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DYNAMICS OF PLANETARY REDUCTOR

Abstract

Dynamics of planetary reductor is investigated, its mathematical model is worked out. As a result the formula for determination its vibration frequency is given in the article.

Planetary reductors are widely used in moveable units. Investigations of dynamics of planetary reductors let carry in clarity in many important problems. Determination of dependence of natural oscillations on parameters of the system; work security of the system at resonance regime; determination of dynamic forces which origin in elements of the system with following their transition to nodes and conjunctions; determination of parameters of the system which provide its optimization from point of dynamics of the system and etc. are referred to such problem.

From this point of view the working out of mathematical model of the system is the important problem. Mathematical model of dynamics of the mechanical system can be worked out by different methods. Then both integral and differential methods can be used. The integral method is based on application of Lagrange equation of the second genus. Advantage of this method is that the forces of reaction which origin in nodes of ideal connections, are not contained in differential equations immediately describing dynamics of the system. The negative fact of this method is partition of the system into elements when determination of reaction forces of the connections [1] is necessary.

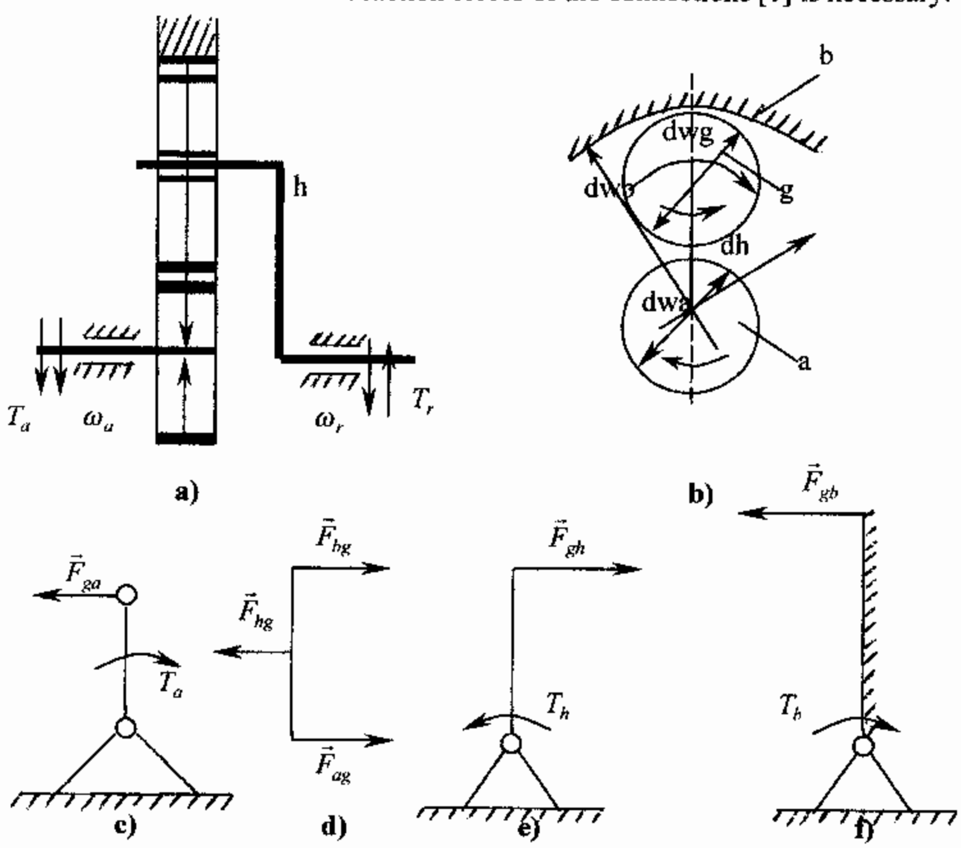

Fig.1. Scheme of loading of planetary transition of type $A^h ha$.

Differential method.

Working out of the mathematical model of dynamics of the system is based on D'Lamber's principle. Advantage of this method is its visualization and possibility for deeper analysis of dynamics of the system. Let's construct the dynamical model of the system on example of planetary reduction $A^b ha$ shown in fig.1 [2]. T_a - torque moment on drive shaft of the reductor; T_r - torque moment on driven shaft; C - rigidity of units; ω is angular velocity; J - moment of inertia of rotating elements; M - mass; V - velocity; r -initial radius; F - forces of reaction in units of conjunction. The differential equation of motion of drive shaft of planetary reductor is written in these denotations in the form:

$$J_0 \frac{d\omega_a}{dt} + C_a \int(\omega_0 - \omega_a)dt = T_a . \tag{1}$$

For the drive central shaft

$$J_a \frac{d\omega_a}{dt} - C_a \int(\omega_0 - \omega_a)dt = -3F_{ga} r_a . \tag{2}$$

Planet pinion (satellite) g makes plane-parallel motion. For this pinion

$$M \frac{dV_g}{dt} = -F_{hg} + F_{bg} + F_{ag} , \tag{3}$$

$$J_g \frac{d\omega_g}{dt} = \left(F_{gb} - F_{ag}\right) r_g . \tag{4}$$

For gear-wheel b with interior gearing

$$J_b \frac{d\omega_b}{dt} = 3F_{gb}\left(r_a + 2r_g\right) - C_b \int \omega_b dt . \tag{5}$$

For h

$$J_h \frac{d\omega_h}{dt} = 3F_{gh}\left(r_a + r_g\right) - C_h \int(\omega_h - \omega_r)\, dt . \tag{6}$$

For the final section of output shaft

$$J_r \frac{d\omega_r}{dt} = C_h \int(\omega_h - \omega_r)dt - T_r . \tag{7}$$

We are to add to these equations the expressions obtained from the kinetics of mechanism. And here the instant velocity, center of a gear-wheel g will not be in engagement with a gear wheel b, because it was adopted that a gear wheel b makes an oscillatory motion with velocity $V_b = \omega_b\left(r_a + 2r_g\right)$ (fig. 2). Denoting the instant velocity center with K we may write

$$V_A = \omega_a r_a = \omega_g x \qquad x = AK$$

$$V_B = \omega_g\left(2r_g - x\right)$$

hence $\frac{\omega_a r_a}{x} = \frac{V}{2r_g - x}$ and

$$x = \frac{2\omega_a r_a r_g}{\omega_a r_a + \omega_b\left(r_a + 2r_g\right)},$$

$$\omega_g = \frac{r_a}{2r_g}\omega_a + \frac{r_a + 2r_g}{2r_g}\omega_h . \tag{8}$$

The velocity of the velocity center g will be

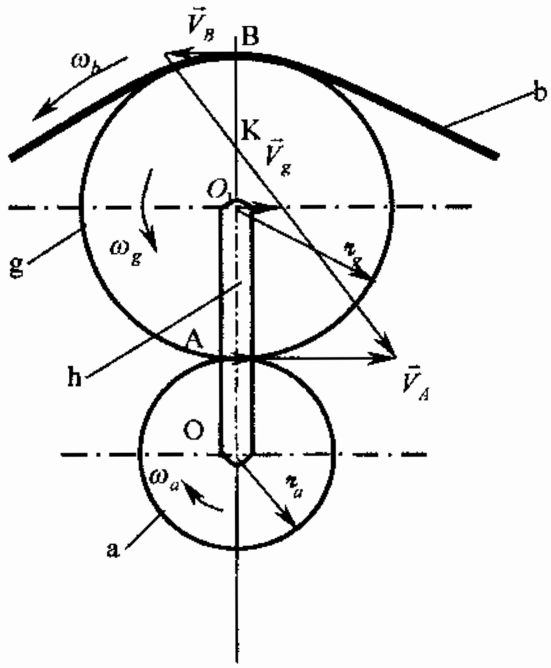

Fig.2. Distribution of velocities in a planetary transition of type $A^b ha$ subject to oscillatory motion.

$$V_g = \omega_g \left(x - r_g\right) = \left[\frac{r_a}{2r_g}\omega_a + \frac{r_a + 2r_g}{2r_g}\omega_g\right] \times$$

$$\times \left[\frac{2\omega r_a r_a r_g}{\omega_a r_a + \omega_b \left(r_a + 2r_g\right)} - r_g\right] = \left[\frac{r_a}{2r_g}\omega_a + \frac{r_a + 2r_g}{2r_g}\omega_b\right] \times \tag{9}$$

$$\times \frac{\left[\omega_a r_a r_g - \omega_b r_g \left(r_a + 2r_g\right)\right]}{\omega_a r_a + \omega_b \left(r_a + 2t_g\right)} = \frac{r_a}{2}\omega_a - \frac{\left(r_a + 2r_g\right)}{2}\omega_h$$

on the other hand

$$V_g = \omega_h \left(r_a + r_g\right),$$

$$\omega_h = \frac{r_a}{2\left(r_a + r_g\right)}\omega_a - \frac{\left(r_a + 2r_g\right)}{2\left(r_a + r_g\right)}\omega_b . \tag{10}$$

The system (1)…(10) represents a mathematical model of dynamics of planetary reductor by the scheme by the scheme $A^b ha$. Here $J_0;J_a;J_b;J_h;J_r;r_a;r_g;C_a;C_b;C_h$ are the principle parameters of the system, T_0 and T_r are the torques in the entry and exit of the reduction. $\omega_0;\omega_a;\omega_r;\omega_b;\omega_h;F_{ga};F_{bg};F_{gh}$ and T_0 are unknown variables.

From the conditions that action equals to reaction:

$$F_{ga} = F_{ag} \qquad F_{bg} = F_{gh} . \tag{11}$$

Taking into account (8), (9), (10), (11) in (1)…(7) we obtain

$$J_0 \frac{d\omega_0}{dt} + C_a \int (\omega_0 - \omega_a) dt = T_0 , \tag{12}$$

$$J_a \frac{d\omega_a}{dt} - C_a \int (\omega_0 - \omega_a) dt = -3F_{ag} r_a , \tag{13}$$

$$\frac{Mr_a}{2}\frac{d\omega_a}{dt} - \frac{M(r_a + 2r_g)}{2}\frac{d\omega_b}{dt} = -F_{gh} + F_{bg} + F_{ag} , \tag{14}$$

$$\frac{J_g}{2}\frac{r_a}{r_g}\frac{d\omega_a}{dt} + \frac{J_g(r_a + 2r_g)}{2r_g}\frac{d\omega_b}{dt} = (-F_{bg} + F_{ag}) r_g , \tag{15}$$

$$J_b \frac{d\omega_b}{dt} + C_b \int \omega_b dt = 3F_{bg}(r_a + 2r_g), \tag{16}$$

$$\begin{aligned} \frac{J_h r_a}{2(r_a + r_g)}\frac{d\omega_a}{dt} - \frac{J_h(r_a + 2r_g)}{2(r_a + 2r_g)}\frac{d\omega_b}{dt} = 3F_{gh}(r_a + r_g) - \\ - C_h \int \left[\frac{r_a}{2(r_a + r_g)}\omega_a - \frac{(r_a + 2r_g)}{2(r_a + r_g)}\omega_b - \omega_r \right] dt , \end{aligned} \tag{17}$$

$$J_r \frac{d\omega_r}{dt} = C_h \int \left[\frac{r_a}{2(r_a + r_g)}\omega_a - \frac{(r_a + 2r_g)}{2(r_a + r_g)}\omega_b - \omega_r \right] dt - T_r . \tag{18}$$

Hence we determine F_{ag}, F_{bg} and F_{hg}

$$F_{ag} = -\frac{J_a}{3r_a}\frac{d\omega_a}{dt} + \frac{C_a}{3r_a}\int (\omega_0 - \omega_a) dt , \tag{19}$$

$$F_{bg} = -\frac{J_b}{3(r_a + 2r_g)}\frac{d\omega_b}{dt} + \frac{C_b}{3(r_a + 2r_g)}\int \omega_b \, dt . \tag{20}$$

$$\begin{aligned} F_{gh} = \frac{J_h r_a}{6(r_a + 2r_g)^2}\frac{d\omega_a}{dt} - \frac{J_h(r_a + 2r_g)}{6(r_a + 2r_g)^2}\frac{d\omega_b}{dt} + \frac{C_h}{3(r_a + 2r_g)} \times \\ \times \int \left[\frac{r_a}{2(r_a + 2r_g)}\omega_a - \frac{r_a + 2r_g}{2(r_a + 2r_g)}\omega_b - \omega_r \right] dt . \end{aligned} \tag{21}$$

Taking into account (19), (20) and (21) in (12)…(18) we get:

$$J_0 \frac{d\omega_0}{dt} + C_a \int (\omega_0 - \omega_a) dt = T_0 , \tag{22}$$

$$\begin{aligned} \left[\frac{Mr_a}{2} + \frac{J_a}{2r_a} + \frac{J_h r_a}{6(r_a + r_g)^2} \right] \frac{d\omega_b}{dt} - \\ - \left[\frac{M(r_a + 2r_g)}{2} + \frac{J_b}{3(r_a + 2r_g)} + \frac{J_h(r_a + 2r_h)}{6(r_a + r_g)^2} \right] \frac{d\omega_b}{dt} + \\ + \frac{C_a}{3(r_a + r_g)} \int \left[\frac{r_a}{2(r_a + r_g)}\omega_a - \frac{r_a + 2r_g}{2(r_a + r_g)}\omega_h - \omega_r \right] dt - \\ - \frac{C_a}{3r_a}\int (\omega_0 - \omega_a) dt - \frac{C_b}{3(r_a + 2r_g)}\omega_b dt = 0, \end{aligned} \tag{23}$$

$$\left[\frac{J_g r_a}{2r_g^2}+\frac{J_a}{3r_a}\right]\frac{d\omega_a}{dt}+\left[\frac{J_g(r_a+2r_g)}{2r_g^2}+\frac{J_b}{3(r_a+2r_g)}\right]\times$$
$$\times\frac{d\omega_b}{dt}-\frac{C_a}{3r_a}\int(\omega_0-\omega_a)dt+\frac{C_b}{3(r_a+2r_g)}\int\omega_b dt=0\,, \tag{24}$$

$$J_r\frac{d\omega_r}{dt}-C_h\int\left[\frac{r_a}{2(r_a+r_g)}\omega_a-\frac{(r_a-2r_a)}{2(r_a+r_g)}\omega_b-\omega_r\right]dt=-T_r\,. \tag{25}$$

Applying the Caurson-Heavyside's transformations

$$F(P)=P\int_0^\infty e^{Pt}g(t)dt\,.$$

in the system (22)…(25) we get:

$$(J_0p^2+C_a)\overline{\omega}_0-C_a\overline{\omega}_a=\overline{T}_0P\,, \tag{26}$$

$$-\frac{C_a}{3r_a}\overline{\omega}_0+\left\{\left[\frac{Mr_a}{2}+\frac{J_a}{3r_a}+\frac{J_hr_a}{6(r_a+r_g)^2}\right]P^2+\left[\frac{C_hr_a}{6(r_a+r_g)^2}+\frac{C_a}{3r_a}\right]\right\}\overline{\omega}_a-$$
$$-\left\{\left[\frac{M(r_a+2r_g)}{2}+\frac{J_b}{3(r_a+2r_g)}+\frac{J_h(r_a+2r_g)}{6(r_a+r_g)^2}\right]P^2+\frac{C_h(r_a+2r_g)}{6(r_a+r_g)^2}+\right. \tag{27}$$
$$\left.+\frac{C_b}{3(r_a+2r_g)}\right\}\overline{\omega}_b-\frac{C_h}{2(r_a+r_g)}\overline{\omega}_r=0,$$

$$-\frac{C_a}{3r_a}\overline{\omega}_0+\left[\left(\frac{J_gr_a}{2r_g^2}+\frac{C_a}{3r_a}\right)P^2+\frac{C_a}{3r_a}\right]\overline{\omega}_a+$$
$$+\left\{\left[\frac{J_g(r_a+2r_g)}{2r_g^2}+\frac{J_b}{3(r_a+2r_g)}\right]P^2+\frac{C_b}{3(r_a+2r_g)^2}\right\}\overline{\omega}_b=0\,, \tag{28}$$

$$-\frac{C_hr_a}{2(r_a+r_g)}\overline{\omega}_a+\frac{C_h(r_a+2r_g)}{2(r_a+r_g)}\overline{\omega}_b+\left[J_{rp^2}+C_h\right]\overline{\omega}_r=-\overline{T}_rP\,. \tag{29}$$

For determination of frequencies of eigen vibration of system it is enough in (26)…(29) accepting $p=i\omega$ to equate to zero the determinant of system [3]

$$-\frac{C_h}{2(r_a+r_g)}\left[-J_0\omega^2+C_a\right]\left[-\left(\frac{J_gr_a}{2r^2g}+\frac{J_a}{3r_a}\right)\omega^2+\frac{C_a}{2r_a}\right]\frac{C_a(r_a+2r_g)}{2(r_a+r_g)}-$$
$$-\frac{C_h^2r_a(-J_0\omega^2+C_a)}{4(r_a+r_g)}\left\{-\left[\frac{J_g(r_a+2r_g)}{2r_g^2}+\frac{J_b}{3(r_a+r_g)}\right]\omega^2+\frac{C_b}{3(r_a+2r_g)}\right\}+$$
$$+\frac{C_aC_h^3(r_a+2r_g)}{12(r_a+r_g)^2r_a}+(-J_r\omega^2+C_h)(-J_0\omega^2+C_a)\times$$
$$\times\left\{-\left[\frac{Mr_a}{2}+\frac{J_a}{3r_a}+\frac{J_hr_a}{6(r_a+r_g)^2}\right]\omega^2+\left[\frac{C_hr_a}{6(r_a+r_g)^2}+\frac{C_a}{3r_a}\right]\right\}\times$$

$$\times\left\{-\left[\frac{J_g(r_a+2r)}{2r_g^2}+\frac{J_b}{3(r_a+r_g)}\right]\omega^2+\frac{C_b}{3(r_a+2r_g)}\right\}-\frac{\left(-J_r\omega^2+C_h\right)C}{3r_a}\times$$

$$\times\left\{-\left[\frac{M(r_a-2r_g)}{2}+\frac{J_b}{3(r_a-2r_g)}+\frac{J_h(r_a+2r_g)}{6(r_a+r_g)^2}\right]\omega^2+\frac{C_h(r_a+2r_g)}{6(r_a+r_g)^2}+\right.$$

$$\left.+\frac{C_b}{3(r_a+2r_g)}\right\}-\left(-J_r\omega^2+C_h\right)\left\{-\left[\frac{J_g(r_a+2r_g)}{2r_g^2}+\frac{J_b}{3(r_a+r_g)}\right]\omega^2+\right.$$

$$\left.+\frac{C_b}{3(r_a+2r_g)}\right\}\frac{C_a^2}{3r}+\left(-J_r\omega^2+C_h\right)\left[-\left(\frac{J_g r_a}{2r^2 g}+\frac{J_a}{3r_a}\right)\omega^2+\frac{C_a}{3r_a}\right]\times$$

$$\times\left\{-\left[\frac{M(r_a+2r_g)}{2}+\frac{J_b}{3(r_a+2r_g)}+\frac{J_h(r_a+2r_g)}{6(r_a+r_g)^2}\right]\omega^2+\frac{C_h(r_a+2r_g)}{6(r_a+r_g)^2}+\frac{C_b}{3(r_a+2r_g)}\right\}\times$$

$$\times\left(-J_0\omega^2+C_a\right)=0.$$

Accepting $\omega^2=z$ for determination Z we'll receive the algebraic equation of the fourth degree.

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