MATHEMATICS

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ON L_p SOLVABILITY IN WEIGHTED SPACES OF THE FIRST BOUNDARY VALUE PROBLEM FOR THE LAPLACE EQUATION IN A DOMAIN WITH A CONIC POINT ON THE BOUNDARY

Abstract

The paper is devoted to the study of L_p - solvability of the first boundary value problem for a Laplace equation in domains with conical point on the boundary. There are found conditions on weight functions under fulfillment of which the two-weight coercive L_p -estimate for a Laplace operator is valid.

The paper is devoted to the study of L_p -solvability of the first boundary value problem for a Laplace equation in a domain with an isolated conic point on the boundary. A great number of papers have been devoted to this theme, and the state of this problem is stated in review [1]. Here we note researches [2-4] directly adherent to this paper. For the first time the asymptotic behavior of solutions and solvability of general boundary value problems for arbitrary order elliptic equations in weighted L_2 -spaces with power weight was established in [2]. The aim of this paper is to find weight functions conditions for twoweight coercive L_p -estimate for the Laplace operator to be valid. Then these results will be used in studying asymptotic behavior of solutions to boundary value problems for elliptic equations in the vicinity of a conic point under minimal requirements with regard to coefficients.

Before passing to further statement, we cite notations and definitions that we'll use.

$B_r^{x^0}$ is a ball $\{|x-x^0|<r\}$, $S_r^{x^0}$ is a sphere $\{x:|x-x^0|=r\}$, K^{x^0} is a cone with a vertex at x^0, $K_r^{x^0}=K^{x^0}\cap B_r^{x^0}$, $D=K^{x^0}\cap S_1^{x^0}$.

Define also a space of functions $H_\omega^{k,p}(K_r^{x^0})$. Let $\omega(t)\geq 0$ be a continuous on $(0,\infty)$ function satisfying for $t\neq 0$ the condition

$$c_2\omega(2t)\leq\omega(t)\leq c_1\omega(2t), \tag{1}$$

where c_1, c_2 are positive constants.

Now denote $k=0,1,...$ and $p>1$ by $H_\omega^{k,p}(K_r^{x^0})$ space of functions, obtained by the completion in the norm

$$\|u\|_{H_\omega^{k,p}(K_r^{x^0})}=\left(\int_{K_r^{x^0}}\sum_{l=0}^{n}\sum_{|m|=l}|x|^{(l-k)p}\omega(|x|)|D^m u|^p dx\right)^{\frac{1}{p}}$$

a set of smooth in $\overline{K}_r^{x^0}$ functions, that equal to zero on the boundary $\partial K_r^{x^0}$.

Consider a problem on the validity of the two-weight coercive estimate

$$\|u\|_{H_{\omega_0}^{2,p}(K_{r_0}^{x^0})}\leq c\|\Delta u\|_{H_{\omega_1}^{0,p}(K_{r_0}^{x^0})} \tag{2}$$

with a constant C that is independent of u.

For the result we denote by λ the first eigen-number of the homogeneour Dirichlet problem in the domain $D \subset S_1^{x^0}$ and put $\chi = \sqrt{(n-2)^2 + 4\lambda}$. Let $y_1(r)$ and $y_2(r)$ be linear independent solutions of the equation

$$y'' + \frac{n-1}{r} y' + \lambda r^{-2} y = 0$$

of the form $y_1(r) = r^{\frac{2-n+\chi}{2}}$, $y_2(r) = r^{\frac{2-n-\chi}{2}}$.

In the sequel everywhere we shall assume that $n > 2$ and the boundary ∂D of the domain D belongs to the class C^2.

Theorem 1. *For the validity of inequality (2) with a constant* $\mathbf{C}$, *not depending on* u, *it is necessary and sufficient that weight functions* ω_0 *and* ω_1 *satisfy the conditions*

$$\sup_{r \in (0,r_0)} \left(\int_r^{r_0} y_2^p(t) t^{n-1-2p} \omega_0(t) dt \right) \left(\int_0^r \omega_1^{-\frac{1}{p-1}}(t) y_1^{\frac{p}{p-1}}(t) t^{n-1} dt \right)^{p-1} < \infty \tag{3}$$

and

$$\sup_{r \in (0,r_0)} \left(\int_0^r y_1^p(t) t^{n-1-2p} \omega_0(t) dt \right) \left(\int_r^{r_0} \omega_1^{-\frac{1}{p-1}}(t) y_2^{\frac{p}{p-1}}(t) t^{n-1} dt \right) < \infty . \tag{4}$$

The proof is based on two subsidiary statements.

Lemma 1. *Let the weight function* $\omega_0(t)$ *satisfy the condition (1). Then for any function* $u \in H_{\omega_0}^{2,p}\left(K_r^{x^0}\right)$ *it is valid the inequality*

$$\|u\|_{H_{\omega_0}^{2,p}\left(K_{r_0}^{x^0}\right)} \le c \left(\|\Delta u\|_{H_{\omega_0}^{0,p}\left(K_{r_0}^{x^0}\right)} + \|u\|_{H_{\omega_2}^{0,p}\left(K_{r_0}^{x^0}\right)} \right), \tag{5}$$

where $\omega_2(t) = t^{-2p}\omega_0(t)$ *and a constant* $\mathbf{C}$ *doesn't depend on* u.

Proof. Consider the set

$$G_k = K_{r_0 2^{-(k-1)}}^{x^0} \setminus K_{r_0 2^{-(k+2)}}^{x^0} , \quad G_k' = K_{r_0 2^{-k}}^{x^0} \setminus K_{r_0 2^{-(k+3)}}^{x^0} ,$$

where $k = 1,2,\ldots$. Since the boundary ∂D of the domain D belongs to the class C^2, then by virtue of a priori estimate in spaces W_p^2 (see for instance [5]) for $k = 1,2,3,\ldots$

$$\sum_{l=0}^{2} \left(r_0 2^{-k}\right)^{(l-2)p} \left\| \sum_{|m|>l} \left|D^m u\right| \right\|_{L_p(G_k)}^p \le c \left(\|\Delta u\|_{L_p(G_k')}^p + \left(r_0 2^{-k}\right)^{-2p} \|u\|_{L_p(G_k')}^p \right)$$

with a constant $\mathbf{C}$, not depending on k and u. That is, by virtue of (1) after multiplying the both sides of the inequality by $\omega_0\left(r_0 2^{-k}\right)$ we shall have

$$\|u\|_{H_{\omega_0}^{2,p}(G_k)}^p \le c \left(\|\Delta u\|_{H_{\omega_0}^{0,p}(G_k')}^p + \|u\|_{H_{\omega_2}^{0,p}(G_k')}^p \right),$$

where the weight ω_2 is defined previously. Summing these inequalities on all $k = 1,2,\ldots$ we find

$$\|u\|_{H_{\omega_0}^{2,p}\left(K_{r_0/4}^{x^0}\right)} \le c \left(\|\Delta u\|_{H_{\omega_0}^{0,p}\left(K_{r_0}^{x^0}\right)} + \|u\|_{H_{\omega_2}^{0,p}\left(K_{r_0}^{x^0}\right)} \right). \tag{6}$$

Besides,

$$\|u\|_{H^{2,p}_{\omega_0}\left(K^{x^0}_{r_0}\setminus K^{x^0}_{r_0/4}\right)} \le c\left(\|\Delta u\|_{H^{0,p}_{\omega_0}\left(K^{x^0}_{r_0}\right)} + \|u\|_{H^{0,p}_{\omega_2}\left(K^{x^0}_{r_0}\right)}\right). \tag{7}$$

To prove this inequality it is sufficient to pass to the polar coordinates (r,ω), where $r=\left|x-x^0\right|$ and continue the initial function in an odd form with respect to r by the surface $S^{x^0}_{r_0}\cap K^{x^0}_{r_0}$. After this, (7) follows from a priori estimate in the space W^2_p. Now by joining (6) and (7) we obtain the required statement. Lemma is proved.

Lemma 2. *Let with respect to the weight functions w_0 and ω_1 the assumptions (1), (3), (4) be fulfilled, and $G(x,z)$ be a Green function of a Laplace operator with respect to the Dirichlet problem in the domain $K^{x^0}_{r_0}$. Then for any function $f\in H^{0,p}_{\omega_1}\left(K^{x^0}_{r_0}\right)$ it is valid the inequality*

$$\left\|\int_{K^{x^0}_{r_0}} G(x,z)f(z)dz\right\|_{H^{0,p}_{\omega_2}\left(K^{x^0}_{r_0}\right)} \le \|f\|_{H^{0,p}_{\omega_1}\left(K^{x^0}_{r_0}\right)}, \tag{8}$$

where the weight ω_2 is determined in the condition of lemma 1, and a constant C is independent of f.

Proof. Not restricting generality we shall consider that x^0 coincides with the origin of coordinates, $f\ge 0$ and $f\in C^\infty_0\left(K^{x^0}_{r_0}\right)$. Use the estimate of Green's function, established in [6]

$$G(x,z)\le C_0\begin{cases} y_1(|z|)y_2(|x|), & \text{if } \quad 2|z|<|x| \\ y_1(|x|)y_2(|z|), & \text{if } \quad |z|>2|x| \\ |x\text{-}z|^{2-n}, & \text{if } \quad \dfrac{|x|}{2}<z<2|x| \end{cases} \tag{9}$$

where the functions $y_1(t)$, $y_2(t)$ have been determined previously.

Consider the sets

$$A_1=\{z:2|z|<|x|\}\cap K^{x^0}_{r_0}, \quad A_2\{z:|z|>2|x|\}\cap K^{x^0}_{r_0},$$

$$A_3=\left\{z:\frac{|x|}{2}\le|z|\le 2|x|\right\}\cap K^{x^0}_{r_0}$$

and assume

$$I_i(x)=\int_{A_i} G(x,z)f(z)dz.$$

Since

$$\int_{K^0_{r_0}} G(x,z)f(z)dz=\sum_{i=1}^{3} I_i(x)$$

then to prove the lemma it is sufficient to get an estimate of the form (8) for each of functions $I_i(x)$. By virtue of (9) after we pass to spherical coordinates we have

$$I_1(x)\le c_0 y_2(|x|)\int_{A_1} y_1(|z|)f(z)dz = cy_2(|x|)\int_0^{|x|/2} y_1(t)\left(\int_{D_t} f(z)dS_z\right)dt,$$

where $D_t = K_{r_0}^0 \cap S_t^0$. That is, with regard to the Hölder inequality

$$I_1(x)\le cy_2(|x|)\int_0^{x/2} y_1(t)\left(\int_{D_t} ds\right)^{\frac{1}{q}}\left(\int_{D_t} f^p(z)ds_z\right)^{\frac{1}{q}} dt \le$$

$$\le cy_2(|x|)\int_0^{x/2} y_1(t)t^{\frac{n-1}{q}}\left(\int_{D_t} f^p(z)ds_z\right)^{\frac{1}{q}} dt,$$

since $\int_{D_t} ds \le const\ t^{n-1}$.

Thus

$$\|I_1\|^p_{H^{0,p}_{\omega_2}\left(K^{x^0}_{r_0}\right)} \le const\int_0^{r_0} y_2^p(r)\omega_0(r)r^{n-1-2p}\left(\int_0^r y_1(t)t^{\frac{n-1}{q}} g(t)dt\right)^p dr, \tag{10}$$

where $g(t)=\left(\int_{D_t} f^p(z)ds\right)^{\frac{1}{p}}$.

Give a similar estimate for $I_2(x)$. From (9), as earlier, we have

$$I_2(x)\le c_0 y_1(|x|)\int_{2|x|}^{r_0} y_2(t)\left(\int_{D_t} f(z)ds\right)dt \le cy_1(|x|)\int_{2|x|}^{r_0} y_2(t)t^{\frac{n-1}{q}} g(t)dt.$$

So

$$\|I_2\|^p_{H^{0,p}_{\omega_2}\left(K^0_{r_0}\right)} \le const\int_0^{r_0} y_1^p(r)\omega_0(r)r^{n-1-2p}\left(\int_r^{r_0} y_2(t)t^{\frac{n-1}{q}} g(t)dt\right)^p dr. \tag{11}$$

It is well-known that (see [7]) the Hardy inequality

$$\int_r^{r_0}\left(\int_0^r f(t)dt\right)^p \omega_0(r)dr \le c\int_0^{r_0} f^p(r)\omega_1(r)dr$$

is fulfilled at $p>1$ for all non-negative on $(0,r_0)$ functions f at fixed weights $\omega_0 \ge 0$ and $\omega_1 \ge 0$ if and only if ω_0 and ω_1 satisfy the condition

$$\sup_{r\in(0,r_0)}\left(\int_r^{r_0}\omega_0(t)dt\right)\left(\int_0^r[\omega_1(t)]^{-\frac{1}{p-1}}dt\right)^{p-1} < \infty.$$

Similarly, the dual inequality

$$\int_0^{r_0}\left(\int_{r_0}^r f(t)dt\right)^p \omega_0(r)dr \le c\int_0^{r_0} f^p(r)\omega_1(r)dr$$

is valid for all non-negative on $(0,r_0)$ functions f if and only if

$$\sup_{r\in(0,r_0)}\left(\int_0^r\omega_0(t)dt\right)\left(\int_r^{r_0}[\omega_1(t)]^{-\frac{1}{p-1}}dt\right)^{p} < \infty.$$

Since, with respect to ω_0 and ω_1, assumptions (3) and (4) have been fulfilled, then according to the cited result from (10) and (11) it follows that

$$\|I_i\|^p_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le c\int_0^{r_0}\left(\int_{D_t} f^p(z)ds_z\right)\omega_1(t)dt,$$

where $i=1,2$. That is, it is easy to see

$$\|I_i\|_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le c\|f\|_{H^{0,p}_{\omega_1}(K^0_{r_0})},\ \ i=1,2. \tag{12}$$

Pass to the estimate of $I_3(x)$. According to (9) and Hölder's inequality

$$I_3(x)\le c_0\int_{A_3}\frac{f(z)}{|x-z|^{n-2}}dz \le c\left(\int_{A_3}\frac{dz}{|x-z|^{n-2}}\right)^{\frac{1}{q}}\left(\int_{A_3}\frac{f^p(z)}{|x-z|^{n-2}}dz\right)^{\frac{1}{p}}.$$

So

$$\|I_3\|^p_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le c\int_{K^0_{r_0}}\left(\int_{A_3}\frac{dz}{|x-z|^{n-2}}\right)^{p-1}\left(\int_{A_3}\frac{f^p(z)}{|x-z|^{n-2}}dz\right)\frac{\omega_0(|x|)}{|x|^{2p}}dx.$$

Since for $z\in A_3$, $|x-z|\le|x|+|z|\le 3|x|$, then

$$\int_{A_3}\frac{dz}{|x-z|^{n-2}} \le \int_{B^x_{3|x|}}\frac{dz}{|x-z|^{n-2}} \le const|x|^2.$$

Therefore, after passing to spherical coordinates we get

$$\|I_3\|^p_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le const\int_{K^0_{r_0}}\frac{\omega_0(|x|)}{|x|^2}\left(\int_{\frac{|x|}{2}}^{2|x|}\left(\int_{D_t}\frac{f^p(z)}{|x-z|^{n-2}}ds_z\right)dt\right)dx=$$

$$=const\int_0^{r_0}\frac{\omega_0(r)}{r^2}\left(\int_{r/2}^{2r}\left(\int_{D_t}f^p(z)\left(\int_{D_r}\frac{ds_x}{|x-z|^{n-2}}\right)ds_z\right)dt\right)dr.$$

Now we observe that

$$\int_{S^0_r}\frac{ds_x}{|x-z|^{n-2}} \le const\ r.$$

That is, we can easily see from (1),

$$\|I_3\|^p_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le const\int_0^{r_0}\left(\int_{r/2}^{2r}\frac{\omega_0(\rho)}{\rho}\left(\int_{D_\rho}f^p(z)ds_z\right)d\rho\right)dr.$$

By changing the integration order in the given iterated integral we can show that

$$\|I_3\|^p_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le const\int_0^{r_0}w_0(\rho)\left(\int_{D_\rho}f^p(z)ds_z\right)d\rho = const\int_{K^0_{r_0}}f^p(z)\omega_0(|z|)dz.$$

Since from conditions (4) and (1) it follows the inequality

$$\omega_0(t)\le const\,\omega_1(t), \tag{13}$$

then

$$\|I_3\|_{H^{0,p}_{\omega_2}(K^0_{r_0})} \le c\|f\|_{H^{0,p}_{\omega_1}(K^0_{r_0})}.$$

By joining these inequalities with (11) we pass to the required estimate (8). The lemma is proved.

Proof of Theorem 1. Sufficiency. To prove the sufficiency we use inequality (5) of lemma 1, estimate (13) and the statement of lemma 2, hence (2) follows.

Necessity. As earlier, without restricting generality, we'll assume that the point x^0 coincides with the origin of coordinates. Let $\varphi_1(\omega)$ be the first eigen function of Laplace-Beltrami in D, corresponding to the first eigen-number λ, determined previously.

Assume $\tilde{y}_1(r)=y_1(r)/y_1(r_0)$, $\tilde{y}_2(r)=y_2(r)/y_2(r_0)$, $\hat{y}_2(r)=\tilde{y}_2(r)-\tilde{y}_1(r)$ and

$$u(r,\omega)=\varphi_1(\omega)\left[\hat{y}_2(r)\int_0^r f(t)t^{n-1}y_1(t)+y_1(r)\int_r^{r_0} f(t)t^{n-1}\hat{y}_2(t)dt\right].$$

We can see that

$$\Delta u=f(r)\varphi_1(\omega)\left[(2-n+\chi)r_0^{-\chi}r^{\chi}-\chi\right]r_0^{\frac{n-2+\chi}{2}},$$

$$u\Big|_{\partial K_{r_0}^0\setminus\{0\}}=0,$$

where $\chi>0$ is determined previously.

We shall consider that $f\in C_0^\infty(0,r_0)$ and $f\geq 0$. According to the assumption for $u(r,\omega)$, inequality (2) is fulfilled. Hence it follows that

$$\int_0^{r_0}\hat{y}_2^p(r)r^{n-1}\left(\int_0^r f(t)t^{n-1}y_1(t)\right)^p r^{-2p}\omega_0(r)dr\leq c(r_0,\chi)\int_0^{r_0} f^p(t)t^{n-1}\omega_1(t)dt$$

and

$$\int_0^{r_0}y_1^p(r)r^{n-1-2p}\omega_0(r)\left(\int_r^{r_0} f(t)t^n\hat{y}_2(t)dt\right)^p dr\leq c(r_0,\chi)\int_0^r f^p(t)t^{n-1}\omega_1(t)dt$$

with a constant C not depending on f. Since a function f is arbitrary, then from the result [7], stated in lemma 2 it follows that the weight functions $\omega_0(t)$ and $\omega_1(t)$ must satisfy the inequalities

$$\sup_{r\in(0,r_0)}\left(\int_r^{r_0}\hat{y}_2^p(t)t^{n-1-2p}\omega_0(t)dt\right)\left(\int_0^r[\omega_1(t)]^{-\frac{1}{p-1}}y_1^{\frac{p}{p-1}}(t)t^{n-1}dt\right)^{p-1}<\infty \tag{14}$$

and

$$\sup_{r\in(0,r_0)}\left(\int_0^r y_1^p(t)t^{n-1-2p}\omega_0(t)dt\right)\left(\int_r^{r_0}[\omega_1(t)]^{-\frac{1}{p-1}}\hat{y}_2^{\frac{p}{p-1}}(t)t^{n-1}dt\right)^{p-1}<\infty. \tag{15}$$

Show that conditions (3) and (4) follow from (14) and (15). Give the proof on the example of (14).

If $r\geq r_0/2$, then by virtue of (1)

$$\int_r^{r_0}\hat{y}_2^p(t)t^{n-1-2p}\omega_0(t)dt\geq c(r_0,\omega_0)\int_r^{r_0}y_2^p(t)t^{n-1-2p}\omega_0(t)dt. \tag{16}$$

But if $r<r_0/2$, then again by virtue of (1) and explicit form of the function $\hat{y}_2(t)$

$$\int_r^{r_0}\hat{y}_2^p(t)t^{n-1-2p}\omega_0(t)dt=\int_r^{r_0/2}\hat{y}_2^p(t)t^{n-1-2p}\omega_0(t)dt+\int_{r_0/2}^{r_0}\hat{y}_2^p(t)t^{n-1-2p}\omega_0(t)dt\geq$$
$$\geq c(r_0,\omega_0)\left[\int_r^{r_0/2}y_2^p(t)t^{n-1-2p}\omega_0(t)dt+\int_{r_0/2}^{r_0}y_2^p(t)t^{n-1-2p}\omega_0(t)dt\right]=$$
$$=c(r_0,\omega_0)\int_r^{r_0}y_2^p(t)t^{n-1-2p}\omega_0(t)dt. \tag{17}$$

Now, from (14), (16), (17) it follows that

$$\sup_{r\in(0,r_0)}\left(\int_r^{r_0}y_2^p(t)t^{n-1-2p}\omega_0(t)dt\right)\left(\int_0^r[\omega_1(t)]^{-\frac{1}{p-1}}y_1^{\frac{p}{p-1}}(t)t^{n-1}dt\right)^{p-1}<\infty.$$

Q.E.D. Similarly it is established that condition (3) follows from (15). The theorem is proved.

Cite a corollary from theorem (1), which is standard.

Theorem 2. *Let weight functions ω_0 and ω_1 satisfy the condition (3) and (4). Then the problem $\Delta u=f$ in $K_{r_0}^{x^0}$, $u\big|_{\partial K_{r_0}^{x^0}}=0$ is solvable in the space $H_{\omega_0}^{2,p}\left(K_{r_0}^{x^0}\right)$ and for its solution it is valid the estimate*

$$\|u\|_{H_{\omega_0}^{2,p}\left(K_{r_0}^{x^0}\right)}\leq c\|f\|_{H_{\omega_1}^{0,p}\left(K_{r_0}^{x^0}\right)}$$

with a constant c, not depending on f and u.

Remark. By a standard coefficients freezing method we can easily select a class of operators with variable coefficients of the form

$$\mathcal{L}=\sum_{i,k=1}^{n}a_{i,k}(x)\frac{\partial^2}{\partial x_i\partial x_k},$$

for which a Dirichlet problem

$$\mathcal{L}u=f \quad \text{in} \quad K_{r_0}^{x^0}, \quad u\big|_{\partial K_{r_0}^{x^0}}=0$$

is solvable in the space $H_{\omega_0}^{2,p}\left(K_{r_0}^{x^0}\right)$ for any right side $f\in H_{\omega_1}^{0,p}\left(K_{r_0}^{x^0}\right)$, if with respect to ω_0 and ω_1 conditions (3) and (4) are fulfilled. Note that similar problems for heatconductivity equations in a special form non-cylindrical domains have been considered in [8].

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