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POTENTIAL METHOD IN SOLUTION OF MULTIVARIATE PROBLEMS OF TWO-PHASE FILTRATION IN HETEROGENEOUS FORMATIONS

Abstract

Combined numerical methods being effective from the point of view of realisation in computers (finite difference method, border elements method, least squares method, multicriterial optimisation, nonlinear programming) have been developed in heterogeneous formations for multidimensional two-phase filtration problems.

Many applied problems of filtration theory, aerodynamics, electrical physics, optimal developments of oil and gas fields [1] are reduced to the unknown moving boundary problems on which different rigid restrictions are imposed.

For instance, in [2] the problems on mutual fluid displacement in schematisation of filtration process by Barkley-Laveretta, Rapaport-Lees, and also in mixing of fluids have been reduced to Cauchy's initial boundary value problems. The isosat (isocone) motion problems have been reduced to the system of functional (integrodifferential) equations. Star like restrictions are imposed on isolines during the process.

In the given paper, for indicated problems the invariant mathematical model is suggested with respect to dynamics of isolines

$$\Gamma(t)=\bigcup_{j=0}^{N}\gamma_j(t), \tag{1}$$

which are described by Jordanian curves.

Let at the initial time $t=0$ in the domain $D_0(0)$ saturation (concentration) of displacing phase $s(0)$ has minimal value $s=\underline{s}\ (C=\underline{C})$, and in the domain - maximal $s=\overline{s}\ (C=\overline{C})$. In a transient domain with the boundary $\Gamma(0)$ the saturation (concentration) adopts the value $\underline{s}<s<\overline{s}\ (\underline{C}<C<\overline{C})$. Then, by using the zone linearizastion method (see fig.1), divide the transient domain by isocats (isocones)

$$\underline{s}=s_0<s_1<s_2<\ldots<s_{N-1}<s_N=\overline{s}\quad \left(\underline{C}=C_0<C_1<C_2<\ldots<C_{N-1}<C_N=\overline{C}\right)$$

to N subdomains $D_j(0)$ with regard to that the boundaries $\gamma_0(0)$ and $\gamma_N(0)$ coincide with isosats (isocones) s_0 and s_N (C_0 and C_N) respectively. At each subdomain $D_j(0)$ between $\gamma_{j-1}(0)$ and $\gamma_j(0)$ contours in case of immiscible liquids average filtration resistance

$$c_j=kf_1(\widetilde{s}_j)\mu_i^{-1}+kf_2(\widetilde{s}_j)\mu_2^{-1}=k(K_1)_j+k(K_2)_j=k(K)_j$$

where f_i is relative phase permeability of the i-th phase;

$\widetilde{s}_j$ is average saturation;

k is absolute permeability.

In case of joint filtration mixed fluids, concentration $\widetilde{C}_j$ is averaged, and viscosity and density of mixture are counted to be known functions of concentration $\mu_j=\mu(\widetilde{C}_j);\ \rho_j=\rho(\widetilde{C}_j)$.

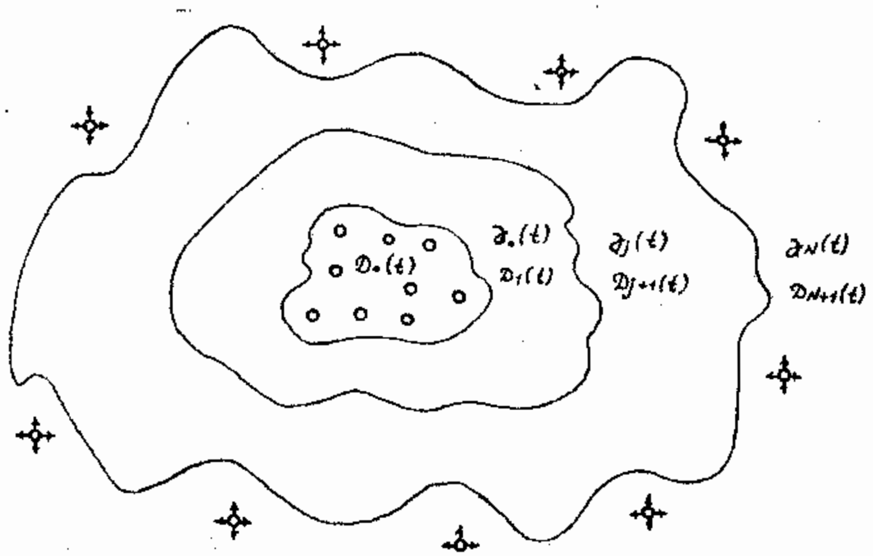

Fig.1.

The problems on motion tracing of isolines analogously [3] are reduced to the following system of equation (invariant mathematical model)

$$V(\sigma,t)-a\int_{\Gamma(t)}\left(V(\sigma,t)-dV(\xi,t)\right)\frac{\partial}{\partial n}\ln R^{-1}(\sigma,\xi,t)d\xi=K\frac{\partial\varphi}{\partial n}, \qquad (2)$$

$$\frac{\partial x(\sigma,t)}{\partial t}=-V(\sigma,t)\frac{\partial y(\sigma,t)}{\partial\sigma}, \qquad (3)$$

$$\frac{\partial y(\sigma,t)}{\partial t}=V(\sigma,t)\frac{\partial x(\sigma,t)}{\partial\sigma}, \qquad (4)$$

$$x(\sigma,0)=x^0(\sigma);\ y(\sigma,0)=y^0(\sigma), \qquad (5)$$

where $V(\sigma,t)$ is the motion velocity of isolines along internal normal n;

$$R(\sigma,\xi,t)=\left((x(\sigma,t)-x(\xi,t))^2+(y(\sigma,t)-y(\xi,t))^2\right)^{1/2};$$

$(x(\sigma,t),y(\sigma,t))$ are parametric coordinates of isolines; σ,ξ are arch coordinates of the contour $\Gamma(t)$;

$$\varphi(x,y,t)=(2\pi Hc_0)^{-1}\sum_{i=1}^{M_1(t)}Q_i(t)\ln R_i^{-1}(t)+(2\pi Hc_{N+1})^{-1}\sum_{i=M_1(t)+1}^{M(t)}Q_i(t)\ln R_i^{-1}(t)$$

is potential of external field representing the sum of pressure fields created by each well;

H is formation thickness; $Q_i(t)$ are the production rates of producing wells, whose quantity $M_1(t)$ depends on time t; $Q_i(t)$ are production rates of $M(t)-M_1(t)$ of injection wells; $M(t)$ is a general quantity of wells;

$$R_i(t)=\left((x-x_i)^2+(y-y_i)^2\right)^{1/2},$$

$x^0(\sigma)$, $y^0(\sigma)$ are known initial states of isolines.

The values of the coefficient $a = (c_j - c_{j+1})/2\pi c_{j+1}$ is the same for all three displacement schemes, therefore, write the values of coefficients d and k.

1) Schematisation of Barkley-Leverett displacement process (two-phase filtration of immiscible and uncompressible fluids)

$$d = c_j c_{j+1}(c_q - c_{q+1})(c_j - c_{j+1})^{-1} c_q^{-1} c_{q+1}^{-1} F_j' / F_q';$$
$$K = -c_j m^{-1} F_j;\ \ (q = \overline{0,N})$$

where F is a Leverett function; m is the formation porosity.

2) Schematisation of Rappaport-Lee's displacement process (two-phase filtration with regard to capillary and gravitational forces)

$$d = c_j c_{j+1}(c_q - c_{q+1})(c_j - c_{j+1})^{-1} c_q^{-1} c_{q+1}^{-1} m_q m_j^{-1} F_j' / F_q';$$
$$m_j = m - \nabla\left(kK_1K_2K^{-1}\frac{dP_c}{ds}\nabla S\right)\gamma_j(t)\left(\frac{\partial s_j}{\partial t}\right)^{-1};$$
$$K = -c_j m^{-1} F_j';\ \ (q = \overline{0,N})$$

where P_c is the capillary (interface) pressure.

We are to note that the exceed of the point $h(x,y,z)$ over the plane of zero gravitational potential here and further is constant.

We also note that the system of equations (2)-(5) for schemes 1 and 2 is valid for $F_N' \neq 0$. If $F_N' = 0$, then the contour $\gamma_N(t)$ is known, but it doesn't change by the time. In this case, additional conditions must be done on the fixed contour γ_N: the pressure or its derivative

$$P(x,y,t) = P^0(x,y,t),$$
$$\frac{\partial P(x,y,t)}{\partial n} = g(x,y,t).$$

Then according to these two conditions instead of integral equation (2) we get two (similar to [4]), integral equations, but now with respect to the motion velocity of N isosats

3) At mutual displacement of two miscible fluids

$$d = c_j c_{j+1}(c_q - c_{q+1})(c_j - c_{j+1})^{-1} c_q^{-1} c_{q+1}^{-1} m_q m_j^{-1};$$
$$m_j = m - D(\nabla^2 C)\gamma_j(t)\left(\frac{\partial s_j}{\partial t}\right)^{-1};$$
$$K = -c_j m^{-1} F_j';\ \ (q = \overline{0,N}),$$

where D is a dispersion coefficient.

Effective combined numerical method from the point of view of its realisation in computer has been developed to solve a system of equations (2)-(5). In integral equation (2) with a describe operator

The obtained nonlinear system of algebraic equations is solved by iteration method.

To illustrate the effectiveness of the suggested method, the following problem on optimal rate choice of producing wells on the basis of regulation of isosats is solved. Let two producing wells be posed in the oil region representing 100 m radius area at initial moment. The coordinates of the well equal to $(-40m;0m)$ and $(60m;0m)$ respectively.

The planned oil-withdrawal that is constant and equals to $Q_{pl}(t)=16m^3/day$ by these two producing wells. Ultimate possibilities of the wells are the same and equal to $Q_1^{ul}(t)=Q_2^{ul}(t)=16m^3/day$. Constant pressure of $100m$ is maintained on a external reservoir boundary of $200m$ radius periphery.

The problem is reduced under the following initial data [2]

$$\mu_1\mu_2^{-1}=3;\ f_1(s)=\left[(1-s+\alpha)^2\right]\left[(1+\alpha)^2-\alpha^2\right]^{-1}$$
$$\alpha=0.015464;\ f_2(s)=s^2;\ \underline{s}=0;\ \bar{s}=1;\ \ N=1;$$

$$F_0'=F'(s_\phi)=1.488854;\ \tilde{s}_1=<s_1>_{D_1}=0.671657;\ F_0'=0.01 \tag{6}$$
$$k=3;\ c_0=1;\ c_1=1.467795;\ c_2=3\,.$$

To solve a standard problem of quadratic programming by Bill's method [5] the following model is used

$$\min\left(\frac{1}{2\pi Hc_0}\sum_{i=1}^{M_1(t)}Q_i(t)\ln R_i^{-1}(t)+\frac{1}{2\pi Hc_{N+1}}\sum_{i=M_1(t)+1}^{M(t)}Q_i(t)\ln R_i^{-1}(t)+\right.$$
$$+K^{-1}\int_{\Gamma_0}\mu(\sigma_0,t)\frac{\partial}{\partial n}\frac{\partial}{\partial n_0}\ln R_0^{-1}d\sigma_0- \tag{7}$$
$$\left.-k^{-1}V(\sigma,t)+aK^{-1}\int_{\Gamma(t)}\left(V(\sigma,t)-dV(\xi,t)\right)\frac{\partial}{\partial n}\ln R^{-1}(\sigma,\xi,t)d\xi\right)^2$$

under restrictions

$$\sum_{i=1}^{M_1(t)}Q_i(t)=Q_{pl}(t), \tag{8}$$
$$0\le Q_i(t)\le Q_i^{ul}(t),\ \ \left(i=\overline{1,M_1(t)}\right). \tag{9}$$

In fig.2 the positions of frontal isosat at various time under optimal production conditions of well (in view of summetry the results of upper half-plane are cited) are given by continuous line. In this variant water inrush is observed in 1283 days after the operation begins, moreover, simultaneous approach of frontal isosat to producing wells happens. In the same figure the position of a frontal isosat under inoptimal control when the rate of the first well was $10m^3/day$ and at the second - $6m^3/day$ is given by dotted line. As calculations show, water inrush happens in 931days, but only in the second well consider a problem on two phase filtration of incompressoble fluids without regard to capillary effect in heterogeneous formation with variable permeability $K(x,y)$, $\left(\Delta\sqrt{K(x,y)}=0\right)$.

Producing and injection working at given production rate conditions wells (see fig.1) are respectively arranged in domains with minimal $\underline{s}$ and maximal $\bar{s}$ values of water - saturation. The initial distribution of formation water saturability is known.

The pressure function must satisfy the equation

$$\frac{\partial}{\partial x}\left(K(x,y)\frac{\partial P}{\partial y}\right)+\frac{\partial}{\partial y}\left(K(x,y)\frac{\partial P}{\partial y}\right)=0\,. \tag{10}$$

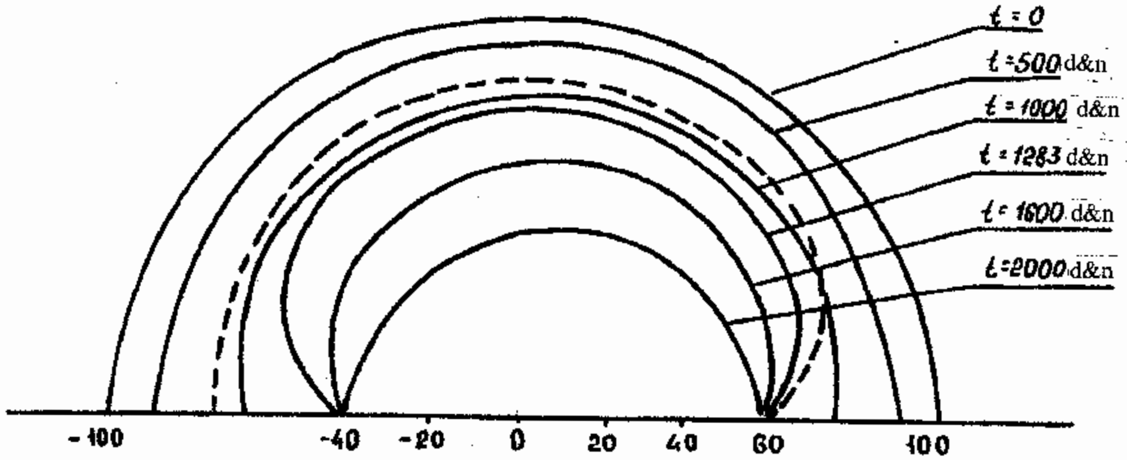

Fig.2. Position of a frontal isosat under optimal values of producing wells production rates.

On the boundary of phase division (1) the following conditions are fulfilled

$$P^+(x,y,t) = P^-(x,y,t), \tag{11}$$

$$c_j \frac{\partial P^+(x,y,t)}{\partial n} = c_{j+1} \frac{\partial P^{-1}(x,y,t)}{\partial n}. \tag{12}$$

As a method for obtaining the equation system admitting to trace the isosat motion is unique, we show its application to the problem when on the exterior reservoir boundary Γ_0 it is given the condition

$$P(x,y,t)\big|_{\Gamma_0} = P^0(x,y,t). \tag{13}$$

Then pressure function is seeked in the form

$$P(x,y,t) = \varphi(x,y,t) + \frac{1}{\sqrt{K(x,y)}} \int_{\Gamma(t)} \rho(\sigma,t) \ln R^{-1} d\sigma + \frac{1}{\sqrt{K(x,y)}} \int_{\Gamma(t)} \mu(\sigma_0,t) \frac{\partial}{\partial n_0} \ln R_0^{-1} d\sigma_0, \tag{14}$$

where

$$\varphi(x,y,t) = \left(2\pi H c_0 \sqrt{K(x,y)}\right)^{-1} \sum_{i=1}^{M_1(t)} Q_i(t)/\sqrt{K_i} \ln R_i^{-1}(t) + \left(2\pi H c_{N+1} \sqrt{K(x,y)}\right)^{-1} \sum_{i=M_1(t)+1}^{M(t)} Q_i(t)/\sqrt{K_i} \ln R_i^{-1}(t).$$

Here $c_0 = f_1(\underline{s})\mu_1^{-1}$; $c_{N+1} = f_2(\bar{s})\mu_2^{-1}$; K_i is the permeability in the bottom-hole of the i -th well.

The isosat motion tracing problem in an inhomogeneous formation in active hydractic head condition evidence is reduced to the following system of equation

$$V(\sigma,t)-a\int_{\Gamma(t)}\left(V(\sigma,t)-cV(\xi,t)\right)\frac{\partial}{\partial n}\ln R^{-1}(\sigma,\xi,t)d\xi -$$
$$-a\sqrt{K(\sigma,t)}\left(\frac{\partial}{\partial n}\frac{1}{\sqrt{K(\sigma,t)}}\right)\int_{\Gamma(t)}cV(\xi,t)\ln R^{-1}d\xi = \tag{15}$$
$$=K\frac{\partial\varphi}{\partial n}+K\frac{\partial}{\partial n}\left(\frac{1}{\sqrt{K(\sigma,t)}}\int_{\Gamma_0}\mu(\sigma_0,t)\frac{\partial}{\partial n_0}\ln R_0^{-1}d\sigma_0\right),$$

$$\mu(\sigma_0,t)=-(2\pi)^{-1}\left(\sqrt{K(\sigma_0,t)}\left(P^0(\sigma_0,t)\right)-\varphi(\sigma_0,t)\right)+$$
$$+\frac{m(c_j-c_{j+1})}{2\pi c_j c_{j+1}F_j'}\int_{\Gamma(t)}\left(V(\sigma,t)/\sqrt{K(\sigma,t)}\ln R^{-1}d\sigma -\right. \tag{16}$$
$$\left.-\int_{\Gamma(t)}\left(\mu(\sigma_0,t)-\mu(\xi_0,t)\right)\frac{\partial}{\partial n_0}\ln R_0^{-1}(\sigma_0,\xi_0,t)d\xi_0\right),$$

$$\frac{\partial x(\sigma,t)}{\partial t}=-V(\sigma,t)\frac{\partial y(\sigma,t)}{\partial\sigma}, \tag{17}$$
$$\frac{\partial y(\sigma,t)}{\partial t}=-V(\sigma,t)\frac{\partial x(\sigma,t)}{\partial\sigma}, \tag{18}$$
$$x(\sigma,0)=x^0(\sigma);\quad y(\sigma,0)=y^0(\sigma), \tag{19}$$
$$x_0=x_0(\sigma_0),\ y_0=y_0(\sigma_0), \tag{20}$$

where

$$c=c_j c_{j+1}(c_q-c_{q+1})(c_j-c_{j+1})^{-1}c_q^{-1}c_{q+1}^{-1}F_j^{-1}\sqrt{K(\sigma(t))}/F_q^{-1}\sqrt{K(\xi(t))};\ \left(q=\overline{0,N}\right)$$

$x^0(\sigma), y^0(\sigma), x_0(\sigma_0), y_0(\sigma_0)$ are given functions.

In solving the equations system (15)-(20) unlike (2)-(5) it is necessary to calculate a logarithmic potential of a simple stratum in multiphase domains (the second integral in equation (15)), for which convergent quadratic processes have been structured.

To illustrate the effectiveness of the suggested model we solve the following problem on optimal spacing of producing wells with production rate of $16m^3/day$ in a circular radius $R=100m$ in heterogeneous formation of $10m$ thickness. On a circular external contour reservoir of radius $R_0=200m$ the $100ath$ constant pressure is maintained. It is needed to space the producing well so that waterless period of well operation be remained as long time as possible. Consequently, it is necessary to minimise the functional

$$\min_{(x_1,y_1)\in D_0}\int_{\Gamma(t)}\left(V(\sigma,t)-\overline{V}(\sigma,t)\right)^2 d\sigma, \tag{21}$$

where D_0 is an oil-content domain: $V(\sigma,t)$ is isosat motion velocity and it is defined from (15)-(20);

$\overline{V}(\sigma,t)$ is the optimal velocity of isosat motion that is in backward-proportional dependence on permeability value (the higher is permeability the lower is velocity of isosat) the distance to the well.

For numerical calculation we take the following permeability function

$$K=(R\cos\Theta+2R_0)^2$$

under initial data (6) with only difference that

$$c_0 = 0.33(3);\ c_1 = 0.489265;\ c_2 = 1$$

optimal point of producing well spacing with coordinates $(-13.63m;0m)$ for 12 iterations have been found by direct search method [5]. The search of origin is carried out from point $(50m;48m)$ with initial increments $\Delta x = 2m;\ \Delta y = 3m$.

In fig.3 the frontal isosat positions in exploitation of producing well spaced in an optimal point are given by continuous lines. Here water inrush advances in the 2617 day. In spacing the well in the reservoir centre water inrush happens in 1869 days (frontal isosats position is shown by a dotted line). The second dotted line indicates the frontal isosat position after encroachment in 2617-th day (in unoptimal spacing of spacing well, i.e. in the centre of reservoir).

Another direction of works based on the investigations data is in [6,7]. Regressive models of homogeneous and heterogeneous oil formations have been obtained. Approximation algorithms of interface considering the errors of input and output coordinates have been developed. Recurrent identification algorithms of interinfluence and permeability on bottom hole pressure measuring and well production rates in the presence of measurement noise have also been developed. As calculations show, the developed methods and algorithms do not require significant computer resources for realisation that is very important in constructing permeability fields and isolines on the basis of smoothing splines and the method based on Poincare's theorem.

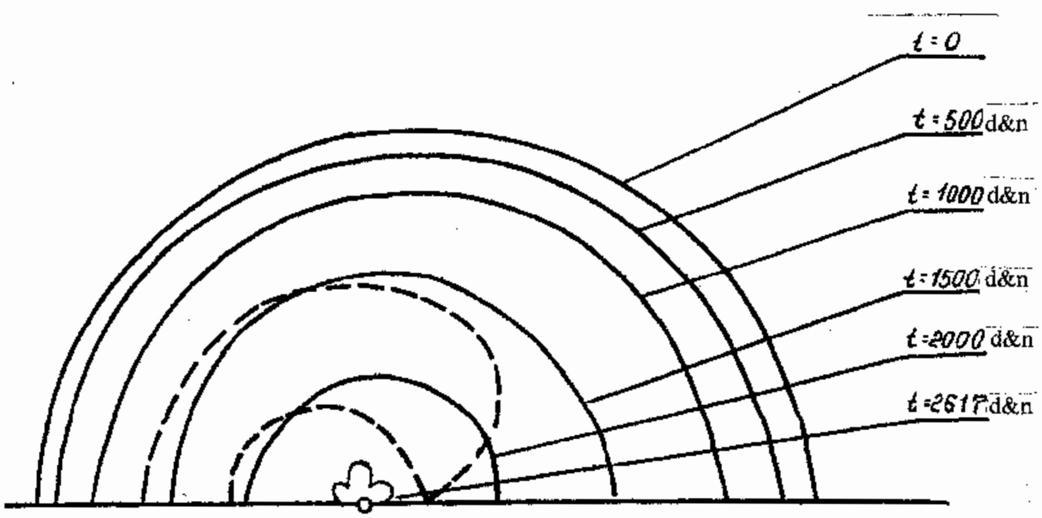

Fig.3. Frontal isosat position in optimal spacing of producing well.

Unlike the existing methods, not considering multiply connectedness in its wide sense, a complex system of multiply connected control by different sources and drainage have been developed with regard to self-acting change of the object connectedness, interference and optimal dynamics with minimal energy loss with observance of reservation, and planned-economical, industrial-technological and phase restrictions. Optimality of guaranteed result is achieved at the expense of positional strategies (feedback principle control) and suggested optimality criteria. The obtained mathematical model is easily and economically adapted in exploitation of the object (practical admissible accuracy is achieved on 4-5 data gauging).

Solution algorithms of problems are based on ideas of multicriterial optimisation methods (including the case when LPR has no sufficient experience), dynamic, nonlinear, (various search methods), quadratic (Bill's modification method) programming, MNK with restrictions and is realised in two stage. At the first stage the choice strategy of controlling variables with regard to phase and technological restrictions are defined. At the second stage the values of control parameters with regard to technological, industrial and planned restrictions are defined.

We are to note that on a successful choice of control strategy step we can speak after completing the second stage. The solution of the problem is achieved infinite number of steps. Real state of the object and its properties (by automated system) are defined at each time step, since found spacing and working conditions of sources and drainage's may not supply the given or calculated according to criteria optimal dynamics of the object (spacing of sources and drainage's is of discrete character and their amount is finite).

Developed algorithms and applied programmes package were tested in computes of IBM PC type. They are realised in FORTRAN-programmes by using modular technology and don't use library subprograms from SPO of particular computer.

The obtained results have been successfully used in automated projecting and controlling the behaviour of high temperature plasma in installations with external magnetic sources of IN TOP type, in automated designing of multisechtional acoustic linings for gondola of turboreactive engines supplying the most damping of sound in various type channels [8].

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Received January 12, 2000; Revised May 24, 2000.
Translated by Aliyeva E.T.