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DIRECT AND INVERSE PROBLEMS OF RELAXATIONAL FILTRATION

Abstract

The solution of the problem on relaxational filtration of viscoelastic fluid to the central well in a circular elastic bed, and correlations admitting to determine all parameters of the bed and fluid including relaxation times of velocity and pressure have been obtained in the paper.

The interest to direct and inverse problems for different relaxational filtration models recently has been significantly arisen. One of frequently discussed ones is also the model

$$\frac{\partial}{\partial t}\left[p+\tau_u\frac{\partial p}{\partial t}\right]=\chi\Delta\left[p+\tau_p\frac{\partial p}{\partial t}\right], \qquad (1)$$

describing in particular, isothermal filtration process of homogeneous viscoelastic fluid (further, oil) in a weakly deformable homogeneous and isotropic porous medium [1].

Here t is a time; Δ is a Laplace operator; p is a formation pressure; $\chi=\frac{k}{\mu\beta^*}$ is a coefficient of a formation piezoconductivity; k is absolute permeability of a porous medium; μ is dynamic viscosity of oil; β^* is a coefficient of formation elastic capacity; τ_u and τ_p are relaxation time of velocity and pressure respectively.

In the considered case, equation (1) is the consequence of two equations-equation of flow continuity [7]

$$\beta^*\frac{\partial p}{\partial t}+\nabla\cdot\vec{u}=0 \qquad (2)$$

and oil filtration equation (law) [1]

$$\vec{u}+\tau_u\frac{\partial\vec{u}}{\partial t}=-\frac{k}{\mu}\nabla\left[p+\tau_p\frac{\partial p}{\partial t}\right], \qquad (3)$$

where $\vec{u}$ is a filtration velocity; ∇ (nabla) is a vector differential operator.

In solving concrete problems, the passage from boundary-value problems for equations (2) and (3) to equivalent problems for equation (1) may be found nontrivial. As investigations show, in such cases it is appropriate to solve these problems with respect to equations (2), (3).

For the first time such an approach has been realized in [3], where exact solutions of one-dimensional direct problems on oil inflow to rectilinear gallery and central point drainage in closed formations under given constant production rate by means of which equivalent initial boundary value problems for equation (1) have been justified. In the given paper the results of investigations of another typical problem on filtration of oil to a central well in a circular formation under constant production rate are given. In addition, we consider the case when pressure relaxation time is more than time of velocity relaxation $\tau_p>\tau_u>0$.

The direct problem. Let a well with radius R_c, draining weakly deformable homogeneous and isotropic circular formation of radius R_k and of constant thickness h, after long work with a constant production rate Q of homogeneous viscoelastic oil at moment $t=0$ be stopped. Initial formation pressure p_0 on the exterier layer of formation is kept.

Mathematical problem is reduced to the determination on the domain $(R_c \le r \le R_k;\, 0 \le t)$ of functions $p(r,t)$ and $u(r,t)$ satisfying the equations

$$\beta^* \frac{\partial p(r,t)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left[ru(r,t)\right] = 0, \tag{4}$$

$$u(r,t) + \tau_u \frac{\partial u(r,t)}{\partial t} = -\frac{k}{\mu}\frac{\partial}{\partial r}\left[p(r,t) + \tau_p \frac{\partial p(r,t)}{\partial t}\right], \tag{5}$$

$$(R_c < r < R_k;\ 0 < t)$$

and conditions

$$p(r,0) = p_0 - \frac{\mu Q}{2\pi kh}\ln\frac{r}{R_k} \qquad (R_c \le r \le R_k), \tag{6}$$

$$u(r,0) = \frac{Q}{2\pi rh} \qquad (R_c < r \le R_k), \tag{7}$$

$$u(R_c,t) = 0; \quad p(R_k,t) = p_0 \qquad (0 \le t). \tag{8}$$

Briefly give main moments for the construction of the solution of problem (4)-(8). We seek the solution of the problem in the form of

$$p(r,t) = p_0 + \frac{\mu Q}{2\pi kh}\sum_{m=1}^{\infty} c_m(t) U_0(\lambda_m r); \quad u(r,t) = \frac{Q}{2\pi h}\sum_{m=1}^{\infty} d_m(t) U_1(\lambda_m r)$$

$$(R_c \le r \le R_k;\ 0 \le t),$$

where

$$U_0(\lambda_m r) \equiv Y_0(\lambda_m R_k) J_0(\lambda_m r) - J_0(\lambda_m R_k) Y_0(\lambda_m r);$$

$$U_1(\lambda_m r) \equiv Y_0(\lambda_m R_k) J_1(\lambda_m r) - J_0(\lambda_m R_k) Y_1(\lambda_m r);$$

J_0 and J_1 are the zero and first order Bessel's functions;

Y_0 and Y_1 are the zero and first order Neimann's functions;

λ_m is the m-th positive root of the equation $U_1(\lambda_m R_c) = 0$.

In addition, using the known expansion

$$\ln\frac{r}{R_k} = \frac{\pi}{R_c}\sum_{m=1}^{\infty}\frac{J_0(\lambda_m R_k) J_1(\lambda_m R_c) U_0(\lambda_m r)}{\lambda_m\left[J_1^2(\lambda_m R_c) - J_0^2(\lambda_m R_k)\right]} \qquad (R_c \le r \le R_k),$$

$$\frac{1}{r} = -\frac{\pi}{R_c}\sum_{m=1}^{\infty}\frac{J_0(\lambda_m R_k) J_1(\lambda_m R_c) U_1(\lambda_m r)}{J_1^2(\lambda_m R_c) - J_0^2(\lambda_m R_k)} \qquad (R_c < r \le R_k)$$

for determination $c_m(t)$ and $d_m(t)$ $(m = 1,2,\ldots)$ we get a system of ordinary differential equations

$$\dot c_m(t) = -\chi\lambda_m d_m(t) \qquad \tau_u \dot d_m(t) = \lambda_m c_m(t) - \left(1 + \tau_p \chi \lambda_m^2\right) d_m(t) \qquad (t > 0) \tag{9}$$

with initial conditions

$$c_m(0) = q_m; \quad d_m(0) = \lambda_m q_m, \tag{10}$$

where $q_m = -\dfrac{\pi J_0(\lambda_m R_k) J_1(\lambda_m R_c)}{R_c \lambda_m\left[J_1^2(\lambda_m R_c) - J_0^2(\lambda_m R_k)\right]}$.

The solution of problem (9)-(10) for $\tau_p > \tau_u > 0$ has the form

$$c_m(t)=q_m p_m(t),\ \ d_m(t)=\lambda_m q_m u_m(t)\ , \tag{11}$$

where

$$\left.\begin{aligned}
p_m(t)&\equiv\frac{b_m(1-\tau_u a_m)e^{-a_m t}-a_m(1-\tau_u b_m)e^{-b_m t}}{b_m-a_m}\\
u_m(t)&\equiv\frac{(1-\tau_u a_m)e^{-a_m t}-(1-\tau_u b_m)e^{-b_m t}}{\tau_u(b_m-a_m)}\\
a_m&=\frac{1}{2}\left(\alpha_m-\sqrt{\alpha_m^2-4\beta_m}\right);b_m=\frac{1}{2}\left(\alpha_m+\sqrt{\alpha_m^2-4\beta_m}\right);\alpha_m=\frac{1+\tau_p\chi\lambda_m^2}{\tau_u};\beta_m=\frac{\chi\lambda_m^2}{\tau_u}
\end{aligned}\right\}$$

Thus, the solution of initial problem (4)-(8) has the form

$$p(r,t)=p_0-\frac{\mu Q}{2khR_c}\sum_{m=1}^{\infty}\frac{p_m(t)J_0(\lambda_m R_k)J_1(\lambda_m R_c)U_0(\lambda_m r)}{\lambda_m\left[J_1^2(\lambda_m R_c)-J_0^2(\lambda_m R_k)\right]}, \tag{12}$$

$$u(r,t)=-\frac{Q}{2hR_c}\sum_{m=1}^{\infty}\frac{u_m(t)J_0(\lambda_m R_k)J_1(\lambda_m R_c)U_1(\lambda_m r)}{J_1^2(\lambda_m R_c)-J_0^2(\lambda_m R_k)}. \tag{13}$$

Note that, by substituting into (12) and (13) $p_m(t)$ and $u_m(t)$ to $e^{-\chi\lambda_m^2 t}$ we get corresponding solution to the problem of classic elastic filtration regime [5].

Calculating in (12) $\frac{\partial p}{\partial t}$ and formally passing to the limit for $t\to+0$, we get

$$\left.\frac{\partial p(r,t)}{\partial t}\right|_{t=+0}=\frac{Q}{2\beta^* hR_c}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m R_k)J_1(\lambda_m R_c)}{J_1^2(\lambda_m R_c)-J_0^2(\lambda_m R_k)}U_0(\lambda_m r). \tag{14}$$

The series $\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m R_k)J_1(\lambda_m R_c)}{J_1^2(\lambda_m R_c)-J_0^2(\lambda_m R_k)}U_0(\lambda_m r)$ only at a point $r=R_k$ of the segment $R_c\le r\le R_k$ converges in an ordinary sense, moreover, at this point $\left.\frac{\partial p(R_k,t)}{\partial t}\right|_{t=0}=0$ that must be by virtue of boundary condition (8). However, in the sense of convergence of generalized functions it holds

$$\delta(r-R_c)=-\frac{\pi}{2}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m R_k)J_1(\lambda_m R_c)}{J_1^2(\lambda_m R_c)-J_0^2(\lambda_m R_k)}U_0(\lambda_m r)\quad (R_c\le r\le R_k)$$

considering it (14) has the form

$$\left.\frac{\partial p(r,t)}{\partial t}\right|_{t=+0}=-\frac{Q}{\pi\beta^* hR_c}\delta(r-R_c)\quad (R_c\le r\le R_k).$$

Here δ is a singular generalized Dirac's function.

By direct verification we can be convinced that (12) is also a solution of the following problem

$$\frac{1}{\chi}\frac{\partial}{\partial t}\left[p+\tau_u\frac{\partial p}{\partial t}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(p+\tau_p\frac{\partial p}{\partial t}\right)\right](R_c<r<R_k;0<t), \tag{15}$$

$$p(r,0)=p_0-\frac{\mu Q}{2\pi kh}\ln\frac{r}{R_k};\ \left.\frac{\partial p(r,t)}{\partial t}\right|_{t=0}=-\frac{Q}{\pi\beta^* hR_c}\delta(r-R_c)\quad (R_c\le r\le R_k), \tag{16}$$

$$\frac{\partial}{\partial r}\left[p(r,t)+\tau_p\frac{\partial p(r,t)}{\partial t}\right]\Bigg|_{r=R_c}=0\,;\quad p(R_k,t)=p_0\quad (0<t). \tag{17}$$

As we see from (16), the second initial condition, i.e. $\left.\frac{\partial p}{\partial t}\right|_{t=0}$ is expressed by a singular generalized Dirac's function that is the result of instant well shut off working up to stoppage with a constant rate.

Thus, under known $p_0,\chi,\varepsilon=\frac{kh}{\mu},\tau_u,\tau_p,R_k$ and R_c by means of formulas (12) and (13) we can determine the values of pressure and velocity of filtration at any point of formation at any time. However, in practice some of these parameters, and often all of them are unknown and are subjected to preliminary determination.

Note the following easily verifiable fact: under known pressure buildup curve (PBC) in the well bottom hole, i.e. under given $p(R_c,t)$ $(t\geq 0)$, initial formation pressure p_0, coefficient of hydroconductivity of formation ε, velocity and pressure relaxation times τ_u and τ_p and relations $\frac{R_c}{R_k}$ and $\frac{\chi}{R_k^2}$ (or, that, equivalent, relations $\frac{\chi}{R_k^2}$ and $\frac{\chi}{R_c^2}$) are definable. We reduce their definition method.

The inverse problem. Let besides conditions (6)-(8), additional condition be given in the form of PBC in the well bottom hole, i.e.

$$p(R_c,t)=p_c(t)\quad (t\geq 0) \tag{18}$$

and it is required to determine the parameters $\tau_u,\tau_p,\varepsilon,\frac{R_c}{R_k}$ and $\frac{\chi}{R_k^2}$.

From (12) it follows, that $p(r,\infty)=\lim\limits_{t\to\infty}p(r,t)=p_0$ $(R_c\leq r\leq R_k)$ and therefore, supposing p_0 is known, calculate determined moments $M_n(r)$ definable by the next manner [2]

$$M_n(r)=\int_0^\infty[p_0-p(r,t)]t^n dt\quad \left(R_c\leq r\leq R_k\,;\ n=\overline{0,N}\right), \tag{19}$$

where N is some natural number.

Note that, difference $p_0-p(r,t)$ has an exponential character of attenuation for $t\to\infty$, and integrals in (19) at any final n converge sufficiently rapid.

We have from (12) and (19)

$$M_n(r)=-\frac{Q\cdot n!}{2\pi\varepsilon}\left\{(\tau_p-\tau_u)\sum_{j=0}^{n}\left[\varphi_j^{(n)}(\tau_p,\tau_u)\left(\frac{R_k^2}{\chi}\right)^j A_j\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)\right]+\right.$$
$$\left.+\left(\frac{R_k^2}{\chi}\right)^{n+1}A_{n+1}\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)\right\}, \tag{20}$$

where

$$\varphi_j^{(n)}(\tau_p,\tau_u)=\sum_{l=0}^{L}(-1)^l C_{n+1-l}^{n+1-j}C_{n-j}^{l}\tau_p^{n-j-l}\tau_u^l\,;\quad L=\min(j;n-j);$$

$$A_n\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)=\sum_{j=0}^{n}\left[a_j^{(n)}\left(\frac{R_c}{R_k}\right)+b_j^{(n)}\left(\frac{R_c}{R_k}\right)\ln\frac{r}{R_k}\right]\left(\frac{r}{R_k}\right)^{2j},$$

moreover, the coefficients $a_j^{(n)}\left(\frac{R_c}{R_k}\right)$ and $b_j^{(n)}\left(\frac{R_c}{R_k}\right)$ are defined from recurrent relations

$a_0^{(0)}=0$; $b_0^{(0)}=-1$;

$$a_0^{(n)}=\frac{1}{4}\sum_{l=0}^{n-1}\left[\frac{a_l^{(n-1)}}{(l+1)^2}-\frac{b_l^{(n-1)}}{(l+1)^3}\right];\ b_0^{(n)}=\frac{1}{4}\sum_{l=0}^{n-1}\left\{\frac{2a_l^{(n-1)}}{l+1}-\frac{b_l^{(n-1)}}{(l+1)^2}\left[1-2(l+1)\ln\frac{R_c}{R_k}\right]\right\}\left(\frac{R_c}{R_k}\right)^{2(l+1)}$$

$$a_{i+1}^{(n)}=-\frac{1}{4}\left[\frac{a_i^{(n-1)}}{(i+1)^2}-\frac{b_i^{(n-1)}}{(i+1)^3}\right];\ b_{i+1}^{(n)}=-\frac{1}{4}\frac{b_i^{(n-1)}}{(i+1)^2}\quad \left(n=\overline{1,l};\ i=\overline{0,n-1}\right).$$

Thus, for instance, the expressions for $M_0(r)$, $M_1(r)$ and $A_0\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)$, $A_1\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)$, $A_2\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)$ have the form

$$M_0(r)=-\frac{Q}{2\pi\varepsilon}\left[(\tau_p-\tau_u)A_0\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)+\frac{R_k^2}{\chi}A_1\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)\right],$$

$$M_1(r)=-\frac{Q}{2\pi\varepsilon}\left\{(\tau_p-\tau_u)\left[\tau_p A_0\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)+2\frac{R_k^2}{\chi}A_1\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)\right]+\left(\frac{R_k^2}{\chi}\right)^2A_2\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)\right\},$$

$$A_0\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)=-\ln\frac{r}{R_k},$$

$$A_1\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)=\frac{1}{4}\left[1+\left(\frac{R_c}{R_k}\right)^2\left(1-2\ln\frac{R_c}{R_k}\right)\ln\frac{r}{R_k}\right]-\frac{1}{4}\left(1-\ln\frac{r}{R_k}\right)\left(\frac{r}{R_k}\right)^2,$$

$$A_2\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)=\frac{1}{128}\left[5-8\left(\frac{R_c}{R_k}\right)^2\left(1-2\ln\frac{R_c}{R_k}\right)\right]+\frac{1}{64}\left\{8\left(\frac{R_c}{R_k}\right)^2-\left(\frac{R_c}{R_k}\right)^4\times\right.$$

$$\left.\times\left[5-4\ln\frac{R_c}{R_k}+4\left(1-2\ln\frac{R_c}{R_k}\right)^2\right]\right\}\ln\frac{r}{R_k}-\frac{1}{16}\left[1-\left(\frac{R_c}{R_k}\right)^2\left(1-2\ln\frac{R_c}{R_k}\right)+\left(\frac{R_c}{R_k}\right)^2\times\right.$$

$$\left.\times\left(1-2\ln\frac{R_c}{R_k}\right)\ln\frac{r}{R_k}\right]\left(\frac{r}{R_k}\right)^2+\frac{1}{128}\left[3-2\ln\frac{r}{R_k}\right]\left(\frac{r}{R_k}\right)^4.$$

Analogously determined moments $M_n^{(e)}(r)$ for a classic elastic filtration regime case have the form

$$M_n^{(e)}(r)=-\frac{Q\cdot n!}{2\pi\varepsilon}\left(\frac{R_k^2}{\chi}\right)^{n+1}A_{n+1}\left(\frac{r}{R_k},\frac{R_c}{R_k}\right)\qquad (R_c\le r\le R_k).$$

It is shown that by using the first five relations (20) for $r = R_c$, we can uniquely determine the unknown parameters $\tau_u, \tau_p, \varepsilon, \frac{\chi}{R_k^2}$ and $\frac{R_c}{R_k}$. Here, we give the relations admitting under known $\varepsilon, \frac{\chi}{R_k^2}$ and $\frac{R_c}{R_k}$ to determine the values of parameters τ_p and τ_u

$$\tau_p = \frac{\frac{2\pi\varepsilon}{Q} M_1(R_c) + \left(\frac{R_k^2}{\chi}\right)^2 A_2\left(\frac{R_c}{R_k}, \frac{R_c}{R_k}\right)}{\frac{2\pi\varepsilon}{Q} M_0(R_c) + \frac{R_k^2}{\chi} A_1\left(\frac{R_c}{R_k}, \frac{R_c}{R_k}\right)} + 2\frac{R_k^2}{\chi} \frac{A_1\left(\frac{R_c}{R_k}, \frac{R_c}{R_k}\right)}{\ln\left(\frac{R_c}{R_k}\right)}, \tag{21}$$

$$\tau_u = \tau_p - \frac{\frac{2\pi\varepsilon}{Q} M_0(R_c) + \frac{R_k^2}{\chi} A_1\left(\frac{R_c}{R_k}, \frac{R_c}{R_k}\right)}{\ln\left(\frac{R_c}{R_k}\right)}. \tag{22}$$

Give main results of calculations on determination of parameters τ_p and τ_u on calculation formulas (21) and (22). To carry out the calculations we use hypothetical PBC, corresponding to solution (12) of direct problem (4)-(8) under various values of parameters $p_0, \chi, \varepsilon, \tau_u, \tau_p, R_k$ and R_c. Here determined moments $M_0(R_c)$ and $M_1(R_c)$ in view of restrictedness continuities of well shut off T were calculated as

$$M_n(R_c) \equiv \int_0^T [p_c(T) - p_c(t)] t^n dt \qquad (n = 1,2). \tag{23}$$

In table 1 the values of quantities τ_p, τ_u calculated by (21) and (22), and their relative on exact values deviation for different values of T are given. These results correspond to PBC calculated under the following initial data

$Q = -10^{-3} \frac{\text{m}^3}{\text{sec}}$; $p_0 = 3 \cdot 10^7 \text{Pa}$; $\varepsilon = 2 \cdot 10^{-10} \frac{\text{m}^3}{\text{Pa} \cdot \text{sec}}$; $\chi = 0.55 \frac{\text{m}^2}{\text{sec}}$;

$R_c = 0.1$ m; $R_k = 100$ m; $\tau_p = 3600$ sec; $\tau_u = 1080$ sec.

For above-mentioned initial data integrals (23) were calculated by a quadratic formula of trapezium on an uniform frame with the step $\Delta t = 60$ sec.

Table 1.

T, hour	τ_p, sec	Deviation %	τ_u, sec	Deviation %
12	3527	2	1012	6,3
14	3564	1	1044	3,3
16	3580	0,56	1058	2

Calculation results showed that under sufficient continuity of well shut off (output of PBC to stationary) and high accuracy of numerical integration (23), it is also achieved a high accuracy for determination of values of parameters τ_p and τ_u. It is obvious that the requirements imposed on T and Δt yield from the applied determined moments method.

Note that the applied determined moments method is well approved, for instance, for a classic elastic filtration regime case [2]. In [2] this method has been *successfully* applied and for the cases $\tau_u > 0, \tau_p = 0$ and $\tau_u = 0, \tau_p > 0$. However the papers devoted to the development of definition methods of formation and fluid parameters under $\tau_u > 0, \tau_p > 0$ are unique, and present the methods admitting by means of impulsive methods of well investigation to determine absolute values of relaxation time τ_p and τ_u are absent. And the existing impulsive methods admit to determine either their relation [6], or it is assumed that between parameters τ_p and τ_u there is a known relation [4]. Filtration wave pressure method [6] admitting to determine also the values of relaxation times τ_u and τ_p differ by the complexity of carrying out field investigations.

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