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NECESSARY CONDITIONS FOR HIGH-ORDER OPTIMALITY IN SYSTEMS WITH DELAY

Abstract

A new representation of the second variation of a functional is obtained. On the basis of that representation, the concept of a singular control of any order is introduced (in the case of zero order, the control is singular in the classical sense).

A necessary conditions for the optimality of singular controls (in the sense indicated) are obtained.

Optimal control problem in systems with delay is considered in present paper. The analogous of theorems from paper [1] are proved. It must be noted, that the scheme of proof here have principal new aspects besides the formal moments.

1. Consider optimal control problem

$$\dot{x}(t) = f(x(t), x(t-h), u(t), t), t \in T = [t_0, t_1], \tag{1}$$

$$x(t) = \varphi(t), t \in T_0 = [t_0 - h, t_0],$$

$$u(t) \in U \subset E^r, t \in T, \tag{2}$$

$$J(u) = \Phi(x(t_1)) \to \min_u , \tag{3}$$

where $x = (x_1,...,x_n)'$ is vector of phase coordinates, $u = (u_1,...,u_r)'$ is r-vector of control action, and prime is transposition sign, $h - Const > 0, U$ is open set of r-dimensional Euclidian space E^r.

Vector-functions $f(x,y,u,t),(x,y,u,t) \in E^n \times E^n \times U \times T$, $\varphi(t), t \in T_0 = [t_0 - h, t_0]$, we will consider as enough smooth, and considered (admissible) controls $u(t), t \in T$ as enough piece-wise smooth on T (exact propositions on its analytic properties would follow from the form of representation of finite results).

Function $\Phi(x), x \in E^n$, which determine the functional (3), supposed to be continuous with its partial derivatives up to the second order inclusively.

Admissible control $u(t), t \in T$, which is a solution of problem (1)-(3), we will call optimal control (extremal).

It is known (see for example, [2,3]), that if optimal control in problem (1)-(3) takes place, then it necessary should satisfy to the conditions:

$$\delta^1 J(u, \delta u) = -\int_{t_0}^{t_1} H_u'(t) \delta u(t) dt = 0, \forall \delta u(t) \in \widetilde{C}(T, E^r), \tag{4}$$

$$\delta^2 J(u;\delta u) = -\int_{t_0}^{t_1} \{\delta x'(t)[H_{xx}(t) + H_{yy}(t+h)]\delta x(t) + 2\delta x'(t) H_{xy}(t) \delta y(t) +$$
$$+ 2\delta x'(t) H_{xu}(t) \delta u(t) + 2\delta y'(t) H_{yu}(t) \delta u(t) + \delta u'(t) H_{uu}(t) \delta u(t)\} dt + \tag{5}$$
$$+ \delta x'(t_1) \Phi_{xx}(x(t_1)) \delta u(t_1) \geq 0, \forall \delta u(t) \in \widetilde{C}(T, E^r).$$

Here $\delta^1 J(.)$ is first and $\delta^2 J(.)$ is second variation of functional $J(u)$, $\delta u(t)$ is control variation $u(t), t \in T$, $\delta x(t)$ is corresponding variation of trajectory $x(t), t \in T$ of system (1), which is the solution of system

$$\delta\dot{x}(t)=f_x(t)\delta x(t)+f_y(t)\delta y(t)+f_u(t)\delta u(t),\ t\in T,$$
$$\delta x(t)=0,\ t\le t_0\,\delta y(t)=\delta x(t-h),\ t\le t_1, \tag{6}$$
$$H(\psi,x,y,u,t)=\psi' f(x,y,u,t),\ (\psi,x,y,u,v)\in E^n xE^n xE^n xUxT,$$
$$H(t)=\begin{cases}H(\psi(t),x(t),y(t),u(t),t),t\in T\\ 0,t>t_1\end{cases},\quad f_\mu(t)=\begin{cases}f_\mu(x(t),y(t),u(t),t),t\in T\\ 0,t>t_1\end{cases},$$
$$H_\mu(t)=\begin{cases}H_\mu(\psi(t),x(t),y(t),u(t),t),t\in T\\ 0,t>t_1\end{cases},\quad H_{\nu\mu}(t)=\begin{cases}H_{\nu\mu}(\psi(t),x(t),y(t),u(t),t),t\in T\\ 0,t>t_1\end{cases},$$

where $\mu,\nu\in\{x,y,u\},\psi(t)$ is solution of conjugate system [4]:

$$\dot{\psi}(t)=-H_x(t)-H_y(t+h),t\in T,\ \psi(t_1)=-\Phi_x(x(t_1)),\psi(t)=0,t>t_1$$

$\widetilde{C}(T,E^r)$ is class of piece-wise continuous vector-functions $\delta u(t),T\to E^r$.

2. High order expansion of second variation of functional, caused by needle variation. At first, we transform last member in (5).

The following identity is obvious

$$\sum_{i=0}^{n(t_0)}\delta x'(t)\Psi_i(t)\delta x(t-h)\Big|_{t=\tau_i}^{t=t_1}=\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\frac{d}{dt}[\delta x'(t)\Psi_i(t)\delta x(t-ih)]dt, \tag{7}$$

where $\delta x(t),t\le t_1$ is a solution of systems (6), $\tau_i=t_0+ih,n(t_0)$ is integer number, which satisfies to condition $t_1-h\le t_0+n(t_0)h<t_1$, $\Psi_i(t)$, $t[\tau_i,+\infty),i=\overline{0,n(t_0)}$ is enough piece-wise smooth matrix functions of size $(n\times n)$.

We suppose

$$\Psi_0(t_1)=-\Phi_{xx}(t_1),\ \Psi_0(t)=0,t>t_1,\Psi_i(t)=0,t\ge t_1,\ i=\overline{1,n(t_0)}. \tag{8}$$

Then from (7), taking account of $\delta x(t)=0$ for $t\le t_0$, we have

$$\delta x'(t_1)\Phi_{xx}(x(t_1))\delta x(t_1)=-J_1-J_2-\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\delta x'(t)\dot{\Psi}_i(t)\delta x(t-ih)dt, \tag{9}$$

where

$$J_1=\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\delta\dot{x}'(t)\Psi_i(t)\delta x(t-ih)dt,$$
$$J_2=\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\delta x'(t)\Psi_i(t)\delta\dot{x}(t-ih)dt.$$

Introducing into consideration two zero $(n\times n)$ matrices with denotations $\Psi_{-1}(t),\Psi_{n(t_0+1)}(t)$ and taking account of (6), we obtain following expressions

$$J_1=\int_{\tau_1}^{t_1}\delta x'(t)\Psi_0'(t)f_y(t)\delta x(t-h)dt+\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\{\delta u(t)f_u't\Psi_i(t)\delta x(t-ih)dt+$$
$$+\delta x'(t)[f_x'(t)\Psi_i(t)+f_y'(t+h)\Psi_{i+1}(t+h)]\delta x(t-ih)\}dt; \tag{10}$$

$$J_2=\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\{\delta x'(t)\Psi_i(t)f_u(t-ih)\delta u(t-ih)+\delta x'(t)[\Psi_i(t)f_x(t-ih)+ \tag{11}$$

Substitute (10), (11) into (9). Then we will have

$$\delta x'(t_1)\Phi_{xx}(x(t_1))\delta x(t_1) = -\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}[\delta u'(t)f'(t)\Psi_i(t)\delta x(t-ih) + \delta x'(t)\Psi_i(t)f_u(t-ih)\delta u(t-ih)]dt -$$

$$-\int_{t_0}^{t_1}\delta x'(t)[\dot\Psi_0(t) + f_x'(t)\Psi_0(t) + \Psi_0(t)f_x(t) + f_y'(t+h)\Psi_1(t+h)]\delta x(t)dt -$$

$$-\int_{t_0+h}^{t_1}\delta x'(t)[\dot\Psi_1(t) + f_x'(t)\Psi_1(t) + \Psi_1(t)f_x(t-h) + \Psi_0'(t)f_y(t) + \Psi_0(t)f_y(t) + f_y'(t+h)\Psi_2(t+h)]\times$$

$$\times\,\delta x(t-h)dt - \sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}\delta u'(t)[\dot\Psi_i(t) + f_x'(t)\Psi_i(t) + \Psi_i(t)f_x(t-ih) + f_y'(t+h)\Psi_{i+1}(t+h) +$$

$$+\Psi_{i-1}(t)f_y(t-(i-1)h)]\delta x(t-ih)dt\,. \tag{12}$$

Now, if matrix functions $\Psi_i(t),\ t\in[\tau_i,+\infty),\ i=\overline{0,n(t_0)}$ we define as a solution of linear problems of the form:

$$\begin{gathered}
\dot\Psi_0(t) = -f_x'(t)\Psi_0(t) - \Psi_0(t)f_x(t) - f_y'(t+h)\Psi_1(t+h) - H_{xx}(t) - H_{yy}(t+h),\ t\in[t_0,t_1),\\
\dot\Psi_1(t) = -f_x'(t)\Psi_1(t) - \Psi_1(t)f_x(t-h) - [\Psi_0'(t) + \Psi_0(t)]f_y(t) - f_y'(t+h)\Psi_2(t+h) -\\
-2H_{xy}(t),\ t\in[t_0+h_{,t_1}],\\
\dot\Psi_i(t) = -f_x'(t)\Psi_i(t) - \Psi_i(t)f_x(t-ih) - \Psi_{i-1}(t)f_y(t-(i-1)h) - f_y'(t+h)\Psi_{i+1}(t+h),\\
t\in[t_0+ih,t_1),\ i=\overline{2,n(t_0)},\\
\Psi_0(t_1) = -\Phi_{xx}(x(t_1)),\ \Psi_0(t) = 0,\ t>t_1\\
\Psi_i(t) = 0,\ t\ge t_1,\ i=\overline{1,n(t_0)}.
\end{gathered}\tag{13}$$

By the help of (12), (13) we can represent in the following form the second variation of (5)

$$\begin{gathered}
\delta^2 J(u;\delta u) = -\sum_{i=0}^{n(t_0)}\int_{\tau_i}^{t_1}[\delta u'(t)f_u(t)\Psi_i(t)\delta x(t-ih) + \delta x'(t)\Psi_i(t)f_u(t-ih)\delta u(t-ih)]dt -\\
-2\int_{t_0}^{t_1}[\delta x'(t)H_{xu}(t) + \delta x'(t-h)H_{yu}(t)]\delta u(t) - \int_{t_0}^{t_1}\delta u'(t)H_{uu}(t)\delta u(t)dt.
\end{gathered}\tag{14}$$

Suppose, that $\delta u(t), t\in T$ is needle variation

$$\delta u(t) = \begin{cases} u\in E^r,\ t\in[\theta,\theta+\varepsilon),\\ 0,\ t\in T\setminus[\theta,\theta+\varepsilon),\end{cases}\tag{15}$$

Here $\theta\in T_1,\ \varepsilon\in(0,t_1-\theta_{n(\theta)})$, moreover, $[\theta_i,\theta_i+\varepsilon)\subset T_1,\ i=\overline{0,n(\theta)}$, where $\theta_{n(\theta)} = \theta + n(\theta)h$ and T_1 is a set of points $\theta\in T\setminus\{t_1-ih, i=0,1,2,...\}$ such that $\theta=\theta_0,...,\theta_{n(\theta)}$ are points of smoothness of functions $u(t), x(t), \psi(t), t\in T$. Note, that $T\setminus T_1$ is finite set.

It is obvious, that $\varepsilon<h$. Formula (14) in variation (15) with (6), (8) and $\varepsilon<h$ takes from:

$$\delta^2 J(u;\delta u) = -\int_{\theta}^{\theta+\varepsilon}u'H_{uu}(t)u\,dt - 2\sum_{i=0}^{n(\theta)}\int_{\theta}^{\theta+\varepsilon}u'q_i'(t)\delta x(t+ih)dt,\tag{16}$$

where

$$q_0(t)=H(t)+\frac{1}{2}[\Psi_0(t)+\Psi_0'(t)]f_u(t)$$
$$q_i(t)=\frac{1}{2}\Psi_i(t+ih)f_u(t),\ i=\overline{1,n(\theta)}, \tag{17}$$

we will find expansion of second integral in (16) by powers of parameter ε. For this aim we will prove following auxiliary facts.

Supposition 1. *For solution $\delta x(t), t\in T$ of system (6), corresponding to variation (15) takes place expansion*

$$\delta x(\theta_i)=signi\sum_{j=0}^{k}\frac{d^j}{ds^j}[\lambda(s,\theta_i)f_u(s)]\Big|_{s=\theta+0} u\frac{\varepsilon^{j+1}}{(j+1)!}+signio(\varepsilon^{k+1}),$$
$$\theta\in T_1,\ u\in E^r,\ i\in\{0,1,...,n(\theta)\},\ k\in\{0,1,...\}, o(\varepsilon^{k+1})\sim\varepsilon^{k+2},$$

where $\theta_i=\theta+ih, \lambda(s,t), t_0\le s<t\le t_1$ is solution of system of the form

$$\lambda_t(s,t)=f_x(t)\lambda(s,t)+f_y(t)\lambda(s,t-h),\ t_0\le s<t\le t_1 \tag{18}$$

$\lambda(s,t)=0, s>t$, $\lambda(s,s)=E$, *(E is unit $(n\times n)$ matrix).*

Proof. Let $k\in\{0,1,...\}$ is arbitrary fixed number. Validity of supposition 1 is obvious for $i=0$.

If $i\in\{1,2,...,n(\theta)\}$, then 1) using Cauchy formula on presentation of solution, 2) applying Teylor formula and 3) taking account of smoothness of $\lambda(s,t)$ by $s\in[t_0,t)\cap T_1$ we have

$$\delta x(\theta_i)=\int_\theta^{\theta+\varepsilon}\lambda(s,\theta_i)f_u(s)uds=\int_\theta^{\theta+\varepsilon}\left\{\sum_{j=0}^{k}\frac{d^j}{ds^j}[\lambda(s,\theta_i)f_u(s)]\Big|_{s=\theta+0}\frac{(s-\theta)^j}{j!}+\right.$$
$$\left.+\frac{d^{k+1}}{ds^{k+1}}[\lambda(s,\theta_i)f_u(s)]\Big|_{s=\tilde\theta}\frac{(s-\theta)^{k+1}}{(k+1)!}\right\}uds=$$
$$=\sum_{j=0}^{k}\frac{d^j}{ds^j}[\lambda(s,\theta_i)f_u(s)]\Big|_{s=\theta+0}u\frac{\varepsilon^{j+1}}{(j+1)!}+o(\varepsilon^{k+1}),\ \theta\in T_1,$$

where $\tilde\theta\in[\theta,\theta+\varepsilon)$, $o(\varepsilon^{k+1})\sim\varepsilon^{k+2}$.

Therefore, supposition 1 is proved.

By reasoning, analogous to conclusions was done for proof of lemma from [1], we prove

Supposition 2. *If $\delta x(t), t\in T$ is a solution of system (6), corresponding to variation (15), then following presentations take place:*

$$\frac{d^m}{dt^m}\delta x(t)=\sum_{j=0}^{n(\theta)}\left[a_{m-1}^{(j)}(t)\delta x(t-jh)+b_{m-1}^{(j)}(t)\delta_0u(t-jh)\right],\ t\in T_{\theta\varepsilon}=\bigcup_{j=0}^{n(\theta)}[\theta_j,\theta_j+\varepsilon),\ m=1,2,...$$

where $\delta_0u(\tau)=\begin{cases}\delta u(\tau),\ \tau\in T,\\ 0,\ \tau\notin T\end{cases}$

$$a_m^{(j)}=a_{m-1}^{(j)}(t)+a_{m-1}^{(j)}(t)a_0^{(0)}(t-jh)+a_{m-1}^{(j-1)}(t)a_0^{(1)}(t-(j-1)h),\ j=0,1,...;m=1,2,...$$
$$a_0^{(0)}(t)=\begin{cases}f_x(t),\ t\in T\\ 0,\ t\notin T,\end{cases}\qquad a_0^{(1)}=\begin{cases}f_y(t),\ t\in T\\ 0,\ t\notin T,\end{cases} \tag{19}$$
$$a_0^{(j)}=0, t\in(-\infty;+\infty),\ j=\overline{2,n(\theta)},...a_r^{(-1)}(t)=0, r=0,1,...,$$

$$b_m^{(j)}(t)=\dot b_{m-1}^{(j)}(t)+b_{m-1}^{(j)}(t)b_0^{(0)}(t-jh),\ j=0,1,...,$$
$$b_0^{(0)}=\begin{cases} f_u(t),\ t\in T\\ 0, t\notin T,\end{cases}\qquad b_0^{(j)}(t)=0,\ t\in(-\infty,+\infty),\ j=1,2,\dots . \tag{20}$$

Supposition 3. *For the solution* $\delta x(t), t\in T$ *of system (6), corresponding to variation (15) and its derivatives, the following expressions at the points* $\theta_i=\theta+ih, i=\overline{0,n(\theta)}$ *take place:*

$$\left.\frac{d^m}{dt^m}\delta x(t)\right|_{t=\theta_i+0}=sign\,i\,b_{m-i}^{(i)}(\theta_i)u+sign\,i\sum_{j=0}^{k}D_{m-1}^{(i)}(\theta;j)u\frac{\varepsilon^{j+1}}{(j+1)!}+sign\,i\,\theta(\varepsilon^{k+1}),$$
$$k\in\{0,1,2,...\},\ m=0,1,...,$$

where

$$D^{(i)}(\theta;j)=\sum_{l=1}^{i}a_r^{(i-l)}(\theta_i)D_{-1}^{(l)}(\theta;j),\ i=\overline{1,n(\theta)},\ j=\overline{0,k}, r=0,1,...,$$
$$D_{-1}^{(l)}(\theta;j)=\frac{d^j}{ds^j}\left[\lambda(s,\theta_l)f_u(s)\right]_{s=\theta+0},\ l=\overline{1,n(\theta)},\ j=\overline{0,k} \tag{21}$$

We continue investigation of the second variation of functional on variation (15). From (16), applying Teylor formula at the point $\theta\in T_1$ and Leibnitz formula on differentiation of product, we obtain

$$\delta^2J(u;\delta u)=-\int_\theta^{\theta+\varepsilon}u'H_{uu}(t)u\,dt-2\sum_{i=0}^{n(\theta)}\Big[q_i'(\theta)\delta x(\theta_i)\varepsilon+$$
$$+\sum_{m=1}^{k+1}\sum_{j=0}^{m}C_m^j\left.\frac{d^{m-j}}{dt^{m-j}}q_i'(t)\right|_{t=\theta+0}\left.\frac{d^j}{dt^j}\delta x(t-ih)\right|_{t=\theta+0}\frac{\varepsilon^{m+1}}{(m+1)!}+o(\varepsilon^{k+2})\Big],$$

where $k\in\{0,1,...\}, C_m^j=\dfrac{m!}{J!(m-j)!}$.

Therefore, supposing $C_0^0=1$ and taking account of supposition 3, we have:

$$\delta^2J(u;\delta u)=-\int_\theta^{\theta+\varepsilon}u'H_{uu}(t)u\,dt-2u'\sum_{i=0}^{n(\theta)}\sum_{m=0}^{k+1}\sum_{j=1}^{m}C_m^j\frac{d^{m-j}}{dt^{m-j}}q_i'(t)\Big|_{t=\theta+0}b_{j-1}^{(i)}(\theta_i)u\frac{\varepsilon^{m+1}}{(m+1)!}-$$
$$-2u'\sum_{i=0}^{n(\theta)}\sum_{l=1}^{k}\sum_{m=1}^{k+1}\sum_{j=0}^{m}sign\,i\,C_m^j\frac{d^{m-j}}{dt^{m-j}}q_i'(t)\Big|_{t=\theta+0}D_{j-1}^{(i)}(\theta;l)u\frac{\varepsilon^{l+m+2}}{(m+1)!(l+1)!}+o(\varepsilon^{k+2}). \tag{22}$$

Let as simplify member before last one. Introduce new index $v=l+m+1,\ v=1,2,...,k+1$, exclude from consideration index $l=v-m-1\ge 0$. Suppose also $m+1+\mu,\ \mu=1,2,...,v,m,=\mu-1\ge 0$. Then we will have:

$$-2u'\sum_{i=0}^{n(\theta)}\sum_{l=0}^{k}\sum_{m=0}^{k+1}\sum_{j=0}^{m}sign\,i\,C_m^j\frac{d^{m-j}}{dt^{m-j}}q_i'\Big|_{t=\theta+0}D_{j-1}^{(i)}(\theta;l)u\frac{\varepsilon^{l+m+2}}{(m+1)!(l+1)!}=$$
$$=2u'\sum_{i=0}^{n(\theta)}\sum_{v=1}^{k}\sum_{\mu=1}^{k+1}\sum_{j=0}^{m}sign\,i\,C_{\mu-1}^j\frac{d^{\mu-j-1}}{dt^{\mu-j-1}}q_i'\Big|_{t=\theta+0}D_{j-1}^{(i)}(\theta;v-\mu)uC_{v+1}^{\mu}\frac{\varepsilon^{v+1}}{(v+1)!}+o(\varepsilon^{k+2}).$$

Finally, introducing into consideration sequence of quadratic forms

$$K_\nu(\theta)[u,u] = u'\sum_{\mu=1}^{\nu}\sum_{i=0}^{n(\theta)}\Big[C_\nu^\mu \frac{d^{\nu-\mu}}{dt^{\nu-\mu}} q_i'(t)\Big|_{t=\theta+0} b_{\mu-1}^{(i)}(\theta_i) +$$
$$sign\, i\sum_{j=0}^{\mu-1} C_{\mu-1}^j C_{\nu+1}^\mu \frac{d^{\mu-j-1}}{dt^{\mu-j-1}} q_i'(t)\Big|_{t=\theta+0} D_{j-1}^{(i)}(\theta,\nu-\mu)\Big]u, \quad (23)$$
$$K_0(\theta)[u,u] = u'H_{uu}(\theta)u,\ \theta\in[t_0,t_1],\ \nu=1,2,...;$$

we rewrite (22) in compact form:

$$\delta^2 J(u;\delta u) = -\int_\theta^{\theta+\varepsilon} K_0(t)[u,u]dt - 2\sum_{\nu=1}^{k+1} K_\nu(\theta)[u,u]\frac{\varepsilon^{\nu+1}}{(\nu+1)!} + o(\varepsilon^{k+2}),\ u\in E^r, \quad (24)$$

3. High-order optimality conditions.

From (4), (5) follows classical necessary conditions of optimality (analogous of Euler equation and Lagrange-Clebsch conditions):

$$H_u(t) = 0,\ u'H_{uu}(t)u = 0,\ \forall t\in T_1,\ \forall u\in E^r. \quad (25)$$

Definition 1. *Admissible control* $u(t), t\in T$ *, which satisfies to the condition (25), we will call singular of the order zero, or, simply, singular (in classical sense) at the point* θ *, if* $\alpha > 0$ *is such that*

$$E_0(\theta,\alpha) \equiv \bigcap_{t\in[\theta,\theta+\alpha)} Ker K_0(t) \neq \{0\} \subset E^r,$$

where $Ker K_0(t)$ *is kernel of quadratic form* $K_0(t)[u,u]$*, determined by (23),* $[\theta,\theta+\alpha)\subset T_1$.

Definition 2. *Admissible control* $u(t), t\in T$ *which is singular at the point* θ *, we will call singular of the order* $k\,(k>0)$ *at the point* θ *, if following correlations hold*

$$E_0(\theta,\alpha)\cap E(\theta;k)\neq\{0\},\ E_0(\theta;\beta)\cap E(\theta,k+1) = \{0\},\ \forall\beta\in(0,\alpha],$$

where

$$E(\theta,m) = \bigcap_{i=1}^{m} Ker K_i(0).$$

If for the control $u(t),\ t\in T$ *, which is singular at the point* θ *, holds correlation*

$$E_0(\theta,\alpha)\cap E(\theta,k)\neq\{0\},\ \forall k\in\{1,2,...\},$$

then we suppose the order of singularity to be equal to infinity of these definitions, from representation (24) according to condition (5), we obtain validity of following statement.

Theorem 1. *Let admissible control* $u(t), t\in T$ *is singular of the order* $k, k\in\{0,1,...\}$ *at the point* $\theta\in T_1$ *. Then for optimality* $u(t),\ t\in T$ *it is necessary validity of inequality*

$$K_1(t)[u,u]\le 0,\ \forall t\in[\theta,\theta+\alpha)\cap T_1,\ \forall u\in E_0(\theta,\alpha), \quad (26)$$
$$K_{\nu+1}(\theta)[u,u]\le 0,\ \forall u\in E_0(\theta,\alpha)\cap E(\theta;\nu),\ \nu=1,2,...,k, \quad (27)$$

where quadratic forms $K_\nu(\tau)[u,u],\ \nu=1,2,...$ *are determined here by formulas (23), (21)-(19), (17).*

Note, that this theorem is analogue of theorem from [1]. Conditions of the type (26) with the help of another method was obtained in [3-6].

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