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GLOBALLY LOCAL APPROXIMATION ON ARCS

Abstract

In the paper the results connected with the theory of approximation on arcs in the complex plane are given. Some of these results were formulated in form of hypothesises and problems in D. Newman, V.K. Dyadik and the author works in 1974-76. The rather general results reduced in this work let solve also D.Newman's problem on the arcs for which Jackson's theorem is valid.

In the present paper the proofs of the theorems, which were formulated by the author at the International Collocvium in 1976 in Budapest [1] are given. Moreover, the new results in this direction are given which help in solving of the Newman's problem connected with global polynomial approximation and formulated in 1974 [2]. We will give of D.Newman's problem in detail with the proof of the straight and inverse theorem in next paper.

Some of these results were developed further in works of V.I. Beliy [3] and V.V.Andriyevski [4].

Let us denote by A_k (see [9]) the class of the curves Γ which are the prototype of the circumference for some K- quasiconform onto mapping of the plane. The natural class coming in A_k is the class of K - curves for which for any z_1 and z_2 on Γ the least part of Γ connecting them has the length of the same order, often the length of the chord too.

Let us prove the following confirmation.

Theorem 1. *Let* $\mu \in H_2(\Gamma)$ $(0 < \alpha \leq 1)$ *(H_α is the class of Hölder of order α). Then for any natural n there exist a polynom $P_n(z)$ of degree $\leq n$ such that*

$$|f(z) - p_n(z)| \leq const d_*^\alpha\left(z, \frac{1}{n}\right),$$

where by $d_*\left(z, \frac{1}{n}\right)$ we denote the maximal from z the distances from till two branches of the level line $\Gamma_{1+\frac{1}{n}}$, $d_+\left(z, \frac{1}{n}\right)$ and $d_-\left(z, \frac{1}{n}\right)$, in every bimultiple point $z \in \Gamma$ i.e.

$$d_*\left(z, \frac{1}{n}\right) = \max\left\{d_+\left(z, \frac{1}{n}\right),\ d\left(z, \frac{1}{n}\right)\right\}.$$

Proof. Let for simplicity of the proof Γ be K-curve. We will assume without lossing of the generality that the open curve Γ has its ends in points ± 2 of the plane (z). Let us map the plane (z) to the plane (τ) with help of function

$$z = \tau + \frac{1}{\tau}\left(\tau = \frac{z}{2} + \sqrt{\frac{z^2}{4} - 1}\right). \qquad (*)$$

It is obvious that on this mapping the exterior of the arc Γ will be mapped to the exterior or to the interior of the closed K- curve C containing the points ± 1, i.e. $\pm 1 \in C$. Moreover, we will assume that the point $\tau = 0$ is strongly in C.

Note that by this mapping the function $f(z)$ is transformed to some function

$$f_1(\tau)=f\left(\tau+\frac{1}{\tau}\right)=f(z)\ (\tau\in C).$$

Let us show that if $f(z)\in H_\alpha(\Gamma)$, then $f_1(\tau)\in D_\alpha^\alpha[\pm 1,C)$ (class $D_\alpha^\beta(z_0,\Gamma)$ was introduced in [5] and is determined by the condition $\forall z_1,z_2\in\Gamma$ $|f(z_1)-f(z_2)|\le$ $\le const\max\left\{|z_0-z_1|^\beta,|z_0-z_2|\right\}\times|z_1-z_2|^2$). Indeed, if we make substitution of (*) in the correlation $|f(z_1)-f(z_2)|\le const.\ |z_1-z_2|^\alpha$, $\forall z_1,z_2\in\Gamma$ then will obtain $|f_1(\sigma_1)-f(\sigma_2)|\le const\dfrac{|\tau_1\tau_2-1|^\alpha}{|\tau_1\tau_2|^\alpha}|\tau_1-\tau_2|^\alpha$. Now if we that $|\tau_1-\tau_2|=1$ and $|1-\tau_1\tau_2|=\|-\tau_1|+|\tau_1\|-\tau_2|\le const\ \max\{|1-\tau_1|,|1-\tau_2|\}$ consider (here first the case is considered when τ_1 *and* τ_2 are closer to the point +1), then we will have $|f_1(\tau_1)-f_2(\tau_2)|\le const\ \max\left\{|1-\tau_1|^\alpha,|1-\tau_2|^\alpha\right\}|\tau_1-\tau_2|^\alpha$, i.e. $f_1(\tau)\in D_\alpha^\alpha(1,C)$, $0<\alpha\le 1$.

Further, by virtue of the theorem proved in [6], for $\beta=\alpha$ and $z_0=1$ we will obtain that for $f_1(\tau)$ for any natural n there exists the rational function $R_n(\tau)=\sum\limits_{k=-n}^{n}a_kz^k$ for which $|f_1(\tau)-R_n(\tau)|\le|1-\tau|^\alpha\left\{d^\alpha\left(\tau,\frac{1}{n}\right)+d_1^\alpha\left(\tau,\frac{1}{n}\right)\right\}+d^{2\alpha}\left(\tau,\frac{1}{n}\right)+d_1^{2\alpha}\left(\tau,\frac{1}{n}\right)$ $\left(d_1\left(\tau,\frac{1}{n}\right)\right.$ is the distance from point τ to the internal line of the level $\left.\Gamma^{(1)}_{1+\frac{1}{n}}\right)$. It is obvious this correlation can be written in the form:

$$\left|f\left(\tau+\frac{1}{\tau}\right)-R_n(\tau)\right|\le|1-\tau|^\alpha\left\{|\widetilde{\tau}-\tau|^\alpha+|\widetilde{\tau}_1-\tau|^\alpha\right\}+|\tau_1-\tau|^{2\alpha}+|\widetilde{\tau}-\tau|^{2\alpha},\qquad (1)$$

where $\widetilde{\tau}=\Psi_1\left(\left(1+\frac{1}{n}\right)\varphi_1(\tau)\right)$, $\widetilde{\tau}_1=\widetilde{\Psi}_1\left(\left(1-\frac{1}{n}\right)\widetilde{\varphi}_1(\tau)\right)$, and the functions φ_1 and Ψ_1 $\left(\widetilde{\varphi}_1\widetilde{\Psi}_1\right)$ map exterior (interiour) of curve C to exteriour (interiour) of the unitary circumference γ if to consider that for curves $\Gamma\in K$ (or $\Gamma\in A_k$) the correlation [3] has place:

$$d\left(\tau,\frac{1}{n}\right)\underset{\cap}{\cup}|\widetilde{\tau}-\tau|,\ d_1\left(\tau,\frac{1}{n}\right)\underset{\cap}{\cup}|\widetilde{\tau}-\tau|.$$

Now substituting τ in (1) for $\dfrac{1}{\tau}$ we obtain

$$\begin{gathered}\left|f\left(\tau+\frac{1}{\tau}\right)-R_n\left(\frac{1}{\tau}\right)\right|\le const\left|1-\frac{1}{\tau}\right|^\alpha\left\{\left|\frac{1}{\widetilde{\tau}}-\frac{1}{\tau}\right|^\alpha+\left|\frac{1}{\widetilde{\tau}}-\frac{1}{\tau}\right|^\alpha\right\}+\\+\left|\frac{1}{\widetilde{\tau}}-\frac{1}{\tau}\right|^{2\alpha}+\left|\frac{1}{\widetilde{\tau}_1}-\frac{1}{\tau}\right|^{2\alpha}\le const|1-\tau|^\alpha\left\{|\widetilde{\tau}-\tau|^\alpha+|\widetilde{\tau}_1-\tau|^\alpha\right\}+\end{gathered}\qquad (2)$$

$$+\left|\widetilde{\tau}_1-\tau\right|^{2\alpha}+\left|\widetilde{\tau}-\tau\right|^{2\alpha}.$$

Finally, adding the correlation's (1) and (2) we obtain

$$\left|f\left(\tau+\frac{1}{\tau}\right)-\frac{R_n(\tau)+R_n\left(\frac{1}{\tau}\right)}{2}\right|\le const\left|1-\tau\right|^{\alpha}\left\{\left|\widetilde{\tau}-\tau\right|^{\alpha}+\left|\widetilde{\tau}_1-\tau\right|^{\alpha}\right\}+ \tag{3}$$

$$+\left|\widetilde{\tau}_1-\tau\right|^{2\alpha}+\left|\widetilde{\tau}-\tau\right|^{2\alpha}.$$

It is not difficult to notice, that $\frac{R_n(\tau)+R_n\left(\frac{1}{\tau}\right)}{2}$ is the polynomial of degree n by $\tau+\frac{1}{\tau}$, i.e. $P_n(\tau)+\frac{1}{\tau}$.

Thus, the correlation (3) can be written in following form:

$$\left|f\left(\tau+\frac{1}{\tau}\right)-P_n\left(\tau+\frac{1}{\tau}\right)\right|\le const\left|1-\tau\right|^{\alpha}\left\{\left|\widetilde{\tau}-\tau\right|^{\alpha}+\left|\widetilde{\tau}_1-\tau\right|^{\alpha}\right\}+$$

$$+\left|\widetilde{\tau}-\tau\right|^{2\alpha}+\left|\widetilde{\tau}_1-\tau\right|^{2\alpha}.$$

And if to take into account that for $\Gamma\in K$ or A_k

$$\left|\widetilde{\tau}-\tau\right|\le const\left|1-\tau\widetilde{\tau}\right|,\qquad \left|\widetilde{\tau}_1-\tau\right|\le const\left|1-\tau\widetilde{\tau}_1\right|,$$

$$\left|1-\tau\right|\underset{\cap}{\cup}\left|1-\tau^2\right|\le\left|1-\tau\widetilde{\tau}\right|+\left|\tau\right|\left|\widetilde{\tau}-\tau\right|\le\left|1-\tau\widetilde{\tau}\right|$$

and

$$\left|1-\tau\right|^{\alpha}\left\{\left|\widetilde{\tau}-\tau\right|^{\alpha}+\left|\tau_1-\tau\right|^{\alpha}\right\}\le\left|\frac{1-\tau\widetilde{\tau}}{\tau\widetilde{\tau}}\right|^{\alpha}\left|\widetilde{\tau}-\tau\right|^{\alpha}+\left|\frac{1-\tau\widetilde{\tau}}{\tau\widetilde{\tau}}\right|^{\alpha}\left|\widetilde{\tau}_1-\tau\right|^{\alpha}=$$

$$=\left|\widetilde{\tau}+\frac{1}{\widetilde{\tau}}-\left(\tau+\frac{1}{\tau}\right)\right|^{\alpha}+\left|\widetilde{\tau}_1+\frac{1}{\widetilde{\tau}_1}-\left(\tau+\frac{1}{\tau}\right)\right|^{\alpha},$$

then we will have

$$\left|f\left(\tau+\frac{1}{\tau}\right)-P_n\left(\tau+\frac{1}{\tau}\right)\right|\le const\left|\widetilde{\tau}+\frac{1}{\tau}-\left(\tau+\frac{1}{\tau}\right)\right|^{\alpha}+\left|\widetilde{\tau}_1+\frac{1}{\tau_1}-\left(\tau+\frac{1}{\tau}\right)\right|^{\alpha}. \tag{4}$$

Now it is sufficient to show that the mapping $\tau+\frac{1}{\tau}$ implies the following correlations

$$\widetilde{z}=\widetilde{\tau}+\frac{1}{\widetilde{\tau}}\ \ and\ \ z_-=\widetilde{\tau}_1\left(\frac{1}{n+1}\right)+\widetilde{\tau}^{-1}\left(\frac{1}{n+1}\right), \tag{5}$$

where

$$\widetilde{z}_-\left(\frac{1}{m}\right)=\Psi\left(\left(1+\frac{1}{m}\right)\varphi^{-1}(z)\right),\quad z\in\Gamma,$$

and functions φ and Ψ with the certain normalizing map the exterior of arc Γ to the exterior of the unitary circumference γ_0 of the plane ω and contrary.

Let us first show that the mapping $z=\tau+\tau^{-1}$ implies $\widetilde{z}=\widetilde{\tau}+\widetilde{\tau}^{-1}$. With this purpose let us find the correlation between functions φ and φ_l, Ψ and Ψ_l.

It is obvious that they will be the following.

$$\varphi_1(\tau)=\varphi\left(\tau+\frac{1}{\tau}\right)=\varphi(z),\ \Psi(\omega)=\Psi_1(\omega)+\frac{1}{\Psi_1(\omega)}. \tag{6}$$

Indeed, let $\tau\in C$. This point can be mapped to γ_0 by two methods.

1^0. Immediately with help of function $\omega=\varphi_1(\tau)$;

2^0. At the result of subsequental mappings first to plane (i.e. to arc Γ) with help of function $z=\tau+\tau^{-1}$, and then to γ_0 with help of function $\omega=\varphi(z)$.

At the result we obtain

$$\varphi_1(\tau)=\varphi\left(\tau+\frac{1}{\tau}\right)\quad\left(z=\tau+\frac{1}{\tau}\right).$$

The first formula of the correlation (6) has been proved. In order to prove the second formula of (6) let us take the point $\omega\in\gamma_0$ and map it by two methods to arc Γ.

1^0. Immediately with help of $z=\Psi(\omega)$;

2^0. At the result of subsequental mappings first to plane τ (i.e. to curve C) with help of function $\tau=\Psi_1(\omega)$, and then to plane z (i.e. to the arc Γ) with help of function $z=\tau+\tau^{-1}$.

At the result we obtain $\Psi(\omega)=\Psi_1(\omega)+\Psi_1^{-1}(\omega)$.

The correlation (6) has been completely proved by it. Now let us show that $\tilde z=\tilde\tau+\tilde\tau^{-1}$ implies $z=\tau+\tau^{-1}$. Indeed,

$$z=\Psi\left(\left(1+\frac{1}{n}\right)\varphi(z)\right)=\Psi_1\left(\left(1+\frac{1}{n}\right)\varphi(z)\right)+\Psi_1^{-1}\left(\left(1+\frac{1}{n}\right)\varphi(z)\right)=$$

$$=\Psi_1\left(\left(1+\frac{1}{n}\right)\varphi_1(\tau)\right)+\Psi_1^{-1}\left(\left(1+\frac{1}{n}\right)\varphi_1(\tau)\right)=\tilde\tau+\frac{1}{\tilde\tau}.$$

Thus, the first correlation of (5) has been proved. Now in order to prove the second correlation of (5) let us note that function $z=\tau+\tau^{-1}$ maps the exterior of arc Γ to the exterior of curve C. Then let us establish the connection between the functions $\tilde\Psi_1$ and Ψ_1 and also $\tilde\varphi_1$ and φ_1 which it is obvious will be expressed in the form:

$$\varphi_1(\tau)=\tilde\varphi_1^{-1}\left(\tau^{-1}\right),\ \Psi_1(\omega)=\tilde\Psi_1^{-1}\left(\frac{1}{\omega}\right). \tag{7}$$

Indeed, using two methods of mapping of the point $\omega\in\gamma_0$ to curve C, we obtain $\omega=\omega_1^{-1}=\tilde\varphi_1^{-1}\left(\tau^{-1}\right)=\varphi_1(\tau)$.

And mapping the point $\tau\in C$ *to* γ_0, we obtain $z=z_1^{-1}=\tilde\Psi^{-1}\left(\omega^{-1}\right)=\Psi_1(\omega)$.

Now using (6) and (7) we will have

$$\tilde z_-\left(\frac{1}{n}\right)=\Psi\left(\left(1+\frac{1}{n}\right)\varphi^{-1}(z)\right)=\tilde\Psi_1^{-1}\left(\frac{1}{\left(1+\frac{1}{n}\right)\varphi^{-1}(z)}\right)+$$

$$+\tilde{\Psi}_1\left(\frac{1}{\left(1+\frac{1}{n}\right)\varphi^{-1}(z)}\right)=\tilde{\Psi}_1^{-1}\left(\left(1-\frac{1}{n+1}\right)\tilde{\varphi}_1\left(\tau^{-1}\right)\right)+$$

$$+\tilde{\Psi}_1\left(\left(1-\frac{1}{n+1}\right)\tilde{\varphi}_1\left(\tilde{\tau}^{-1}\right)\right)=\tilde{\tau}_1\left(\frac{1}{n+1}\right)+\tilde{\tau}_1^{-1}\left(\frac{1}{n+1}\right).$$

Thus, the second correlation from (5) has been proved. Now making substitution $z=\tau+\tau^{-1}$ in (4) and using (5) we find $|f(z)-P_n(z)|\le\left|\tilde{z}_+\left(\frac{1}{n}\right)-z\right|^{\alpha}+\left|\tilde{z}_-\left(\frac{1}{n+1}\right)-z\right|^{\alpha}$,

where (see [4]) the value of $\left|\tilde{z}_+\left(\frac{1}{n+1}\right)-z\right|$ is equal to the distance from point z to one of two branches of the line of level $\Gamma_{1+\frac{1}{n}}$, i.e. $\left|\tilde{z}+\left(\frac{1}{n+1}\right)-z\right| \overset{\cup}{\underset{\cap}{}} d_+\left(z,\frac{1}{n}\right)$, and the value $\left|\tilde{z}_-\left(\frac{1}{n+1}\right)-z\right|$ is equal to the distance from point z to other branch of the line of level $\Gamma_{1+\frac{1}{n}}$, i.e. $\left|\tilde{z}_-\left(\frac{1}{n+1}\right)-z\right| \overset{\cup}{\underset{\cap}{}} d_-\left(z,\frac{1}{n}\right)$. Hence it follows immediately the confirmation of Theorem 1.

Further we will say that the quasiconform arc Γ belongs to class C^* if for it the correlation has place:

$$d_+\left(z,\frac{1}{n}\right)\overset{\cup}{\underset{\cap}{}} d_-\left(z,\frac{1}{n}\right)\overset{\cup}{\underset{\cap}{}} d\left(z,\frac{1}{n}\right). \tag{8}$$

From Theorem 1 it follows an important corollary for the arcs from class C^*.

Corollary 1. *Let* Γ *be the arc from class* C^* *and* $f(z)\in H_\alpha(\Gamma)$ $(0<\alpha\le 1)$. *Then for any natural* n *there exists the polynomial* $P_n(z)$ *of power* $\le n$ *such that* $|f(z)-P_n(z)|\le const\ d^\alpha\left(z,\frac{1}{n}\right)$. *This confirmation with the inverse theorem of N.A. Lebedev and P.M. Tamrazov [7] reduces to the following conclusion:*

Theorem 2. *Let* $\Gamma\in C^*$. *In order to* $f(z)\in H_\alpha(\Gamma)$ $(0<\alpha<1)$ *it is necessary and sufficient for any natural* n *the polynomial with power* n $P_n(z)$ *to exist such that*

$$|f(z)-P_n(z)|\le const\ d^\alpha\left(z,\frac{1}{n}\right).$$

Remark 1. Watching the method of the proof of Theorem 1, it is not difficult to be persuaded in that it can be generalized also in terms of modulus of continuity. Now let us prove the following theorem:

Theorem 3. *Let the arc* $\mu\in A_k$ *and* $f(z)\in C(\Gamma)$. *Then for every natural* n *there exists a polynomial* $P_n(z)$ *degree* n *for which* $|f(z)-P_n(z)|\le\omega\left(f_0,\frac{1}{n}\right)$, *where*

$\omega\left(f_0,\frac{1}{n}\right)$ *is the modulus of continuity of function* $f_0(\omega)=f(\Psi(\omega))$ *determined on the unitary circumference* γ_0.

Proof. As in Theorem 1 we will consider that the arc $\Gamma\in K$ has its ends in the points -2; +2 in plane z and let us map plane z to plane τ with help of function $z=\tau+\frac{1}{\tau}$. On this mapping, as it was said in the proof of Theorem1, the exterior of arc Γ will be mapped to the exterior or interior of some closed K - curve C containing points ±1, i.e. $\pm1\in C$. Moreover, we will consider that point $z=0$ is strongly in C. Moreover, on this mapping, function $f(z)$ will be transformed to some function $g(\tau)=f\left(\tau+\tau^{-1}\right)=f(z)\ \ (\tau\in C)$.

It is not difficult to show that function $g(\tau)$ will be continuous on C and for the moduluses of continuity $\omega(g_1,\delta)$ *and* $\omega(g_2,\delta)$, where $g_1(\omega)=g(\Psi_1(\omega))$ and $g_2(\omega)=g\left(\widetilde{\Psi}_1(\omega)\right)$, the following correlation has place:

$$\omega(g_1,\delta)=\omega(f_0,\delta),\ \omega(g_2,\delta)=\omega(f_0,\delta), \tag{9}$$

where $f_0(\omega)=f(\Psi(\omega))$.

Indeed, by virtue of (6) it is not difficult to show that the mapping $z=\tau+\tau^{-1}$ entails $z_h=\tau_h+\tau_h^{-1}$ where $z_h=\Psi\left(\varphi(z)e^{ih}\right)=\Psi\left(\omega e^{ih}\right)$, $\tau_h=\Psi_1\left(\varphi_1(\tau)e^{ih}\right)=\Psi_1\left(\omega e^{ih}\right)$. It immediately follows from the following correlations:

$$z_h=\Psi\left(\varphi(z)e^{ih}\right)=\Psi_1\left(\varphi(z)e^{ih}\right)+\Psi_1^{-1}\left(\varphi(z)e^{ih}\right)=$$
$$=\Psi_1\left(\varphi_1(\tau)e^{ih}\right)+\Psi_1^{-1}\left(\varphi_1(\tau)e^{ih}\right)=\tau_h+\tau_h^{-1}.$$

Further by using the correlation (7), we can show that the mapping $z=\tau+\tau^{-1}$ entails also the mapping $z_{-h}=\tau_h^*+\frac{1}{\tau_h^*}$, $\tau_h^*=\widetilde{\Psi}_1\left(\widetilde{\varphi}_1(\tau)e^{ih}\right)$. These reasoning will prove the correlation (9) if to take into account that

$$\omega(g_1,\delta)=\sup_{|h|\le\delta}\left|g(\tau_h)-g(\tau)\right|,$$

$$\omega(g_2,\delta)=\sup_{|h|\le\delta}\left|g\left(\tau_h^*\right)-g(\tau)\right|,$$

$$\omega(f,\delta)=\sup_{|h|\le\delta}\left|f(z_h)-f(z)\right|.$$

Further, by virtue of [6] for the continuous function $g(\tau)$ on C and any real n the rational function $R_n(\tau)$ will be found for which $\left|g(\tau)-R_n(\tau)\right|\le\omega\left(g_1,\frac{1}{n}\right)+\omega\left(g_2,\frac{1}{n}\right)$. It is obvious, this correlation can be written in following form:

$$\left|f\left(\tau+\tau^{-1}\right)-R_n(\tau)\right|\le const\sup_{|h|\le\frac{1}{n}}\left|f\left(\tau_h+\tau_h^{-1}\right)-f\left(\tau+\tau^{-1}\right)\right|+$$

$$+\sup_{|h|\le\frac{1}{n}}\left|f\left(\tau_h^*+\frac{1}{\tau_h^*}\right)-f\left(\tau+\tau^{-1}\right)\right|.$$

Hence, substituting τ by $\frac{1}{\tau}$ we will obtain

$$\left|f\left(\tau+\tau^{-1}\right)-R_n\left(\tau^{-1}\right)\right|\le const\sup_{|h|\le\frac{1}{n}}\left|f\left(\tau_h+\tau_h^{-1}\right)-f\left(\tau+\tau^{-1}\right)\right|+$$

$$+\sup_{|h|\le\frac{1}{n}}\left|f\left(\tau_h^*+\frac{1}{\tau_h^*}\right)-f\left(\tau+\frac{1}{\tau}\right)\right|.$$

Adding these two correlation's we find

$$\left|f\left(\tau+\tau^{-1}\right)-\frac{R_n(\tau)+R_n\left(\tau^{-1}\right)}{2}\right|\le const\ \sup_{|h|\le\frac{1}{h}}\left|f\left(\tau_h+\tau_h^{-1}\right)-f\left(\tau+\tau^{-1}\right)\right|+$$

$$+\sup_{|h|\le\frac{1}{h}}\left|f\left(\tau_h^*+\frac{1}{\tau_h^*}\right)-f\left(\tau+\tau^{-1}\right)\right|.$$

In Theorem 1 we have already showed that $\frac{R_n(\tau)+R_n\left(\tau^{-1}\right)}{2}$ is the polynomial of degree n by $\tau+\tau^{-1}$. Taking it into account, making substitution $z=\tau+\tau^{-1}$ and using (9), we obtain $\left|f(z)-R_n(z)\right|\le const\ \omega\left(f_0,\frac{1}{n}\right)$, Q.E.D.

Remark. It is not difficult to notice that it is possible to deduce from these rather general theorems the fact which we gave in [8] and which gives the answer at the solution of following D. Newman's problem [2]:

«Which necessary and sufficient conditions shall the arc Γ have to satisfy that Jackson's theorem to be valid».

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