left[S_n^{(m)}(f;x)\right] = S_n^{(m)}\left(\Delta_h^l f; x\right)$ for any $x, h \in R^m$ and $\Delta_h^l f(x) \in L_p(T^m)$ for $\forall h \in R^m$, then $\left\|\Delta_h^l\left[S_n^{(m)}(f;\cdot)\right]\right\|_{p,m} \le \left\|S_n^{(m)}\right\|_p \cdot \left\|\Delta_h^l f(\cdot)\right\|_{p,m}$, whence

$$\omega_l\left(S_n^{(m)}(f);\delta\right)_{p,m} \le \left\|S_n^{(m)}\right\|_p \omega_l(f;\delta)_{p,m}, \delta \in (0,d],$$

and consequently

$$\omega_l\left(f - S_n^{(m)}(f);\delta\right)_{p,m} \le \left(1 + \left\|S_n^{(m)}\right\|_p\right)\omega_l(f;\delta)_{p,m}. \tag{9}$$

Taking into account the obtained estimations (8) and (9) we have (see the completion of proving theorem 1)

$$\left\|f - S_n^{(m)}(f); H_{p,m}^l[\omega]\right\| = \left\|f - S_n^{(m)}(f)\right\|_{p,m} + \sup_{\delta \in (0,d]} \frac{\omega_l\left(f - S_n^{(m)}(f);\delta\right)_{p,m}}{\omega(\delta)} \le$$
$$\le \left\|f - S_n^{(m)}(f)\right\|_{p,m} + \sup_{\delta \in (0,d/n)} \frac{\omega_l\left(f - S_n^{(m)}(f);\delta\right)_{p,m}}{\omega(\delta)} + \sup_{\delta \in [d/n,d]} \frac{\omega_l\left(f - S_n^{(m)}(f);\delta\right)_{p,m}}{\omega(\delta)} \le$$
$$\le \left\|f - S_n^{(m)}(f)\right\|_{p,m} + \left(1 + \left\|S_n^{(m)}\right\|_p\right) \sup_{\delta \in (0,d/n)} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} + 2^l \frac{\left\|f - S_n^{(m)}(f)\right\|_{p,m}}{\omega(d/n)} \le$$
$$\le \left[2^l + \omega(d)\right] \frac{\left\|f - S_n^{(m)}(f)\right\|_{p,m}}{\omega(d/n)} + \left(1 + \left\|S_n^{(m)}\right\|_p\right) \sup_{\delta \in (0,d/n)} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le$$
$$\le C_1(l,m)\left[2^l + \omega(d)\right]\left(1 + \left\|S_n^{(m)}\right\|_p\right) \frac{\omega_l(f;d/n)_{p,m}}{\omega(d/n)} + \left(1 + \left\|S_n^{(m)}\right\|_p\right) \sup_{\delta \in (0,d/n)} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)} \le$$
$$\le \left\{C_1(l,m)\left[2^l + \omega(d)\right] + 1\right\}\left(1 + \left\|S_n^{(m)}\right\|_p\right) \sup_{\delta \in (0,d/n]} \frac{\omega_l(f;\delta)_{p,m}}{\omega(\delta)}.$$

Theorem 2 is proved.

Assuming in the estimation (7) $\omega(\delta) = \delta^\beta$ and taking into account the known one-dimensional inequalities (see, f.e., [6], inequality (36.1) on p.117, theorem 1 on p.593; [2], inequality (20) on p.183; [7; p.112]): $\left\|S_n^{(1)}(f;\cdot)\right\|_{p,1} \le C_9 \|f\|_{p,1} \ln n$ for $p=1$, $p=\infty$ and $\left\|S_n^{(1)}(f;\cdot)\right\|_{p,1} \le C_{10}(p) \cdot \|f\|_{p,1}$ for $1 < p < \infty$, we obtain

Corollary 3. *Let* $1 \le p \le \infty,\ m \ge 1,\ l \in N,\ f \in L_p(T^m),\ 0 < \beta < \alpha \le l$ *and* $\omega_l(f;\delta)_{p,m} \le M_2 \delta^{\alpha},\ \delta \in (0,d],\ M_2 = M_2(f)$ *be a positive constant; then*

$$\left\| f - S_n^{(m)}(f); H_{p,m}^l\left[\delta^{\beta}\right] \right\| \le C_{11}(l,m,\alpha,\beta,M_2) n^{-(\alpha-\beta)} \times$$
$$\times \left\{ 1 + C_9^m (\ln n)^m,\ \ p = 1,\ p = \infty;\ \ 1 + C_{10}^m(p),\ \ 1 < p < \infty \right\}.$$

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