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LOADED SINGULAR EQUATIONS WITH TWO OPERATORS

Abstract

Regularization problem for the equation

$$u_1x + u_2x_\alpha + u_3S_1x + u_4S_{1\alpha}x + u_5S_2x + u_6S_{12}x + kx = y$$

in the linear loaded normalized ring R is considered in the paper.

So as $S_1^2 = S_2^2 = E$ and for any ψ from R the operators $S_j\psi - \psi S_j$ $(j = 1,2)$ are completely continuous operators.

Using the properties as of singular operators S_1, S_2 and loaded operators $S_{1\alpha}, S_{2\alpha}$, an equivalent equation with completely continuous operator corresponds to this equation.

Let R be linear normalized loaded ring, i.e. according to the some principle to each element x from R associates element x_α also from R, where following axioms hold

$$(x+y)_\alpha = x_\alpha + y_\alpha$$
$$(\lambda x)_\alpha = \lambda x_\alpha$$
$$(xy)_\alpha = x_\alpha y_\alpha$$
$$(x_\alpha)_\alpha = x_\alpha$$
$$\|x_\alpha\| \le \|x\|.$$

Consider equation of the form

$$u_1x + u_2x_2 + u_3S_1x + u_4S_{1\alpha}x + u_5S_2x + u_6S_{2\alpha}x + Kx = y, \qquad (1)$$

where u_j $(i = \overline{1,6})$, y are given elements from R, operator K is completely continuous, S_1, S_2 are singular operators, $S_1^2, S_2^2 = E$ and for any ψ from R operators $S_j\psi - \psi S_j$ $(j = 1,2)$ are completely continuous.

If B is linear bounded operator, then loaded operator B_α is determined by relation

$$B_\alpha x = (Bx)_\alpha, \quad x \in R.$$

It is easy to see, that B_α is also linear bounded operator. For regularization of equation (1) consider two expressions

$$L_1 = u_1x + u_3S_1x + u_5S_2x + Kx,$$
$$L_2 = u_2x_\alpha + u_4S_{1\alpha}x + u_6S_{2\alpha}x.$$

Firstly, note that from condition $S_1S_2 = S_2S_1$ it follows that $\sigma = [s_1, s_2]$ is singular operator of the second order. We will use Hilbert identity for two elements u, v

$$S_1uS_1v = uv + P_1v, \qquad (2)$$
$$S_2uS_2v = uv + P_2v, \qquad (3)$$
$$S_1S_2uS_1S_2v = uv + P_3v, \qquad (4)$$

where P_j $(j = \overline{1,3})$ are completely continuous operators. We will act to the equation by operators S_1, S_2, S_1S_2. For this consider the expressions

$S_1L_1, S_2L_1, S_1S_2L_1, S_1L_2, S_2L_2, S_1S_2L_2$. Firstly consider the expression $S_1L_1 = S_1u_1x + S_1u_3S_1x + S_1u_5S_2x + S_1Kx$. Hence, we have $S_1L_1 = S_1u_1S_1S_1x + + S_1u_3S_1x + S_1u_5S_1S_1S_2x + S_1KS_1S_1x$.

Let $x = \varphi_1, S_1x = \varphi_2, S_2x = \varphi_3, S_1S_2x = \varphi_4$, then $S_1L_1 = S_1u_1S_1\varphi_2 + S_1u_3S_1\varphi_1 + + S_1u_5S_1\varphi_4 + S_1KS_1\varphi_2$. By virtue of Hilbert identity

$$S_1u_1S_1\varphi_2 = u_1\varphi_2 + P_1^{(1)}\varphi_2, \tag{5}$$

$$S_1u_3S_1\varphi_1 = u_3\varphi_1 + P_2^{(1)}\varphi_1, \tag{6}$$

$$S_1u_5S_1\varphi_4 = u_5\varphi_4 + P_3^{(1)}\varphi_4, \tag{7}$$

where $P_j^1\ \left(j=\overline{1,3}\right)$ are completely continuous operators. Thus, symmetric records S_1L_1 after re-denotation have form

$$S_1L_1 = \sum_{i=1}^{4} u_i^{(1)}\varphi_i + \sum_{i=1}^{4} P_i^{(1)}\varphi_i, \tag{8}$$

where $u_i^{(1)}\ \left(i=\overline{1,4}\right)$ are elements from R, operators $P_i^{(1)}\ \left(i=\overline{1,4}\right)$ are completely continuous, in particular, are zero elements or zero operators. By the similar way could be found symmetric record for two expressions in the form:

$$S_2L_1 = \sum_{i=1}^{4} u_i^{(2)}\varphi_i + \sum_{i=1}^{4} P_i^{(2)}\varphi_i, \tag{9}$$

$$S_1S_2L_1 = \sum_{i=1}^{4} u_i^{(3)}\varphi_i + \sum_{i=1}^{4} P_i^{(3)}\varphi_i. \tag{10}$$

Then L_1 could be rewritten in the form:

$$L_1 = \sum_{i=1}^{4} u_i^{(4)}\varphi_i + \sum_{i=1}^{4} P_i^{(4)}\varphi_i. \tag{11}$$

From these expressions it is seen, that to the vector $\left[L_1, S_1L_1, S_2L_1, S_1S_2L_1\right]$ could corresponds expression in the vector form

$$A_1\omega + T_1\omega, \tag{12}$$

where A_1 is matrix with elements from R, operator T_1 is completely continuous in R^4.

Now consider expression L_2. If x_α is loaded, then so as $x_\alpha = S_{1\alpha}S_1x$ taking into account the properties of stress loading and above mentioned denotations, we obtain

$$x_\alpha = S_{1\alpha}S_1\varphi_1,\ x_\alpha = S_{1\alpha}\varphi_2.$$

Similarly, if $x_\alpha = S_{2\alpha}S_2x$, then $x_\alpha = S_{2\alpha}S_2\psi_1,\ x_\alpha = S_{2\alpha}\varphi_2$. Then we also obtain $x_\alpha = \left(S_1S_2\right)_\alpha S_1S_2\varphi_1,\ x_\alpha = \left(S_1S_2\right)_\alpha\varphi_4$.

So, we have shown that loaded element x_α could be expressed by various relations by $\varphi_1, \varphi_2, \varphi_3, \varphi_4$. So as $S_{1\alpha}x, S_{2\alpha}x$ are also loaded elements, then each of these elements could be expressed by $\varphi_j\ \left(j=\overline{1,4}\right)$.

Now, we obtain

$$L_2 = \sum_{i=1}^{4} V_i^{(1)}\varphi_i + \sum_{i=1}^{4} Q_i^{(1)}\varphi_i, \tag{13}$$

$$S_1L_2 = \sum_{i=1}^{4} V_i^{(2)}\varphi_i + \sum_{i=1}^{4} Q_i^{(2)}\varphi_i, \tag{14}$$

$$S_2L_2 = \sum_{i=1}^{4} V_i^{(3)}\varphi_i + \sum_{i=1}^{4} Q_i^{(3)}\varphi_i , \tag{15}$$

$$S_1S_2L_2 = \sum_{i=1}^{4} V_i^{(4)}\varphi_i + \sum_{i=1}^{4} Q_i^{(4)}\varphi_i . \tag{16}$$

Note, that for equalities (14)-(16) we have expressions (2)-(4). Expressions (13)-(16) give possibilities could corresponds expression in the form.

$$A_2\omega + T_2\omega , \tag{17}$$

where A_2 is matrix with elements depended on elements, R and operators S_i, T_2 is completely continuous operator in R^4.

Thus, use the expressions (9)-(11), (13)-(16) to the equation (1) we could compare equation of the following form

$$A_1\omega + T_1\omega + A_2\omega + T_2\omega = h .$$

Let $A = A_1 + A_2, T = T_1 + T_2$, then we can rewrite this equation in the form

$$A\omega + T\omega = h , \tag{18}$$

where T is completely continuous operator in R^4, A is matrix of elements, which depends on elements R and operators S_i.

Let A have bounded inverse operator, then from (18) we have:

$$\omega + T_0\omega = h_0 , \tag{19}$$

where T_0 is completely continuous operator in R^4. And this completes construction of regularization of equation (1). Thus we have shown that to equation (1) we could compare operator equation (19) with completely continuous operators. It must be noted, that such regularization is equivalent to equation (1), i.e. if x is solution of equation (1), then it is obvious, that $\omega = (x, S_1x, S_2x, S_1S_2x)$ is solution of (19), and vice versa, if $\omega = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ is solution of equation (19), then by immediate verification we could check out, that solution of equation (1) is expressed by formula:

$$x = \frac{1}{4}(\varphi_1 + s_1\varphi_2 + s_2\varphi_3 + s_1s_2\varphi_4).$$

So, it has shown that following supposition is valid.

Supposition. *Let R be linear normalized loaded ring, S_1, S_2 are singular operators, which maps in R, satisfy to all mentioned-above conditions, K is completely continuous operator in R. Then if constructed above operator A have bounded inverse operator, then equation (1) is equivalent to equation (19), moreover solutions of these equations are connected with each other by mentioned-above relations.*

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