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NON-CLASSIC MECHANICAL-MATHEMATICAL MODELS OF GEODYNAMICS

Abstract

It is shown that by means of mechanical-mathematical theoretical models one can solve a series of complicated problems of structural geology, geophysics and other sections of the science on Earth. In this context, the bases of non-classic linearized base theoretical model are stated, the statement and solution of some standard problems of geodynamics are reduced. In particular, the solutions of problems on folding in sedimentary basins the problems on propagation of elastic waves in stress anisotropic laminar media in the form of small and large determinations are given.

Introduction. Various problems of geodynamics sch as motion of tectonics plates, formation of folding and their systems in the Earth's depths, formation of geological faults, sinking of basins seismic investigation methods of depths and etc. have been sufficiently theoretically developed due to the integration of possibilities of mathematics, physics, cybernetics, mechanics, geology and geophysics.

The equations and main relations of theoretical models of above said problems are essentially nonlinear. In this connection, their linearization is carried out by various ways. Linearization is processes in small vicinities from initial (natural) undeformed state is one of widely used methods. In this connection, principally, it is made an attempt to study the considered very complicated natural phenomena and processes by means of the methods of linear mechanics and linear theory of partial differential equations with constant coefficients [1]. In scientific references such an approach has received the name «linear theoretic base model» (LTBM) [1,11]. In [1,2,4] theoretic justification of LTBM application for the investigation of different problems of geodynamics has been analyzed in detail, and it is concluded that to investigate these problems, for today, non-classically linearized theoretical base model (NLTBM) is the most justified. The essence of this problem is that its main equations and relations are obtained by means of linearizasion of the main initial nonlinear equations and relations at the small vicinity of the considered actual states, and in spite of the linearity of equations and relations, they describe principally other state. Besides, the coefficients of these equations principally are variable and contain information described by LTBM. Namely these cases allow to investigate the influence of various geodynamic factors on formation of kinematic and dynamic characteristics of different physical field such as stress, strain, elastic waves and etc.

1. Main systems of equations, relations and boundary conditions of NCTBM.

Any natural and artificial process in limits of NCIBM is considered in the form of two qualitatively unlike states - unperturbed (which consists of natural and initial state) and perturbed one. It is considered process, or even if their methods of definition are known. Note that in this connection the quantities of these parameters may be arbitrary, and the problem on their definition - nonlinear. Similar quantities and parameters (including new ones) at perturbed state are unknown, and desired parameters in comparison with non-pertubed ones are sufficiently small (but not infinitely small), that admits to carry out linearization of a differential equation with regard to perturbations, as coefficients contain the parameters of linear and nonlinear physico-mechanical properties of the medium, force and geometric parameters of stress-strain state. At the initial state we can distinguish two stages: natural (underformed) and initially deformed. In this connection, we can consider the problems with small linear and nonlinear and large nonlinear deformations, correlate all measurable quantities per unit area of natural or initially-deformed states. In a unique form for all above mentioned variants of deformation theory in the case of compressible models of medium, the main equation systems of dynamical problems of non-classical linearized theory in a Langrangian coordinate system x_i have the form [3]:

$$\frac{\partial}{\partial x_i}\left(\omega_{ij\alpha\beta}\frac{\partial u_j}{\partial x_j}\right)=\rho\frac{\partial^2 u_j}{\partial t^2},\ \ i,j,\alpha,\beta=\overline{1,4}. \tag{1}$$

Boundary conditions at stresses

$$N_i\omega_{ij\alpha\beta}\frac{\partial u_\alpha}{\partial x_\beta}\bigg|_{S_1}=P_j\,. \tag{2}$$

Boundary conditions at displacements

$$u_j\big|_{S_2}=f_j. \tag{3}$$

Here, u_j are the perturbation vector components, P_j are the external forces vector components applied at perturbed state, referred to the unit of the initial deformed state; ρ is the density of a material, $\omega_{ij\alpha\beta}$ are the fourth rank tensor components characterizing linear, nonlinear physico-mechanical, rheological properties of the medium and the initial various elastic potentials and for the theory of small and large deformations; S_1 and S_2 are the sections of the surface of the considered body; f_j is a given value of perturbations u_j; N_i are the constituents of basis vector of the external normal. In the case of statical problems, at the right hand side of equation (1) the inertional term is omitted. Analogous equations are written in the case of incompressible media with additional conditions of incompressibility.

In (1)-(3) when stresses are absent, formally adopting that u_a are displacements of the main boundary conditions of corresponding classic theorems.

2. Geodynamical aspects.

In above enumerated problems a geodynamic aspect is characterized by the specificity of realization of deformation process in different geological media, geological conditions under the action of various force factors of endogeneous and exogeneous nature. Deformation capability of different rocks forms various litolographic and stratigraphic groups plays an important role. In this connection, in geodynamic problems it is necessary to model deformation laws of rocks. Here by the analogy of deformable solid mechanics, it is assumed that with sufficient accuracy degree these laws may be established by means of elastic potentials for compressible and incompressible media. In limits of such an approach with regard to different stages and degree of deformation the quantities $\omega_{ij\alpha\beta}$ in the case of compressible media are defined by the following relations:

In the case of large initial (final) deformations

$$\omega_{ij\alpha\beta} = \lambda_j \lambda_\alpha \left[\delta_{ij}\delta_{\alpha\beta}A_{i\beta} + (1-\delta_{ij})(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha})\mu_{ij}\right] + \delta_{i\beta}\delta_{j\alpha}S^0_{\beta\beta}, \quad (4)$$

in the case of the first variant of small initial deformations theory (shifts and lengthening are small in comparison with the unit)

$$\omega_{ij\alpha\beta} = \lambda_j \lambda_\alpha \left[\delta_{ij}\delta_{\alpha\beta}A_{i\beta} + (1-\delta_{ij})(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha})\mu_{ij}\right] + \delta_{i\beta}\delta_{j\alpha}\sigma^0_{\beta\beta}, \quad (5)$$

for the second version of small initial deformations in addition to the first variant of small initial deformations it is considered that the components of stress and strain tensors are subjected to the Hook's law

$$\omega_{ij\alpha\beta} = \delta_{ij}\delta_{\alpha\beta}A_{i\beta} + (1-\delta_{ij})(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha})\mu_{ij} + \delta_{i\beta}\delta_{j\alpha}\sigma^0_{\beta\beta}. \quad (6)$$

In formulas (4)-(6) λ_i $(i=1,2,3)$ are the lengthening coordinates along the coordinate axis x_i; δ_{ij} are Kronecker's symbols; $S^0_{\beta\beta}$ are the covariant constituents of Lagrange stress tensor reffered to the base initial state vectors; $\sigma^0_{\beta\beta}$ are the components of the ordinary stress tensor. The quantities $A_{i\beta}, \mu_{ij}$ and $S^0_{\beta\beta}$ (or $\sigma^0_{\beta\beta}$) are found for each of variants (4)-(6) definitely [3]. In particular, in the case of hyperbolic isotropic materials, i.e. under the assumption on the existence of an elastic potential Φ^0, they are defined from

$$A_{i\beta} = \left(\Sigma_{ii}\Sigma_{\beta\beta} + 2\delta_{i\beta}B_{ii}\right)\Phi^0;\ \mu_{ij} = B_{ij}\Phi^0;\ S^0_{\beta\beta} = \Sigma_{\beta\beta}\Phi^0;\ \Phi^0 = \Phi\left(A^0_1, A^0_2, A^0_3\right), \quad (7)$$

where the following differential expressions

$$\Sigma_{ii} = \frac{\partial}{\partial A^0_1} + 2\varepsilon^0_{ii}\frac{\partial}{\partial A^0_2} + 3\left(\varepsilon^0_{ii}\right)^2\frac{\partial}{\partial A^0_3};\ B_{ij} = \frac{\partial}{\partial A^0_2} + \frac{3}{2}\left(\varepsilon^0_{ii} + \varepsilon^0_{jj}\right)\frac{\partial}{\partial A^0_3}. \quad (8)$$

are introduced.

In formulas (7), (8) $A^0_i(i=1,2,3)$ are algebraic invariants of Green's deformation tensor at initial state, that are defined in scopes of small and large deformations theory differently.

In the paper, by analogy with deformable solid mechanics [3,12] to investigate the seismic problems in connection with above mentioned problems it is suggested to use the following type elastic potential

$$\Phi^0 = \frac{1}{2}C_{ijnm}\varepsilon^0_{ij}\varepsilon^0_{mn} + \frac{a}{3}\left(A^0_1\right)^3 + bA^0_1A^0_2 + \frac{c}{3}A^0_3,\ \ i,j,n,m = 1,2,3. \quad (9)$$

Here a,b,c are elasticity modules of the third order isotropic *approximation of a* nonlinear part of anisotropic medium deformation.

The another important moment is the stimulation of action capability, acceptance and transfer of efforts of different layers of the rock in deformation process. By analogy of deformable solid mechanics here we assume that force factors may be divided into two groups: the first «dead» (conservative) forces, (in deformation process they preserve their sizes and directions of initial effect); the second - «tracking» (non-conservative) loadings (in this case the above mentioned conditions are violated). Here, by investigating concrete problems, starting from really existing state, force factors are stimulated either in the form of «dead» or «tracking» loadings. In addition, to the considered deformable systems these force system may affect separately, or may exist in various combinations. These details at each specific problem are investigated specially and justified.

3. Geodynamics of gelatin processes.

One of the most interesting problems of geodynamics is related to the formation of different geological structures of various degree. First of all, we distinguish process of folding, fault formation and tectonic stratification in the crust of Earth.

Numerical investigations of gelation problems in lithosphere on the base of NLTBM have been carried out [1,2,7-10,13]. In oil and gas bearing capability problems, the gelatin problems in sedimentary basins is of great interest. In [10] it is shown that in sedimentation process, in the basin horizontal gradients of effective pressure and density, directed form the center to periphery are formed, i.e. a basin actually generates inner horizontal contrastive forces, independent directly on the influence of exterior forces. The size of such stresses may be sufficient to develop folding.

In particular, for

$$\left(-\sigma_{11}^{0}\right)\approx\frac{P_{эл}}{1+k-2nk}\left\{1-\frac{1}{30A_{22}G_{12}}\left\{2\left(A_{11}A_{22}-A_{12}^{2}+3A_{12}G_{12}\right)+\right.\right.$$
$$+\frac{5}{(1+k-2nk)A_{22}}\left\{2G_{12}\left\{A_{22}\left(A_{22}-A_{12}\right)+kA_{12}\left[A_{12}-A_{22}(1-4n)\right]\right\}+\right. \qquad (10)$$
$$\left.\left.\left.+2\left(1-k\frac{1-n^{2}k}{1+k-2nk}\right)A_{22}\left(A_{11}A_{22}-A_{12}^{2}\right)\right\}\right\}\right\};\quad P_{эл}=\frac{\chi^{2}}{3}\frac{A_{11}A_{22}-A_{12}^{2}}{A_{22}}$$

the stability of the equilibrium of initially plane form of a layer on a bending form or on a form with neck formation happens and it passes to more stable curved equilibrium form with respect to the level and form of acting force system. Here n is a number parameter, that adopts the value $n=0$ in the case of $x_2=\pm h$ «dead» and $n=1$ - «tracking» surface loading; k is a number parameter characterizing the quantity of stress ratio acting in vertical and horizontal directions.

The arising of horizontal contracting stresses and of the problem on linear folding in rapidly sinking basin have been considered in [10], and it was shown that at the expense of the growth of sinking sedimentary basin layers in its structure, the realization of autonomous folding is possible.

4. On the influence of geodynamical factors on seismic waves.

Kinematic characteristics of seismic waves are determined by means of solutions of characteristic equations corresponding to system (1), that have the form [5,6,14]

$$2\rho V_{\alpha}^{2}=\omega_{3333}+\omega_{3113}+\left(\omega_{1111}+\omega_{1331}-\omega_{3333}-\omega_{3113}\right)\sin^{2}\theta_{\alpha}\pm Q_{\alpha};$$
$$Q_{\alpha}=\left\{\left[\left(\omega_{1111}-\omega_{1331}\right)\sin^{2}\theta_{\alpha}-\left(\omega_{3333}-\omega_{3113}\right)\cos^{2}\theta_{\alpha}\right]^{2}+\right. \qquad (11)$$
$$\left.+4\left(\omega_{1133}+\omega_{3131}\right)^{2}\sin^{2}\theta_{\alpha}\cos^{2}\theta_{\alpha}\right\}^{\frac{1}{2}}.$$

In expression (11) the sign «+» before the summand Q corresponds to the quasilongitudinal P wave and here $V_\alpha=V_p$; and the sign «-» - to the quasitransverse SH waves propagating in direction $V_\alpha=V_{SV}$.

For transverse SH waves propagating in direction $(\sin\theta;0;\cos\theta)$

$$\rho V_{SH}^{2}-\omega_{1221}\sin^{2}\theta-\omega_{3223}\cos^{2}\theta=0. \qquad (12)$$

For anisotropic media formulas for $A_{i\beta}, S^0_{i\beta}$ and μ^0_{ij} have somewhat other form [3].

The values of mean velocities of elastic waves is one of the main kinematic characteristics used in seismic survey. Unlike the isotropic case in anisotropic media even in single layer medium there arises necessity to define the values of mean velocities of waves. In the case of linear deformable transversally - isotropic media they have been studied by L.Tomson [5]. First of all, the values of mean velocities of waves in seismic survey, mainly are used in determination of sizes of normal kinematic shears (NMO), and therefore, usually they are marked by the index NMO. In the given paper the values of mean velocities of longitudinal and transverse waves in stress anisotropic media in the case of small and large nonlinear deformations are determined in a general form for arbitrary elastic potentials of contracting models of the form:

$$V_{NMO}(P)=\frac{1}{2}\left\{\rho\left[2\omega_{3333}-\omega_{1331}-\frac{(\omega_{1133}+\omega_{3131})^2}{\omega_{3333}-\omega_{3113}}\right]\right\}^{-\frac{1}{2}},$$

$$V_{NMO}(SV)=\frac{1}{2}\left\{\rho\left[2\omega_{3113}-\omega_{1111}+\frac{(\omega_{1133}+\omega_{3131})^2}{\omega_{3333}-\omega_{3113}}\right]\right\}^{-\frac{1}{2}}, \qquad (13)$$

$$V_{NMO}(SH)=\frac{1}{2}\left[\rho(2\omega_{3223}-\omega_{1221})\right]^{-\frac{1}{2}}.$$

Dynamic characteristics of seismic waves are determined from the system of Septrits type equations that are defined from the continuity condition of displacement perturbations and stresses at the boundary of different media section. In particular, at the boundary of the plane $x_3 = const$ these conditions have the form:

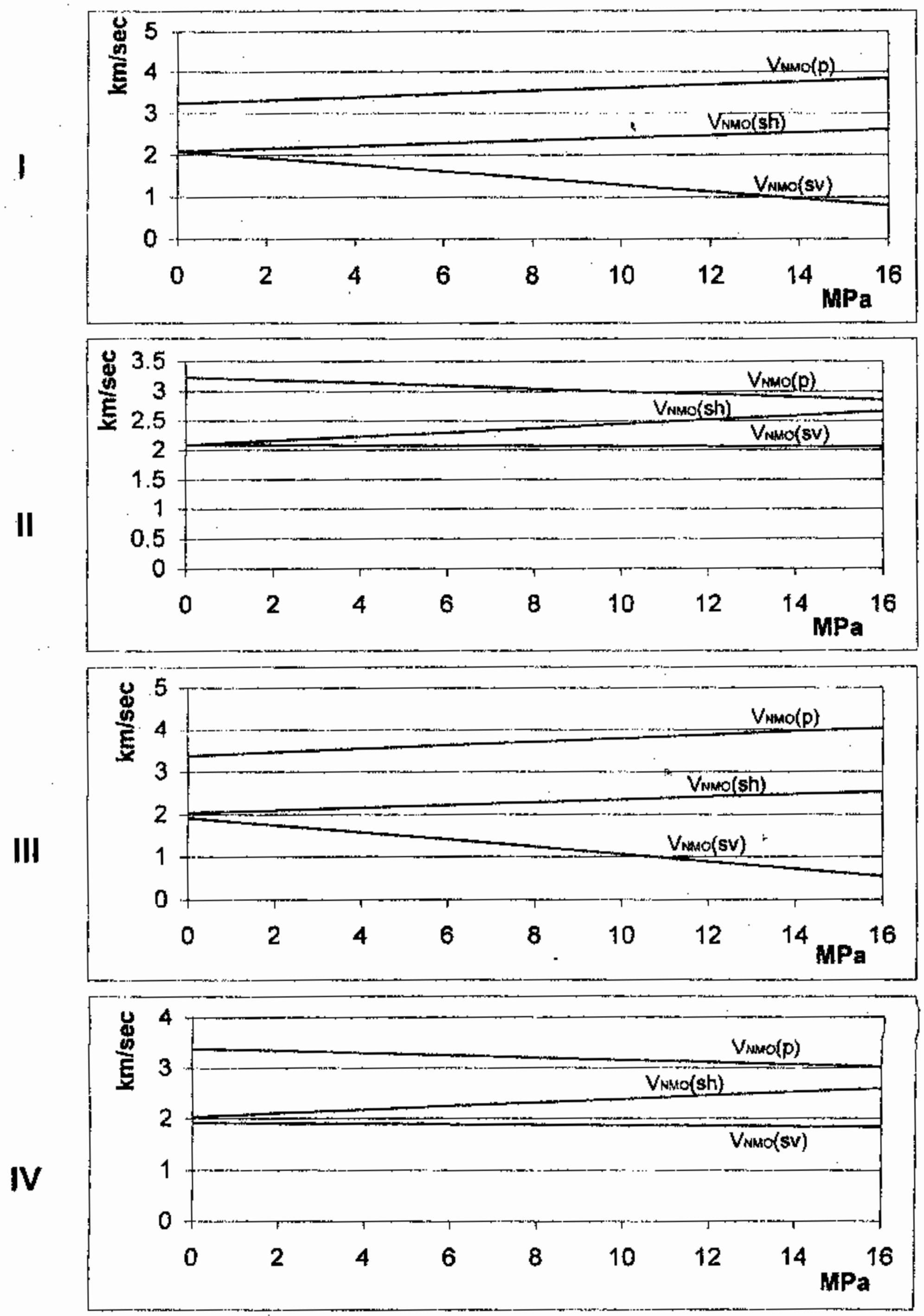
$$u_1^{(1)}=u_1^{(2)};\ u_3^{(1)}=u_3^{(2)};$$

$$\omega^{(1)}_{3113}\frac{\partial u_1^{(1)}}{\partial x_3}+\omega^{(1)}_{3131}\frac{\partial u_3^{(1)}}{\partial x_1}=\omega^{(2)}_{3113}\frac{\partial u_1^{(2)}}{\partial x_3}+\omega^{(2)}_{3131}\frac{\partial u_3^{(2)}}{\partial x_1}; \qquad (14)$$

$$\omega^{(1)}_{1133}\frac{\partial u_1^{(1)}}{\partial x_1}+\omega^{(1)}_{3333}\frac{\partial u_3^{(1)}}{\partial x_3}=\omega^{(2)}_{1133}\frac{\partial u_1^{(2)}}{\partial x_1}+\omega^{(2)}_{3333}\frac{\partial u_3^{(2)}}{\partial x_3}.$$

Here, upper indices 1 and 2 show that the indicated values belong either to the first (upper), or to the second (lower) medium.

Using formulas (14) in transversally - isotropic media case, observing the accuracy with respect to anisotropic parameters and stresses, for mountain rock Sandstone Berea 2000 from Mexico gulf numerical calculations whose results are at the figure, have been carried out. Dependencies of alternation of the mean velocity values of quasilongitudinal SV and SH waves in isotropic (graphs 1 and 2) and transversally - isotropic (graphs III and IV) stress media are reduced. Graphs I and III correspond to comprehensive compression, and graphs II and IV - to the case of single-axis compression along the vertical axis oz.



Theoretical dependence of velocities on pressure
(the sandstone Berea 2000, Mexico gulf)

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