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INVERSE PROBLEM FOR THE STURM-LIOUVILLE OPERATOR ACCORDING TO A SPECTRUM AND NORMALIZING

Abstract

In this article, for given sequences of $\sqrt{\lambda_n}$ and $\{\alpha_n\}$ constructing a differential operator, which has a singularity, has explained. In order to accomplish this, λ_n and α_n have to have some certain asymtotic. This kind of asymptotic formulas depend on the regularity order of the function $q(x)$. This connection can be seen in the solution of the inverse problem which depend on the sequences of $\{\lambda_n\}$ and $\{\alpha_n\}$.

Keywords: *Inverse problem, regulary, singulary.*

Introduction: M.G. Gasimov and B.M. Levitan, in [1], has given on an effective method to construct according to normalizing number and a spectrum of the regular the Sturm-Liouville equation. But this method given in [1], has conditional characteristic such that the sequences $\{\lambda_n\}$ and $\{\alpha_n\}$ are the eigen-values and normalizing numbers of an obvious equation, respectively. Therefore the sequences $\{\lambda_n\}$ and $\{\alpha_n\}$ should have such condition that they will be eigen-values and normalizing numbers of an obvious equation, respectively. This problem which is mentioned above had been solved for regular equation. M.G. Gasimov in [2] has given the solution of the inverse problem which has singularity $l(l+1)/(\pi-x)^2$ (l is a positive integer) and which is singularity at π in the interval $[0,\pi]$ according to a spectrum and normalizing numbers. In $[0,\pi]$, the equation whose singularity is A/x (A is real) at $x=0$ has been investigated by M.G. Gasimov and R.Kh. Amirov [3]. For the equations which have a singularity $\left(\frac{A}{x}+\frac{l(l+1)}{x^2}\right)$ (A is real) at $x=0$ on the interval $[0,\pi]$, it is given the solution of the inverse problem according to a spectrum and normalizing numbers study by R.Kh. Amirov and S. Gülyaz [4].

In this study for Sturm-Liouville operator with $\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}+q(x)$, $q(x)\in L_2[0,\pi]$, in $[0,\pi]$ type potential the solution of the inverse problem is given according to a spectrum and normalizing numbers.

1. Investigating of function $F(x,t)$.

Let $q(x)$ be a real integrable function on the interval $[0,\pi]$ and A,H,δ_i $(i=\overline{1,3})$, be real numbers and $p_1\in\left(1,\frac{5}{4}\right)$, $p_2\in\left(\frac{5}{4},\frac{3}{2}\right)$, $p_3\in\left(\frac{3}{2},2\right)$. We consider the boundary value problem

$$-y''+\left\{\frac{A}{x}+\sum_{i=1}^{3}\frac{b_i}{x^{p_i}}+q(x)\right\}y=\lambda y\,, \tag{1.1}$$

$$y(0)=0,\quad y'(\pi)-Hy(\pi)=0\,, \tag{1.2}$$

We denote the solution (1.1) satisfying the initial conditions

$$\varphi(0,\lambda)=0, \quad \varphi'(0,\lambda)=\sqrt{\lambda} \tag{1.3}$$

by $\varphi(x,\lambda)$. Let $\lambda_0, \lambda_1,...$ be the eigen-values of the boundary value problem (1.1)-(1.3). Then $\varphi(x,\lambda_n)$ $(n=0,1,...)$ are the eigen-functions of this boundary value problem. Let $\varphi_0(x,\lambda_n)$ be a solution of equation (1.1) when $q(x)=0$ satisfying the condition (1.3). $\lambda_0^0, \lambda_1^0,...$ are eigen-values of the boundary value problem (1.1)-(1.3) when $q(x)=0$. The numbers α_n which

$$\alpha_n=\int_0^\pi \varphi^2(x,\lambda_n)dx \qquad (n=0,1,...) \tag{1.4}$$

are called the normalizing constants of the boundary value problem (1.1)-(1.3).

The numbers α_n^0 $(n=0,1,...)$ are called the normalizing constants of the boundary value problem (1.1)-(1.3) when $q(x)=0$.

In [3], it is well known that if $f(x)\in L_2[0,\pi]$ and $g(x)\in L_2[0,\pi]$ then Parseval's equation holds:

$$\int_0^\pi f(x)g(x)dx=\sum_{n=0}^\infty\left[\frac{1}{\alpha_n}\int_0^\pi f(x)\varphi(x,\lambda_n)dx\int_0^\pi g(t)\varphi(t,\lambda_n)dt\right].$$

We will refer to the sequences $\{\lambda_n\}$ and $\{\alpha_n\}$ as the spectral characteristics of the boundary value problem (1.1)-(1.2).

In this paper, we will solve the following problem: what conditions must the sequences $\{\lambda_n\}$ and $\{\alpha_n\}$ satisfy to be the spectral characteristics of some boundary value problem of the type (1.1)-(1.2). For solution of the problem let's constitute

$$F(x,t)=\sum_{n=1}^\infty\left[\frac{1}{\alpha_n}\varphi_0(x,\lambda_n)\varphi_0(t,\lambda_n)-\frac{1}{\alpha_n^0}\varphi_0(x,\lambda_n^0)\varphi_0(t,\lambda_n^0)\right] \tag{1.5}$$

with helping $\{\lambda_n\}$ and $\{\alpha_n\}$ sequences and with helping the function $F(x,t)$ let's write the integral equation which is

$$F(x,t)+K(x,t)+\int_0^x K(x,\xi)F(\xi,t)d\xi=0. \tag{1.6}$$

Here function $K(x,t)$ is not certain.

The constitution an integral equation of the type (1.6) is connected [3]. Therefore, similarly for the problem (1.1)-(1.2) w can write the integral equation (1.6) which is used the solution of the inverse problem.

Now, we investigate existence problem of the solution of integral equation (1.6). To see this, first let's seek properties of function $F(x,t)$. In order to learn the properties of function $F(x,t)$, we will use from the asymptotic expressions of $\varphi_0(x,\lambda_n)$ and $\varphi_0(x,\lambda_n^0)$ functions. Moreover, the asymptotic expressions of these functions are given and they satisfy for $x>0$ and the values of sufficiently large of n: Since

$$\varphi_0(x,\sqrt{\lambda_n})=\sin(n+1/2)x+\sum_{i=1}^3\frac{\delta_i M_{4i}}{2}\frac{\sin(n+1/2)x}{(n+1/2)^{2-p_i}}+\sum_{i=1}^3\frac{\delta_i c_{p_i}}{\pi}(x-\pi)\frac{\cos(n+1/2)x}{(n+1/2)^{2-p_i}}+$$

$$+A_{13}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{4-2p_3}}+A_{23}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{4-2p_3}}+A_{33}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{6-3p_3}}+A_{43}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{6-3p_3}}+$$

$$+\left(\frac{Ax}{2\pi}-\frac{A}{2}\right)\frac{\ln(n+1/2)}{n+1/2}\cos(n+1/2)x+\frac{AN}{2}\frac{\sin(n+1/2)x}{n+1/2}+A_5(x)\frac{\cos(n+1/2)x}{n+1/2}+$$
$$+A_{12}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{4-2p_2}}+A_{22}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{4-2p_2}}+A_{63}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}\sin(n+1/2)x+$$
$$+A_{73}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}\cos(n+1/2)x+A_{83}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{3-p_3}}+A_{93}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{3-p_3}}+$$
$$+A_{103}(x)\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}\sin(n+1/2)x+A_{113}(x)\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}\cos(n+1/2)x+$$
$$+A_{123}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{5-2p_3}}+A_{133}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{5-2p_3}}+A_{62}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}\sin(n+1/2)x+$$
$$+A_{72}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}\cos(n+1/2)x+A_{82}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{3-p_2}}+A_{92}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{3-p_2}}+$$
$$+A_{32}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{6-3p_2}}+A_{42}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{6-3p_2}}+A_{11}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{4-2p_1}}+$$
$$+A_{21}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{4-2p_1}}+A_{61}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}\sin(n+1/2)x+$$
$$+A_{71}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}\cos(n+1/2)x+A_{81}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{3-p_1}}+A_{91}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{3-p_1}}+$$
$$+A_{14}(x)\frac{\ln^2(n+1/2)}{(n+1/2)^2}\sin(n+1/2)x+A_{15}(x)\frac{\ln(n+1/2)}{(n+1/2)^2}\sin(n+1/2)x+$$
$$+A_{16}(x)\frac{\ln(n+1/2)}{(n+1/2)^2}\cos(n+1/2)x+A_{17}(x)\frac{\sin(n+1/2)x}{(n+1/2)^2}+$$
$$+A_{18}(x)\frac{\cos(n+1/2)x}{(n+1/2)^2}+O\left(\frac{\ln n}{n^{5-2p_2}}\right), \tag{1.7}$$

$$\lambda_n^0=(n+1/2)^2+\sum_{i=0}^{2}a_{1i}(n+1/2)^{p_{3-i}-1}+a_3(n+1/2)^{2p_3-3}+\frac{A}{\pi}\ln(n+1/2)+$$
$$+a_8^0(n+1/2)^{3p_3-5}+a_{11}(n+1/2)^{4p_3-7}+a_{14}(n+1/2)^{5p_3-9}+$$
$$+\sum_{i=0}^{2}a_{2i}^0+a_{43}\frac{\ln(n+1/2)}{(n+1/2)^{2-p_3}}+a_{53}^0\frac{1}{(n+1/2)^{2-p_3}}+a_{32}\frac{1}{(n+1/2)^{3-2p_2}}+$$
$$+a_{93}\frac{\ln(n+1/2)}{(n+1/2)^{4-2p_3}}+\gamma_{13}^0\frac{1}{(n+1/2)^{4-2p_3}}+a_{123}\frac{\ln(n+1/2)}{(n+1/2)^{6-3p_3}}+\gamma_{73}\frac{1}{(n+1/2)^{6-3p_3}}+$$
$$+a_{31}\frac{1}{(n+1/2)^{3-2p_1}}+a_{41}\frac{\ln(n+1/2)}{(n+1/2)^{2-p_1}}+a_{51}^0\frac{1}{(n+1/2)^{2-p_1}}+a_{82}^0\frac{1}{(n+1/2)^{5-3p_2}}+$$
$$+a_{42}\frac{\ln(n+1/2)}{(n+1/2)^{2-p_2}}+a_{52}^0\frac{1}{(n+1/2)^{2-p_2}}+a_6\frac{\ln(n+1/2)}{(n+1/2)}+a_7^0\frac{1}{(n+1/2)}+$$
$$+a_{92}\frac{\ln(n+1/2)}{(n+1/2)^{4-2p_2}}+\gamma_{12}^0\frac{1}{(n+1/2)^{4-2p_2}}+a_{103}\frac{\ln^2(n+1/2)}{(n+1/2)^{3-p_3}}+\gamma_{23}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}+$$

$$+\gamma_{33}^{0}\frac{1}{(n+1/2)^{3-p_3}}+a_{133}\frac{\ln^2(n+1/2)}{(n+1/2)^{5-2p_3}}+\gamma_{83}^{0}\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}+\gamma_{93}^{0}\frac{1}{(n+1/2)^{5-2p_3}}+$$

$$+a_{83}^{0}\frac{1}{(n+1/2)^{5-3p_1}}+a_{102}\frac{\ln^2(n+1/2)}{(n+1/2)^{3-p_2}}+\gamma_{22}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}+\gamma_{32}^{0}\frac{1}{(n+1/2)^{3-p_2}}+$$

$$+a_{91}\frac{\ln(n+1/2)}{(n+1/2)^{4-2p_1}}+\gamma_{11}^{0}\frac{1}{(n+1/2)^{4-2p_1}}+a_{112}\frac{1}{(n+1/2)^{7-4p_2}}+$$

$$+a_{101}\frac{\ln^2(n+1/2)}{(n+1/2)^{3-p_1}}+\gamma_{21}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}+\gamma_{31}^{0}\frac{1}{(n+1/2)^{3-p_1}}+a_{22}\frac{\ln(n+1/2)}{(n+1/2)^{6-3p_2}}+$$

$$+\gamma_{72}\frac{1}{(n+1/2)^{6-3p_2}}-\frac{A^3}{12\pi}\frac{\ln^3(n+1/2)}{(n+1/2)^2}+\gamma_4^0\frac{\ln^2(n+1/2)}{(n+1/2)^2}+\gamma_5^0\frac{\ln(n+1/2)}{(n+1/2)^2}+$$

$$+\gamma_6^0\frac{1}{(n+1/2)^2}+O\left(\frac{\ln^3 n}{n^{4-p_3}}\right), \tag{1.8}$$

$$\varphi_0\left(x,\sqrt{\lambda_n^0}\right)=\sin(n+1/2)x+\sum_{i=1}^{3}\frac{\delta_i M_{4i}}{2}\frac{\sin(n+1/2)x}{(n+1/2)^{2-p_i}}+\sum_{i=1}^{3}\frac{\delta_i c_{p_i}}{\pi}(x-\pi)\frac{\cos(n+1/2)x}{(n+1/2)^{2-p_i}}+$$

$$+A_{13}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{4-2p_3}}+A_{23}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{4-2p_3}}+A_{33}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{6-3p_3}}+A_{43}^0(x)\frac{\cos(n+1/2)x}{(n+1/2)^{6-3p_3}}+$$

$$+\left(\frac{Ax}{2\pi}-\frac{A}{2}\right)\frac{\ln(n+1/2)}{n+1/2}\cos(n+1/2)x+\frac{AN}{2}\frac{\sin(n+1/2)x}{n+1/2}+A_5^0(x)\frac{\cos(n+1/2)x}{n+1/2}+$$

$$+A_{12}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{4-2p_2}}+A_{22}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{4-2p_2}}+A_{63}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}\sin(n+1/2)x+$$

$$+A_{73}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}\cos(n+1/2)x+A_{83}^0(x)\frac{\sin(n+1/2)x}{(n+1/2)^{3-p_3}}+A_{93}^0(x)\frac{\cos(n+1/2)x}{(n+1/2)^{3-p_3}}+$$

$$+A_{103}(x)\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}\sin(n+1/2)x+A_{113}(x)\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}\cos(n+1/2)x+$$

$$+A_{123}^0(x)\frac{\sin(n+1/2)x}{(n+1/2)^{5-2p_3}}+A_{133}^0(x)\frac{\cos(n+1/2)x}{(n+1/2)^{5-2p_3}}+A_{62}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}\sin(n+1/2)x+$$

$$+A_{72}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}\cos(n+1/2)x+A_{82}^0(x)\frac{\sin(n+1/2)x}{(n+1/2)^{3-p_2}}+A_{92}^0(x)\frac{\cos(n+1/2)x}{(n+1/2)^{3-p_2}}+$$

$$+A_{32}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{6-3p_2}}+A_{42}^0(x)\frac{\cos(n+1/2)x}{(n+1/2)^{6-3p_2}}+A_{11}(x)\frac{\sin(n+1/2)x}{(n+1/2)^{4-2p_1}}+$$

$$+A_{21}(x)\frac{\cos(n+1/2)x}{(n+1/2)^{4-2p_1}}+A_{61}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}\sin(n+1/2)x+$$

$$+A_{71}(x)\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}\cos(n+1/2)x+A_{81}^0(x)\frac{\sin(n+1/2)x}{(n+1/2)^{3-p_1}}+A_{91}^0(x)\frac{\cos(n+1/2)x}{(n+1/2)^{3-p_1}}+$$

$$+A_{14}(x)\frac{\ln^2(n+1/2)}{(n+1/2)^2}\sin(n+1/2)x+A_{15}^0(x)\frac{\ln(n+1/2)}{(n+1/2)^2}\sin(n+1/2)x+$$

$$+A_{16}(x)\frac{\ln(n+1/2)}{(n+1/2)^2}\cos(n+1/2)x+A_{17}^0(x)\frac{\sin(n+1/2)x}{(n+1/2)^2}+$$

$$+A_{18}^0\frac{\cos(n+1/2)x}{(n+1/2)^2}+O\left(\frac{\ln n}{n^{5-2p_2}}\right). \qquad (1.9)$$

Also,

$$\alpha_n^0=\frac{\pi}{2}+\frac{M_4\pi}{2}\sum_{i=1}^{3}\frac{\delta_i}{(n+1/2)^{2-p_i}}+\nu_{13}\frac{1}{(n+1/2)^{4-2p_3}}+\nu_{73}\frac{1}{(n+1/2)^{6-3p_3}}+$$
$$+\frac{AN\pi}{2}\frac{1}{(n+1/2)}+\nu_{12}\frac{1}{(n+1/2)^{4-2p_2}}+\nu_{23}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}+\nu_{33}\frac{1}{(n+1/2)^{3-p_3}}+$$
$$+\nu_{83}\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}+\nu_{93}\frac{1}{(n+1/2)^{5-2p_3}}+\nu_{22}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}+\nu_{32}\frac{1}{(n+1/2)^{3-p_2}}+$$
$$+\nu_{72}\frac{1}{(n+1/2)^{6-3p_2}}+\nu_{11}\frac{1}{(n+1/2)^{4-2p_1}}+\nu_{21}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}+\nu_{31}\frac{1}{(n+1/2)^{3-p_1}}+$$
$$+\nu_4\frac{\ln^2(n+1/2)}{(n+1/2)^2}+\nu_5\frac{\ln(n+1/2)}{(n+1/2)^2}+\nu_6\frac{1}{(n+1/2)^2}+O\left(\frac{\ln n}{n^{5-2p_2}}\right), \qquad (1.10)$$

$$\alpha_n=\frac{\pi}{2}+\frac{M_4\pi}{2}\sum_{i=1}^{3}\frac{\delta_i}{(n+1/2)^{2-p_i}}+\nu_{13}\frac{1}{(n+1/2)^{4-2p_3}}+\nu_{73}\frac{1}{(n+1/2)^{6-3p_3}}+$$
$$+\frac{AN\pi}{2}\frac{1}{(n+1/2)}+\nu_{12}\frac{1}{(n+1/2)^{4-2p_2}}+\nu_{23}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}+\nu_{33}^0\frac{1}{(n+1/2)^{3-p_3}}+$$
$$+\nu_{83}\frac{\ln\left(n+\frac{1}{2}\right)}{\left(n+\frac{1}{2}\right)^{5-2p_3}}+\nu_{93}^0\frac{1}{(n+1/2)^{5-2p_3}}+\nu_{22}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}+\nu_{32}^0\frac{1}{(n+1/2)^{3-p_2}}+$$
$$+\nu_{72}\frac{1}{(n+1/2)^{6-3p_2}}+\nu_{11}\frac{1}{(n+1/2)^{4-2p_1}}+\nu_{21}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}+\nu_{31}^0\frac{1}{(n+1/2)^{3-p_1}}+$$
$$+\nu_4\frac{\ln^2(n+1/2)}{(n+1/2)^2}+\nu_5^0\frac{\ln(n+1/2)}{(n+1/2)^2}+\nu_6^0\frac{1}{(n+1/2)^2}+O\left(\frac{\ln n}{n^{5-2p_2}}\right). \qquad (1.11)$$

If we replace the expressions (1.7)-(1.8)-(1.9)-(1.10) and (1.11) in (1.5). We have

$$F(x,t)=F_1(x,t)\sum_{n=0}^{\infty}\frac{\sin(n+1/2)(x+t)}{n+1/2}+F_2(x,t)\sum_{n=0}^{\infty}\frac{\sin(n+1/2)(x-t)}{n+1/2}+$$
$$+F_{93}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x+t))}{(n+1/2)^{3-p_3}}+F_{103}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x-t))}{(n+1/2)^{3-p_3}}+$$
$$+F_{113}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x+t))}{(n+1/2)^{3-p_3}}+F_{123}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x-t))}{(n+1/2)^{3-p_3}}+$$
$$+F_{153}(x,t)\sum_{n=0}^{\infty}\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}\sin((n+1/2)(x+t))+$$
$$+F_{163}(x,t)\sum_{n=0}^{\infty}\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}\sin((n+1/2)(x-t))+$$
$$+F_{173}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x+t))}{(n+1/2)^{5-2p_3}}+F_{183}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x-t))}{(n+1/2)^{5-2p_3}}+$$

$$+F_{193}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x+t))}{(n+1/2)^{5-2p_3}}+F_{203}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x-t))}{(n+1/2)^{5-2p_3}}+$$

$$+F_{92}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x+t))}{(n+1/2)^{3-p_2}}+F_{102}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x-t))}{(n+1/2)^{3-p_2}}+$$

$$+F_{112}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x+t))}{(n+1/2)^{3-p_2}}+F_{122}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x-t))}{(n+1/2)^{3-p_2}}+$$

$$+F_{132}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x+t))}{(n+1/2)^{6-3p_2}}+F_{142}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x-t))}{(n+1/2)^{6-3p_2}}+$$

$$+F_{91}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x+t))}{(n+1/2)^{3-p_1}}+F_{101}(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x-t))}{(n+1/2)^{3-p_1}}+$$

$$+F_{111}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x+t))}{(n+1/2)^{3-p_1}}+F_{121}(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x-t))}{(n+1/2)^{3-p_1}}+$$

$$+F_3(x,t)\sum_{n=0}^{\infty}\frac{\ln(n+1/2)}{(n+1/2)^2}\cos((n+1/2)(x+t))+$$

$$+F_4(x,t)\sum_{n=0}^{\infty}\frac{\ln(n+1/2)}{(n+1/2)^2}\cos((n+1/2)(x-t))+$$

$$+F_5(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x+t))}{(n+1/2)^2}+F_6(x,t)\sum_{n=0}^{\infty}\frac{\sin((n+1/2)(x-t))}{(n+1/2)^2}+$$

$$+F_7(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x+t))}{(n+1/2)^2}+F_8(x,t)\sum_{n=0}^{\infty}\frac{\cos((n+1/2)(x-t))}{(n+1/2)^2}+$$

$$+O\left(\frac{\ln n}{n^{5-2p_2}}\right).$$

Here $F_j(x,t)=\sum_{i=1}^{3}F_{ji}(x,t)$, $(j=\overline{1,8})$ are continuous with respect to the variables x and t in interval $[0,\pi]$. Also, since $\sum_{n=1}^{\infty}\frac{\sin nx}{n}=\frac{\pi-x}{2}$, $0<x<2\pi$, $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}=$ $=\frac{3x^2-6\pi x+2\pi^2}{12}$, $0<x<2\pi$, $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}=x\ln\left(2\sin\frac{x}{2}\right)-\frac{1}{2}\int x\cot\frac{x}{2}dx$, $0\le x\le 2\pi$ and $\ln\Gamma(x)=\ln\sqrt{2\pi}+\sum_{n=1}^{\infty}\left(\frac{1}{2n}\cos 2n\pi x+\frac{1}{n\pi}(C+\ln 2n\pi)\sin 2n\pi x\right)$, $0<x<1$, *C -Euler constant.* $F(x,t)$ *have continuously derivatives with respect to variables* x *and* t *where* $0<x<\pi$ *and* $0<t<\pi$.

2. Existence of solution of integral equation with respect to $K(x,t)$ function.

In this section, we will prove that, the solution of integral equation (1.6) exists and has a unique solution. To see this, let's see the homojen integral equation

$$g(t)+\int_0^x F(s,t)g(s)ds=0 \qquad (2.1)$$

has a unique solution $g\equiv 0$ for $x\le\pi$.

Let us assume that this is not so, i.e. that there exists a function $g(t)\neq 0$ satisfying (2.1). Multiplying both sides of (2.1) by $g(t)$ and integrating over the integral $(0,x)$, we find that

$$\int_0^x g^2(t)dt+\int_0^x\int_0^x F(s,t)g(s)g(t)dsdt=0 .$$

Above equality and the expression (1.5) for $F(x,t)$ imply that

$$\sum_{n=0}^{\infty}\frac{1}{\alpha_n}\left(\int_0^x g(t)\varphi_0(t,\lambda_n)dt\right)^2=0$$

for all n, since α_n are positive

$$\int_0^x g(t)\varphi_0(t,\lambda_n)dt=0 . \tag{2.2}$$

We first show that the system of functions $\varphi_0(t,\lambda_n)$ is complete in $L_2[0,\pi]$. In order to show this operation let's take any $f(t)\in L_2[0,\pi]$ and let's show that the equality

$$\int_0^\pi f(t)\varphi_0(t,\lambda_n)dt=0$$

satisfy if and only if $f(t)\equiv 0$.

It was proved that in [3]. If the expression

$$\varphi_0(t,\lambda_n)=\frac{\sin\sqrt{\lambda_n}t}{\sqrt{\lambda_n}}+\int_0^t K_0(t,s)\frac{\sin\sqrt{\lambda_n}s}{\sqrt{\lambda_n}}ds$$

is replaced by $\varphi_0(t,\lambda_n)$ in the last equality then it holds:

$$\int_0^\pi f(t)\left[\frac{\sin\sqrt{\lambda_n}t}{\sqrt{\lambda_n}}+\int_0^t K_0(t,s)\frac{\sin\sqrt{\lambda_n}s}{\sqrt{\lambda_n}}ds\right]dt=\int_0^\pi f(t)\frac{\sin\sqrt{\lambda_n}t}{\sqrt{\lambda_n}}dt+$$

$$+\int_0^\pi f(t)\int_0^t K_0(t,s)\frac{\sin\sqrt{\lambda_n}s}{\sqrt{\lambda_n}}dsdt=\int_0^\pi\left[f(t)+\int_t^\pi K_0(t,s)f(s)ds\right]\frac{\sin\sqrt{\lambda_n}t}{\sqrt{\lambda_n}}dt=0$$

system of functions $\left\{\frac{\sin\sqrt{\lambda_n}t}{\sqrt{\lambda_n}}\right\}$ are linear independent and complete in $L_2[0,\pi]$. For this reason

$$f(t)+\int_t^\pi K_0(t,s)f(s)ds=0 .$$

Here $\sup_{0\le t\le\pi}\int_0^t K_0^2(t,s)ds<\infty$ and $f(t)\in L_2[0,\pi]$ the integral equation which is taken for $f(t)$ is the type of theory of the Volterra integral equation. With helping the type of the Volterra integral equations, $\{\varphi_0(t,\lambda_n)\}$ is linear independent and complete in $L_2[0,\pi]$. From (2.2) $g(t)\equiv 0$ and so it is prove that the solution of integral equation (1.6) exists and has a unique solution.

Clearly as it is seen (1.6) order of derivatives of $K(x,t)$ function with respect to t is same as the function $F(x,t)$. With respect to variable x, investigation of $K(x,t)$ is done by using the following Lemma

Lemma. *Suppose that we are given the integral equation*

$$g(t,a)=h(t,a)+\int_0^t H(t,s,a)h(s,a)ds$$

in which the kernel $H(t,s,a)$ and the inhomogeneous term $g(t,a)$ are continuous functions of the parameter a and the independent variable t. If for $a=a_0$ the homogeneous equation has only the trivial solution then in of t and a. If H and g have m continuous derivatives with respect to a, then so has $h(t,a)$.

The proof of the Lemma is same as the proof of the Lemma [1].

As the reason of this Lemma, since $F(x,t)$ is a continuous function, it follows from Lemma that the function $K(x,t)$ is a continuous function of both variables. Further, it follows from the same Lemma that order of derivatives of $K(x,t)$ function with respect to x is same as the function $F(x,t)$.

3. Determination of boundary conditions of differential equation.

In this section, we will determinate that the boundary conditions of differential equation which $\{\varphi(x,\lambda_n)\}$ functions satisfy using properties of $F(x,t)$ function that is established helping $\{\lambda_n\}$ and $\{\alpha_n\}$ sequences. Suppose that $F(x,t)$ has continuous second derivatives, according to x and t variables, then the function $K(x,t)$ has continuous second derivatives, according to x and t variables from the Lemma

Lemma. *The function $F(x,t)$ which is given (1.5) satisfies the following differential equation*

$$-F_{xx}(x,t)+\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]F(x,t)=-F_{tt}(x,t)+\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}\right]F(x,t). \qquad (3.1)$$

The proof of the Lemma is same as the proof of the Lemma [3].

Theorem 3.1. *The function $K(x,t)$ which is the solution of the integral equation (1.6)satisfies the following partial derivation differential equation*

$$K_{xx}(x,t)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]K(x,t)-2\frac{dK(x,x)}{dx}K(x,t)=K_{tt}(x,t)-\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}\right]K(x,t). \quad (3.2)$$

Proof. If we double differentiate

$$F(x,t)+\int_0^x K(x,\xi)F(\xi,t)d\xi+K(x,t)=0 \qquad (3.3)$$

with respect to variable x, then we obtain

$$\begin{aligned}&F_{xx}(x,t)+K_{xx}(x,t)+\frac{dK(x,x)}{dx}F(x,t)+K(x,x)F_x(x,t)+\\&+K_x(x,t)\big|_{t=x}F(x,t)+\int_0^x K_{xx}(x,\xi)F(\xi,t)d\xi=0.\end{aligned} \qquad (3.4)$$

If we double differentiate (3.3) with respect to variable t, we have

$$F_{tt}(x,t)+K_{tt}(x,t)+\int_0^x K(x,\xi)F_{tt}(\xi,t)d\xi=0.$$

Also if we benefit the equation (3.1) then we obtain

$$F_{tt}(x,t)+\int_0^x\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}-\frac{A}{\xi}-\sum_{i=1}^{3}\frac{\delta_i}{\xi^{p_i}}\right]K(x,\xi)F(\xi,t)d\xi+$$
$$+\int_0^x K(x,\xi)F_{\xi\xi}(\xi,t)d\xi+K_{tt}(x,t)=0.$$

If we benefit from the partial integration in the last equality, we have

$$F_{tt}(x,t)+\int_0^x\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}-\frac{A}{\xi}-\sum_{i=1}^{3}\frac{\delta_i}{\xi^{p_i}}\right]K(x,\xi)F(\xi,t)d\xi+$$
$$+K(x,\xi)F_{\xi}(t,\xi)\Big|_{\xi=0}^{x}-K_{\xi}(x,\xi)F(\xi,t)\Big|_{\xi=0}^{x}+\int_0^x K_{\xi\xi}(x,\xi)F(\xi,t)d\xi+K_{tt}(x,t)=0. \tag{3.5}$$

If we pass to the limit as $t\to 0$ in (3.1). Since the second order derivations of $F(x,t)$ are bounded, we see that

$$F(x,0)=0. \tag{3.6}$$

Moreover, from (3.3) and (3.6) we have $K(x,0)=0$. Thus the equality (3.5) is written by

$$F_{tt}(x,t)+\int_0^x\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}-\frac{A}{\xi}-\sum_{i=1}^{3}\frac{\delta_i}{\xi^{p_i}}\right]K(x,\xi)F(\xi,t)d\xi+$$
$$+K(x,\xi)F_{\xi}(\xi,t)\Big|_{\xi=x}-K_{\xi}(x,\xi)F(x,t)\Big|_{\xi=x}+\int_0^x K_{\xi\xi}(x,\xi)F(\xi,t)d\xi+K_{tt}(x,t)=0. \tag{3.7}$$

Multiplying the equality (3.3) by $\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}-\frac{A}{\xi}-\sum_{i=1}^{3}\frac{\delta_i}{\xi^{p_i}}-2\frac{dK(x,x)}{dx}\right]$ and if we add the equality (3.4) to the equality which obtained new and if we subtract (3.5) from the last expression by using (3.1) equality, we find that

$$K_{xx}(x,t)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]K(x,t)-K_{tt}(x,t)+\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}\right]K(x,t)-2\frac{dK(x,x)}{dx}K(x,t)+$$
$$+\int_0^x\left\{K_{xx}(x,\xi)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]K(x,\xi)-K_{\xi\xi}(x,\xi)+\left[\frac{A}{\xi}+\sum_{i=1}^{3}\frac{\delta_i}{\xi^{p_i}}\right]K(x,\xi)-\right.$$
$$\left.-2\frac{dK(x,x)}{dx}K(x,\xi)\right\}F(\xi,t)d\xi=0. \tag{3.8}$$

Since (3.8) is the homogeneous type integral equation of Volterra, we see that

$$K_{xt}(x,t)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]K(x,t)-$$
$$-K_{tt}(x,t)+\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}\right]K(x,t)-2\frac{dK(x,x)}{dx}K(x,t)=0$$

or

$$K_{xx}(x,t)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}+2\frac{dK(x,x)}{dx}\right]K(x,t)=K_{tt}(x,t)-\left[\frac{A}{t}+\sum_{i=1}^{3}\frac{\delta_i}{t^{p_i}}\right]K(x,t).$$

Now, helping $K(x,t)$, we establish the system of functions $\{\varphi(x,\lambda_n)\}$ in the form

$$\varphi(x,\lambda_n)=\varphi_0(t,\lambda_n)+\int_0^x K(x,t)\varphi_0(t,\lambda_n)\,dt. \qquad (3.9)$$

Here $\{\varphi_0(x,\lambda_n)\}$ is the solution of (1.1) such that it satisfies the conditions (1.3) in the case $q(x)=0$.

Theorem 3.2. $\varphi(x,\lambda_n)$ *which is given (3.9) is the solution of (1.1) and also*

$$q(x)=2\frac{dK(x,x)}{dx}.$$

Proof. Since

$$\varphi_0''(x,\lambda_n)-\left\{\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}-\lambda_n\right\}\varphi_0(x,\lambda_n)=0. \qquad (3.10)$$

If $\varphi(x,\lambda_n)$ in (3.9) is written in the left side of (1.1) we obtain

$$-\varphi''(x,\lambda_n)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}+2\frac{dK(x,x)}{dx}\right]\varphi(x,\lambda_n)+\lambda_n\varphi(x,\lambda_n)=$$

$$=-\frac{dK(x,x)}{dx}\varphi_0(x,\lambda_n)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]\int_0^x K(x,t)\varphi_0(t,\lambda_n)dt-$$

$$-2\frac{dK(x,x)}{dx}\int_0^x K(x,t)\varphi_0(t,\lambda_n)dt+\lambda_n\int_0^x K(x,t)\varphi_0(t,\lambda_n)dt+$$

$$+K(x,x)\frac{d}{dx}\varphi_0(x,\lambda_n)+K_x(x,t)\varphi_0(x,\lambda_n)\Big|_{t=x}+\int_0^x K_{xx}(x,t)\varphi_0(t,\lambda_n)dt.$$

Using (3.2), if we replace $K_{xx}(x,t)$ to $K_{tt}(x,t)$ in the last equality, we find that

$$\varphi''(x,\lambda_n)-\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}+2\frac{dK(x,x)}{dx}\right]\varphi(x,\lambda_n)+\lambda_n\varphi(x,\lambda_n)=-\frac{dK(x,x)}{dx}\varphi_0(x,\lambda_n)+$$

$$+\lambda_n\int_0^x K(x,t)\varphi_0(t,\lambda_n)dt+K(x,x)\frac{d}{dx}\varphi_0(x,\lambda_n)+K_x(x,t)\varphi_0(x,\lambda_n)\Big|_{t=x}+$$

$$+\int_0^x K_{tt}(x,t)\varphi_0(t,\lambda_n)dt-\int_0^x\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]K(x,t)\varphi_0(t,\lambda_n)dt. \qquad (3.11)$$

Moreover, since

$$\int_0^x K_{tt}(x,t)\varphi_0(t,\lambda_n)dt=K_t(x,t)\varphi_0(t,\lambda_n)\Big|_{t=x}-K(x,x)\frac{d}{dx}\varphi_0(x,\lambda_n)+$$

$$+\int_0^x K(x,t)\frac{d^2}{dt^2}\varphi_0(t,\lambda_n)dt.$$

If this expression replace n (3.11) and using (3.10), we have

$$-\varphi''(x,\lambda_n)+\left[\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}+2\frac{dK(x,x)}{dx}\right]\varphi(x,\lambda_n)=\lambda_n\varphi(x,\lambda_n)$$

for $\varphi(x,\lambda_n)$. Helping the reasons of the studies in [3]. It is easily proved that, the family of functions $\{\varphi_0(x,\lambda_n)\}$ which is given as the (3.9) establishes a complete system in $L_2[0,\pi]$. To establish the boundary condition at the point π we observe that

$$\varphi''(x,\lambda_n)+\left[\lambda_n-q(x)-\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]\varphi(x,\lambda_n)=0$$

and

$$\varphi''(x,\lambda_m)+\left[\lambda_m-q(x)-\frac{A}{x}+\sum_{i=1}^{3}\frac{\delta_i}{x^{p_i}}\right]\varphi(x,\lambda_m)=0\,.$$

If we multiply the first equation by $\varphi(x,\lambda_m)$ and the second by $\varphi(x,\lambda_n)$ and subtract one from the other, we obtain the identity

$$\left[\varphi'(x,\lambda_n)\varphi(x,\lambda_m)-\varphi'(x,\lambda_m)\varphi(x,\lambda_n)\right]'=(\lambda_m-\lambda_n)\varphi(x,\lambda_n)\varphi(x,\lambda_m)\,.$$

Integrating this identity over the integral $(0,\pi)$ and using the orthogonality

$$\varphi'(\pi,\lambda_n)\varphi(\pi,\lambda_m)-\varphi'(\pi,\lambda_m)\varphi(\pi,\lambda_n)=0 \text{ or } \frac{\varphi'(\pi,\lambda_n)}{\varphi(\pi,\lambda_n)}=\frac{\varphi'(\pi,\lambda_m)}{\varphi(\pi,\lambda_m)}\,.$$

Thus, the ratio $\dfrac{\varphi'(\pi,\lambda_n)}{\varphi(\pi,\lambda_n)}$ is a constant, and this gives the boundary condition at the point π . Thus, this completes the proof of the following theorem:

Theorem 3.3. *The sequences* $\{\lambda_n\}_{n=0}^{\infty}$ *and* $\{\alpha_n\}_{n=0}^{\infty}$ *are spectral characteristics of some boundary value problem f the (1.1)-(1.2) with a functions* $q''(x)\in L_2[0,\pi]$ *if and only if the following asymptotic estimates hold:*

$$\lambda_n=(n+1/2)^2+\sum_{i=0}^{2}a_{1i}(n+1/2)^{p_{3-i}-1}+a_3(n+1/2)^{2p_3-3}+\frac{A}{\pi}\ln(n+1/2)+a_8(n+1/2)^{3p_3-5}+$$
$$+a_{11}(n+1/2)^{4p_3-7}+a_{14}(n+1/2)^{5p_3-9}+\sum_{i=0}^{2}a_{2i}+a_{43}\frac{\ln(n+1/2)}{(n+1/2)^{2-p_3}}+a_{53}\frac{1}{(n+1/2)^{2-p_3}}+$$
$$+a_{32}\frac{1}{(n+1/2)^{3-2p_2}}+a_{93}\frac{\ln(n+1/2)}{(n+1/2)^{4-2p_3}}+\gamma_{13}\frac{1}{(n+1/2)^{4-2p_3}}+a_{123}\frac{\ln(n+1/2)}{(n+1/2)^{6-3p_3}}+$$
$$+\gamma_{73}\frac{1}{(n+1/2)^{6-3p_3}}+a_{31}\frac{1}{(n+1/2)^{3-2p_1}}+a_{41}\frac{\ln(n+1/2)}{(n+1/2)^{2-p_1}}+a_{51}\frac{1}{(n+1/2)^{2-p_1}}+$$
$$+a_{82}\frac{1}{(n+1/2)^{5-3p_2}}+a_{42}\frac{\ln(n+1/2)}{(n+1/2)^{2-p_2}}+a_{52}\frac{1}{(n+1/2)^{2-p_2}}+a_{1n}\,.$$

and

$$\alpha_n=\frac{\pi}{2}+\frac{M_4\pi}{2}\sum_{i=1}^{3}\frac{\delta_i}{(n+1/2)^{2-p_i}}+\nu_{13}\frac{1}{(n+1/2)^{4-2p_3}}+\nu_{73}\frac{1}{(n+1/2)^{6-3p_3}}+\frac{AN\pi}{2}\frac{1}{(n+1/2)}+$$
$$+\nu_{12}\frac{1}{(n+1/2)^{4-2p_2}}+\nu_{23}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_3}}+\nu_{33}\frac{1}{(n+1/2)^{3-p_3}}+\nu_{83}\frac{\ln(n+1/2)}{(n+1/2)^{5-2p_3}}+$$
$$+\nu_{93}\frac{1}{(n+1/2)^{5-3p_3}}+\nu_{22}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_2}}+\nu_{32}\frac{1}{(n+1/2)^{3-p_2}}+\nu_{72}\frac{1}{(n+1/2)^{6-3p_2}}+$$
$$+\nu_{11}\frac{1}{(n+1/2)^{4-2p_1}}+\nu_{21}\frac{\ln(n+1/2)}{(n+1/2)^{3-p_1}}+\nu_{31}\frac{1}{(n+1/2)^{3-p_1}}+\nu_4\frac{\ln^2(n+1/2)}{(n+1/2)^2}+$$
$$+\nu_5\frac{\ln(n+1/2)}{(n+1/2)^2}+\nu_6\frac{1}{(n+1/2)^2}+O\left(\frac{\ln n}{n^{5-2p_2}}\right), \qquad (1.10)$$

where $\lambda_k\neq\lambda_n$ *for* $k\neq n$*, all the* $\alpha_n>0$.

References

[1]. Gasimov M.G., Levitan B.M. Matem. sborn., №3, 445 (1964).
[2]. Gasimov M.G. Dokl. Akad. Nauk SSSR, 1965, v.161, №2, pp.274-276.
[3]. Gasimov M.G., Amirov R.Kh. Dokl. Akad. Nauk AzSSR, 1985, v.41, №8.
[4]. Amirov R.Kh., S. Gülyaz. Proceeding of the Ninth Int. Colloquium on Differential Equations. Plovdiv, Bulgaria, 18-23 August, 1998, VSP, Utrecht (1999) pp.15-22.

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