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NATURAL OSCILLATIONS OF THE RIGIDLY FIXED RECTANGULAR PLATE FABRICATED FROM THE COMPOSITE MATERIAL WITH THE LOCAL CURVED STRUCTURES

Abstract

In this work the natural oscillations of the rigidly fixed plate fabricated from the composite material with the local curved structures on the axis OX_1 are considered. The Finite Elements Method is used. On the base of the obtained numerical results the influence of curvature parameters on the values of the natural frequencies of the plate is analysied.

One of the main problems in Mechanics of Composite materials is the problem caused by the properties of their structures. One of the main properties of the structure of composite materials is the curving of the armouring elements. As it was expressed in [4] the curvatures in the structure of composite materials can appear as the result of the technological processes making these materials or as the result of the constructive requires to these materials. The form of the curvatures can be local or periodic. For study of the mechanical problems of elements of the constructions made from composite materials with the above pointed curved structures in [1] the continual approach was suggested. In [2] in the frame of [1] and on the base of Hamilton-Ostrogradsky variation principle the approach was developed for investigation of natural oscillations of the elements of the constructions fabricated from composite materials with the curved structures. The concrete investigations of natural oscillations of the composite materials with the periodic curved structures using the Finite Elements Method (FEM) were carried out in [6]. In this work the analogous problem is solved in the case when the curving is local. On the base of the obtained numerical results the influence of the curvature parameters on the values of the natural frequencies of the plate is analyzied.

Formulation of the problem and method of solving. Let's consider the plate fabricated from the many-layer composite material with the periodic curved structures and suppose the plate takes the area $R=\{0\le x_1\le l_1; 0\le x_2\le h; 0\le x_3\le l_3\}$. The armouring layers are arranged in the plane OX_1X_3 and the structure parameters of the material of the plate satisfy the restrictions of the theory [1]. Moreover we suppose that the armouring layers of the plate have curving only in direction of OX_1 axis (i.e. there is not any curving of the armouring layers in direction of axis OX_3) and the curving form in the system of coordinates $OX_1X_2X_3$ is represented by the following function:

$$x_2=F(x_1)=\begin{cases}\varepsilon\lambda^3(x_1-c)^2(x_1-d)^2\exp\left(-\lambda^{2n}(x_1-l_1)^{2n}\right)\\ \times\cos\left(m\pi\lambda(x_1-l_1)\right),\ \ if\ \ x_1\in(c;d)\\ 0,\ if\ \ x_1\in[0,c]\cup[d;1]\end{cases} \qquad (1)$$

In (1) the dimensionless coordinates $\bar{x}_1=x_1/l;\ \bar{x}_2=x_2/l$ and the dimensionless parameters $c=c_1/l;\ d=d_1/l$ have been introduced and the lines above x_1,x_2 have been omitted. Moreover in (1) the following denotations $\lambda=1/(d-c)$, $\varepsilon=A/(d-c)$,

$l_1 = L/l$ have been introduced, where A shows the maximal value of rise of the local curving, L is the distance (in direction of OX_1 axis) between the origin of coordinates and the point where the amplitude of the local curving has the maximal value. Parameters n and m characterize some sides of the form of the local curving. If $\varepsilon = 0$ (i.e. there is not any curving in the structure of plate's material), taking into account that plate's material is homogeneous and orthotrop with the mechanical properties $\overset{0}{\mu}_{ij\alpha\beta}$ (with the main axes of the elastic symmetry along OX_1, OX_2, OX_3), we can write

$$\overset{0}{\mu}_{ij\alpha\beta} = \delta_i^j \delta_\alpha^\beta A_{i\beta}^\theta + \left(1-\delta_i^j\right)\left(\delta_i^\alpha \delta_j^\beta + \delta_i^\beta \delta_j^\alpha\right)G_{ij}^0;$$
$$A_{44}^0 = G_{13}^0;\; A_{55}^0 = G_{23}^0;\; A_{66}^0 = G_{12}^0 \tag{2}$$

Here the denotations from [6] are taken. Note that in the case when $\varepsilon \neq 0$, according to the continual approach [1], the material of the considered plate can be considered continuous homogenous anisotrop with the mechanical parameters $\mu_{ij\alpha\beta}(x_1)$ whose forms were given in [3].

Thus, let's write complete system of the equations for investigation of the problem.

$$\frac{\partial\sigma_{ij}}{\partial x_j} = \rho\frac{\partial^2 u_i}{\partial t^2};\; \sigma_{ij} = \mu_{ij\alpha\beta}(x_1)\varepsilon_{\alpha\beta};$$
$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right);\; i,j,\alpha,\beta = 1,2,3 \;\; x_1, x_2, x_3 \in R\,. \tag{3}$$

In (3) the conventional notation is used. Now let's formulate the boundary conditions in the frame of which we will carry out the investigation. First of all note that the boundary conditions given on upper $(x_2 = h)$ and lower $(x_2 = 0)$ surfaces of the plate will always have the following form:

$$\sigma_{22}\big|_{x_2=h} = 0,\;\; \sigma_{i2}\big|_{x_2=h} = \sigma_{23}\big|_{x_2=h} = 0,$$
$$\sigma_{22}\big|_{x_2=0} = \sigma_{12}\big|_{x_2=0} = \sigma_{23}\big|_{x_2=0} = 0\,. \tag{4}$$

The boundary conditions on the ends in the axis OX_3 will also always have the following form:

$$u_2\big|_{x_3=0;l_3} = 0,\;\; \sigma_{33}\big|_{x_3=0;l_3} = 0\,. \tag{5}$$

However the boundary conditions on the ends in the axis OX_1 for rigid fixing are represented in the form:

$$u_i\big|_{x_1=0;l_1} = 0,\;\; i = 1,2,3\,. \tag{6}$$

Therefore, investigation of natural oscillations of the considered plate is reduced to the solution of the problem (3)-(6) taking (1) into account and for solution of this problem in the frame of Hamilton-Ostrogradsky variation principle the half analytic version of FEM [7,8] is used. The displacement of the plate is represented in the form:

$$u_1 = \vartheta_1(x_1, x_2)\sin\left(\frac{\pi x_3}{l_3}\right)e^{i\omega t},$$

$$u_2 = \vartheta_2(x_1, x_2)\sin\left(\frac{\pi x_3}{l_3}\right)e^{i\omega t},$$

$$u_3 = \vartheta_3(x_1, x_2)\cos\left(\frac{\pi x_3}{l_3}\right)e^{i\omega t}. \quad (7)$$

Substituting (7) into (3)-(6) we find that all the equations (3) and the conditions (4)-(6) with respect to x_3 are satisfied automatically. After that using FEM we begin determining the functions $v_1(x_1, x_2)$ $(i = 1,2,3)$. Using the known procedures we divide the domain $\Omega = \{0 \le x_1 \le l_1;\ 0 \le x_2 \le h\}$ into finite number of elements Ω_j. Suppose, that Ω_j are rectangular quadratic elements of Lagrange family [7,8].

For find of natural oscillations we introduce the equation

$$\left(K - \omega^2 M\right)\vec{a} = 0, \quad (8)$$

where K is the matrix of rigidity; M is the matrix of mass; $\vec{a}$ is the vector with the components of displacement in nodal points. We take the values ω for which the solution of (8) with respect to vector $\vec{a}$ approximate to «infinity», as the natural values of frequencies. The expressed let study the problems of natural and forced oscillations at the same time with use of Finite Elements Method. Thus, we begin to analyse the numerical results relating to the natural oscillations of the plate.

Numerical results. The concrete numerical results have been obtained in the case when the material of the plate consists of the alternating layers of two isotrop homogeneous materials with the elastic properties E_1, E_2 (Young's modulus) and ν_1, ν_2 (Poisson's coefficients). We suppose that $\nu_1 = \nu_2 = 0{,}3$ and the mechanical properties $\overset{0}{A}_{ij}$ in (2) are determined through E_1, E_2, ν_1, ν_2 by the well-known formulas. It is supposed that $\eta_1 = \eta_2 = 0{,}5;\ n = 1$ here η_1 and η_2 show the concentrations of the matrix and the filler correspondingly in the considered material.

Thus, consider the data given in Table 1 where the values of ω_*^2 and Ω_*^2 for the first modes for different h / l_1 are represented for the case when $E^{(2)} / E^{(1)} = 50;\ \gamma = 1;$ $c = 0{,}3;\ d = 0{,}7;\ l_0 = 0{,}5$. It is seen from the data of the table 1 that for all h / l_1 existence of the local curving in the structure of the material of the plate generally reduces to decrease of frequencies of natural oscillations. With increase of h / l_1 the influence of the local curving to the natural frequencies is getting less

Table 1

h/l_1	ε	ω_*^2		
		MOD I	MOD II	MOD III
0.10	0.0	2.62	8.79	20.16
	0.5	2.58	8.76	19.96
0.15	0.0	3.45	10.44	22.84
	0.5	3.43	10.46	22.80
0.20	0.0	3.96	11.35	24.26
	0.5	3.95	11.42	24.32

The influence of change of relation $E^{(2)} / E^{(1)}$ on the frequencies is shown in Table 2. The values of the parameters of the problem which have not been given in the table, were chosen as they were chosen for calculation of data given in Table I. Thus, from the numerical results in Table 2 in follows that with increase of $E^{(2)} / E^{(1)}$ the values ω_*^2 and Ω_*^2 grow monotonely. The influence of the existence of the local curving in the structure on the value of natural frequencies with increase of $E^{(2)} / E^{(1)}$ grows. It should be noted that the analogous results were obtained for the corresponding two-dimensional problems [4].

Table 2

$E^{(2)} / E^{(1)}$	ε	ω_*^2		
		MOD I	MOD II	MOD III
1		0.49	2.43	7.16
10	0.0	1.17	4.99	13.07
	0.1	1.15	4.98	12.97
20	0.0	1.71	6.61	16.28
	0.1	1.69	6.59	16.12
50	0.0	2.62	8.79	20.16
	0.1	2.58	8.76	19.96
100	0.0	3.32	10.20	22.48
	0.1	3.28	10.19	22.37

Consider the numerical results given in Table 3 which were obtained for $E^{(2)} / E^{(1)} = 50$; $h / l_1 = 0{,}1$; $\gamma = 1$; $m = 2$; $\varepsilon = 0{,}5$; $l_0 = 0{,}5$ for different c and d. The difference $(d - c)$ characterize the length of the segment in the direction of the axis OX_1 on which the local curving in structure is. From the mechanics point of view it should that with decrease of the difference $(d - c)$ the influence of the existence of the local curving in the structure of the material of the plate on the values of natural frequencies must get weaker. The expressed is confirmed by the data given in Table 3.

Table 3

$(c;d)$	ω_*^2		
	MOD I	MOD II	MOD III
(0.30;0.70)	2.58	8.76	19.96
(0.35;0.65)	2.58	8.79	19.90
(0.40;0.60)	2.60	8.84	19.95

Finally, consider the influence of change of parameter m, which shows the oscillation character of the local curving, on the values of natural frequencies given in Table 4 where we supposed that $E^{(2)} / E^{(1)} = 50$; $h / l_1 = 0{,}1$; $\gamma = 1$; $l_0 = 0{,}5$; $\varepsilon = 0{,}5$; $c = 0{,}3$; $d = 0{,}7$.

From the numerical results shown in Table 4 it follows that the growth of parameter m generally reduces to the increase of values of natural to the increase of values of natural frequencies.

Table 4

m	ω_*^2		
	MOD I	MOD II	MOD III
0	2.62	8.79	20.16
1	2.60	8.76	20.29
2	2.58	8.76	19.96
3	2.54	8.75	19.60
5	2.48	8.84	19.15
7	2.48	9.20	19.62

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