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**SOLUTION OF THE SPACE ELASTIC-PLASTIC PROBLEM
ON THE STRESS-STRAIN CONDITION IN THE BODY
WITH THE SPHERIC SPACE**

Abstract

The three-dimensional deformable body slackened by the spheric space is considered supposing that the body is subjected to stresses on rather big distance from the bound of the space.

The problem on equilibrium of the symmetric loaded body of rotation is considered as the problem on deformation in the meridional plane when the unknown sought quantities are the components of displacements and stresses.

The effectivity of the suggested method and possibility of its application for the solution of more complex problems in the three-dimensional formulation are proved.

Let's consider the three-dimensional deformable body slackened by the spheric space and assume that on rather big distance from the bound of the space the body is subjected to the tension which is characterized by the general stress condition $\sigma_z^{\infty} = p$. For investigation of distribution of stress in the three-dimensional body (non-elastic medium) which has the space with the radius $\rho = \rho_0 \left(\rho = \frac{r}{r_0} \right)$ we can use the system of the nonlinear solvable equations written in displacements, it is necessary to have their representation in the form of product of some functions each of which satisfies Laplace's equation.

Let's note that the problem on the equilibrium of the symmetric loaded body of rotation can be divided into two independent problems [6]: 1) the problem torsion dealt with determination of the displacement of u_{φ} by the boundary-valued conditions given on this displacement or on the expressed only by it (when u_{ρ} and u_{θ} don't depend on φ) stresses $\tau_{\rho\varphi}, \tau_{z\varphi}$; 2) the problem on deformation in the meridional plane when the sought values are the components of displacement u_{ρ}, u_{θ} stresses $\sigma_{\rho}, \sigma_z, \sigma_{\varphi}, \tau_{z\rho}$.

Further we will consider the second problem for the symmetric loaded sphere. It is natural to introduce in the meridian plane the polar coordinates (ρ, θ) , then the coordinates ρ, θ, φ will be spheric coordinates of the point.

Let's reduce the formulas giving the expressions of deformations of the volume expansion in the spheric coordinates, when the values $\varepsilon_{\rho}, \varepsilon_{\theta}$ and ε_{φ} don't depend on φ . In this case we have [2]

$$\begin{aligned} \varepsilon_{\rho} &= \frac{\partial u_{\rho}}{\partial \rho}, \quad \varepsilon_{\theta} = \frac{1}{r_0 \rho} \frac{\partial u_{\theta}}{\partial \theta} + \frac{1}{\rho} u_{\rho}; \\ \varepsilon_{\varphi} &= \frac{1}{r_0 \rho} (u_{\theta} \cos \theta + u_{\rho}); \\ \gamma_{\rho\theta} &= \frac{1}{r_0 \rho} \frac{\partial u_{\theta}}{\partial \theta} + r_0 \rho \frac{\partial}{\partial \rho} \left(\frac{u_{\theta}}{r_0 \rho} \right) \end{aligned} \quad (1)$$

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and the volume expansion is determined by the formula

$$\Delta = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} (\rho^2 u_\rho) + \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta). \quad (2)$$

As for as the volume spheric harmonic function not depending on φ is represented in the form of one of the products of the form

$$\rho^\Pi \rho_\Pi(\mu), \quad \frac{1}{\rho^{\Pi+1}} \rho_\Pi(\mu), \quad (\mu = \cos \theta),$$

where Π receives entire values ($\Pi = 0, 1, 2, \dots$) and $\rho_\Pi(\mu)$ is Legendr polinom of the Π -th order. Each of these functions satisfies Laplace's equation; the first of them remains finite and continuous (with its derivatives by ρ and θ) in some sphere (for $\rho \leq \rho_0$), and the second is out of such sphere (for $\rho > \rho_0$) and turns into zero for $\rho \rightarrow \infty$.

1. The main dependencies and the solvable equations for the three-dimensional body for the elastic-plastic deformations.

We will consider further the problem on the consideration of the elastic-plastic deformations on the distribution of stresses in the neighbourhood of the spheric space in the three-dimensional body under the action of the axial-symmetric loading in the infinity. For the solution of the elastic-plastic problem we use the principle of all-possible displacements [1,4]. The problem is reduced to the solution of the system of the non-linear algebraic equations.

In order to obtain the non-linear system of equations let's use the equations of continuity of deformations in the spheric coordinates. They have the form [2]

$$\begin{aligned} \frac{\partial^2 \varepsilon_\rho}{\partial \theta^2} - r_0 \rho \frac{\partial \varepsilon_\rho}{\partial \rho} - 2 \frac{\partial^2 (r_0 \rho \varepsilon_{\rho\theta})}{\partial \rho \partial \theta} + \frac{\partial}{\partial \rho} \left(r_0^2 \rho^2 \frac{\partial \varepsilon_\theta}{\partial \rho} \right) &= 0, \\ \frac{\partial}{\partial \theta} \left(\sin^2 \theta \frac{\partial \varepsilon_\varphi}{\partial \theta} \right) + r_0 \rho \sin^2 \theta \frac{\partial \varepsilon_\varphi}{\partial \rho} - 2 \sin \theta \frac{\partial}{\partial \theta} (\sin \theta \varepsilon_{\rho\theta}) + \\ + \frac{\sin^2 \theta}{r_0 \rho} \frac{\partial}{\partial \rho} (r_0^2 \rho^2 \varepsilon_\theta) - \sin \theta \cos \theta \frac{\partial \varepsilon_\theta}{\partial \theta} - 2 \sin^2 \theta \varepsilon_\rho &= 0, \\ \frac{\partial}{\partial \rho} \left(r_0^2 \rho^2 \frac{\partial \varepsilon_\varphi}{\partial \rho} \right) - r_0 \rho \frac{\partial \varepsilon_\rho}{\partial \rho} - 2 \operatorname{ctg} \theta \frac{\partial (r_0 \rho \varepsilon_{\rho\theta})}{\partial \rho} + \operatorname{ctg} \theta \frac{\partial \varepsilon_\rho}{\partial \theta} &= 0, \\ r_0 \rho \sin \theta \frac{\partial^2 (\sin \theta \varepsilon_\varphi)}{\partial \rho \partial \theta} - \sin^2 \theta \frac{\partial \varepsilon_\rho}{\partial \theta} - r_0 \rho \sin \theta \cos \theta \frac{\partial \varepsilon_\theta}{\partial \rho} &= 0. \end{aligned} \quad (3)$$

The components of the tensor of deformations $\varepsilon_\rho, \varepsilon_\theta, \varepsilon_\varphi, \varepsilon_{\rho\theta}$ are expressed in the spheric coordinates by the components of tensor of displacements u_ρ, u_θ by (1). We suppose that in all area the loading process happens when the non-elastic deformations arise in the body. In this case the dependence between the stresses and deformations can be represented in the form [3,4]

$$\sigma_\rho = \frac{\nu E}{(1+\nu)(1-2\nu)} (\varepsilon_\rho + \varepsilon_\theta + \varepsilon_\varphi) + \frac{E}{1+\nu} \varepsilon_\rho - \frac{E \omega(l_i)}{3(1+\nu)} (2\varepsilon_r - \varepsilon_\theta - \varepsilon_\varphi);$$

$$\begin{aligned}\sigma_{\theta} &= \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_{\rho} + \varepsilon_{\theta} + \varepsilon_{\varphi}) + \frac{E}{1+\nu}\varepsilon_{\theta} - \frac{E\omega(l_i)}{3(1+\nu)}(2\varepsilon_{\theta} - \varepsilon_{\rho} - \varepsilon_{\varphi}), \\ \sigma_{\varphi} &= \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_{\rho} + \varepsilon_{\theta} + \varepsilon_{\varphi}) + \frac{E}{1+\nu}\varepsilon_{\varphi} - \frac{E\omega(l_i)}{3(1+\nu)}(2\varepsilon_{\varphi} - \varepsilon_{\rho} - \varepsilon_{\theta}), \\ \tau_{\rho\theta} &= \frac{E}{2(1+\nu)}\varepsilon_{\rho\theta} - \frac{E\omega(l_i)}{2(1+\nu)}\varepsilon_{\rho\theta},\end{aligned}\quad (4)$$

where $\sigma_{\rho}, \sigma_{\theta}, \sigma_{\varphi}, \tau_{\rho\theta}, \varepsilon_{\theta}, \varepsilon_{\varphi}, \varepsilon_{\rho\theta}$ are correspondingly the components of the tensor of stresses and the tensor of deformations; l_i is the intensity of the deformation of shear, $\omega(l_i)$ is A.A.Ilyushin's function.

The equations of equilibrium written in stress without the volume forces have the form [2]

$$\begin{aligned}\frac{\partial\sigma_{\rho}}{\partial\rho} + \frac{1}{r_0\rho}\frac{\partial\sigma_{\rho\theta}}{\partial\theta} + \frac{2\sigma_{\rho} - \sigma_{\varphi}\sigma_{\theta} + \text{ctg}\theta\sigma_{\rho\theta}}{r_0\rho} &= 0, \\ \frac{\partial\sigma_{\rho\theta}}{\partial\rho} + \frac{1}{r_0\rho}\frac{\partial\sigma_{\theta}}{\partial\theta} + \frac{(\sigma_{\theta} - \sigma_{\varphi})\text{ctg}\theta + 3\sigma_{\rho\theta}}{\rho r_0} &= 0.\end{aligned}\quad (5)$$

Putting the physical dependencies (4) into the equation of equilibrium (5), taking into account that the components of deformations are expressed by the displacements by (1) we will obtain the system of two nonlinear differential equations with respect to u_{ρ} and u_{θ} :

$$L_i(u_{\rho}, u_{\theta}) = 0 \quad (i = 1, 2). \quad (6)$$

The expressions $L_1(u_{\rho}, u_{\theta}), L_2(u_{\rho}, u_{\theta})$ because of the big forms are not reduced here.

A.A.Ilyushin's function of plasticity $\omega(l_i)$ coming into (4) and also into the solvable equations (6) is determined as in [3,5]. Represent it in the spheric coordinates in the following form [5]

$$\omega(l_i) = \frac{2a_2}{9} \left[(\varepsilon_{\rho} - \varepsilon_{\theta})^2 + (\varepsilon_{\theta} - \varepsilon_{\varphi})^2 + \frac{3}{2}\varepsilon_{\rho\theta}^2 \right], \quad (7)$$

where the undetermined a_2 is determined on the base of the experimental data on the hardening of the material [4].

If we consider (1) then for Ilyushin's function the displacements we will have expression:

$$\begin{aligned}\omega(l_i) &= \frac{2a_2}{9} \left[2\left(\frac{\partial u_{\rho}}{\partial\rho}\right)^2 + \frac{2}{\rho^2 r_0^2} \left(\frac{\partial u_{\theta}}{\partial\theta}\right)^2 + \frac{2}{r_0\rho^2} u_{\rho}^2 - \frac{2}{\rho r_0} \times \right. \\ &\times \frac{\partial u_{\rho}}{\partial\rho} \frac{\partial u_{\theta}}{\partial\theta} - \frac{4}{r_0\rho} u_{\rho} \frac{\partial u_{\rho}}{\partial\rho} + \frac{2}{\rho^2 r_0^2} u_{\rho} \frac{\partial u_{\theta}}{\partial\theta} - \frac{2\text{ctg}\theta}{\rho r_0} \times \\ &\times u_{\theta} \frac{\partial u_{\theta}}{\partial\theta} \left. \left(\frac{2\text{ctg}^2\theta}{r_0^2\rho^2} + \frac{8}{3r_0^2\rho^2} \right) u^2 + \frac{2\text{ctg}\theta}{\rho^2 r_0^2} u_{\rho} u_{\theta} - \frac{2\text{ctg}\theta}{\rho r_0} \times \right.\end{aligned}$$

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$$\begin{aligned} & \times \frac{\partial u_\rho}{\partial \rho} + \frac{8}{3} \left(\frac{\partial u_\theta}{\partial \rho} \right)^2 + \frac{8}{3\rho^2 r_0^2} \left(\frac{\partial u_\rho}{\partial \theta} \right)^2 + \\ & + \left. \frac{16}{\rho r_0} \frac{\partial u_\theta}{\partial \rho} \frac{\partial u_\rho}{\partial \theta} - \frac{16}{3r_0 \rho} u_\theta \frac{\partial u_\theta}{\partial \rho} - \frac{16}{3r_0 \rho} u_\theta \frac{\partial u_\rho}{\partial \theta} \right]. \end{aligned} \quad (8)$$

Let's note that the right-hand sides as of the function $\omega(l_i)$ itself as its derivatives come into (6) which are calculated by (8).

Therefore, the system (6) represents the system of the non-linear differential equations with respect to the displacements u_ρ and u_θ and it is used for the solution of some boundary valued problems for the boundary conditions on the contour of the space.

2. Distribution of stresses in the neighbourhood of the spheric space for the non-elastic deformations.

Let's investigate the elastic-plastic stress-strain condition the elastic-plastic stress-strain condition in the space outside of the spheric space for the non-linear law of elasticity (4). It's taken that for $\rho \rightarrow \infty$ the conditions are fulfilled

$$\begin{aligned} \sigma_\rho^\infty &= \sigma_z^\infty \cos^2 \theta = P \cos^2 \theta; & \sigma_\theta^\infty &= P \sin^2 \theta; \\ \tau_{\rho\theta}^\infty &= -P \sin \theta \cos \theta; & \sigma_\varphi^\infty &= 0. \end{aligned} \quad (9)$$

Let's note that on the surface of the space ($\rho = \rho_0$) the stresses σ_ρ and $\tau_{\rho\theta}$ must be absent. So it should put onto the solution (9) the stress condition of leading from $\sigma(\theta)$ and $\tau(\theta)$ given in the form of series.

In this case they have (for the external problem $\rho \geq \rho_0$) the form:

$$\begin{aligned} \sigma_\theta &= -P^2 \cos^2 \theta = -\frac{1}{3} P - \frac{2}{3} \rho_2(\mu); \\ \tau(\theta) &= -P \sin \theta \cos \theta = -\frac{P}{3} \frac{d\rho_2(\mu)}{d\theta}. \end{aligned} \quad (10)$$

Taking that the general stress condition is only elastic and the plastic deformations appear as the result of the additional stresses caused with the existence of the space, then the displacement must be represented in the form $u_\rho = u_\rho^0 + u_\rho^*$, $u_\theta = u_\theta^0 + u_\theta^*$ where for the general stress condition we have the formulas:

$$\begin{aligned} u_\rho^0 &= \frac{P}{2E} [1 - \nu + (1 + \nu) \cos 2\theta] r_0 \rho; \\ u_\theta^0 &= -\frac{P}{2E} (1 + \nu) \sin 2\theta r_0 \rho \end{aligned} \quad (11)$$

and the displacements corresponding to the disturbed condition in the neighbourhood of the space, we represent in the form:

$$\begin{aligned} u_\rho^* &= \left(\frac{1}{\rho^2} u_{02} + \frac{1}{\rho^4} u_{04} \right) + \left(\frac{1}{\rho^2} u_{22} + \frac{1}{\rho^4} u_{24} \right) \rho_2(\mu), \\ u_\theta^* &= \left(\frac{1}{\rho^2} v_{22} + \frac{1}{\rho^4} v_{24} \right) \frac{d\rho_2(\mu)}{d\theta}. \end{aligned} \quad (12)$$

Here $\rho_2(\mu)$ is Legendr polinom of the second order; $u_{02}, u_{04}, u_{22}, u_{24}, v_{22}, v_{24}$ are arbitrary unknown constants.

We will consider that the bound of the space is fixed rigidly; then the solution of the system (6) should satisfy these boundary conditions:

$$u_{\rho}^0 + u_{\rho}^* \Big|_{\rho=1} = 0; \quad u_{\theta}^0 + u_{\theta}^* \Big|_{\rho=1} = 0. \quad (13)$$

Satisfying the boundary conditions (13) the number of arbitrary constants becomes less and for the components of displacements we obtain the approximating formulas:

$$u_{\rho} = \left(\frac{1}{\rho^2} - \frac{1}{\rho^4} \right) [u_{02} + u_{22} \rho_2(\mu)] - \frac{P}{E} [1 - \nu + (1 + \nu) \cos 2\theta] \frac{1}{\rho^4} + \frac{P}{E} \times \\ \times [1 - \nu + (1 + \nu) \cos 2\theta] r_0 \rho, \\ u_{\theta} = \left[\left(\frac{1}{\rho^2} - \frac{1}{\rho^4} \right) v_{22} - \frac{P(1 + \nu)}{3E} \frac{1}{\rho^4} \right] \frac{dP_2(\mu)}{d\theta} - \frac{P(1 - \nu) \sin 2\theta}{2E} r_0 \rho. \quad (14)$$

Substituting (14) into (6) we obtain the system of equations which we represent in the short form:

$$L_j(\rho, \theta) = L_j^0(\rho, \theta) - a_2 L_j^*(\rho, \theta), \quad (j = 1, 2) \quad (15)$$

or in the detailed form:

$$L_1(\rho, \theta) = \sum_{i=4,6} A_i^0 \left(\frac{1}{\rho} \right)' - \sum_{i=4,6} B_i^0 \left(\frac{1}{\rho} \right)' \cos 2\theta - a_2 \left[\sum_{i=4}^{16} A_i^* \left(\frac{1}{\rho} \right)' + \sum_{i=4}^{16} B_i^* \left(\frac{1}{\rho} \right)' \times \right. \\ \left. \times \cos 2\theta + \sum_{i=4}^{16} D_i^* \left(\frac{1}{\rho} \right)' \cos 4\theta + \sum_{i=4}^{16} E_i^* \left(\frac{1}{\rho} \right)' \cos 6\theta \right]; \\ L_2 = \sum_{i=4,6} M_i^0 \left(\frac{1}{\rho} \right)' \sin 2\theta + a_2 \left[\sum_{i=4}^{16} M_i^* \left(\frac{1}{\rho} \right)' \sin 2\theta + \right. \\ \left. + \sum_{i=4}^{16} N_i^* \left(\frac{1}{\rho} \right)' \sin 4\theta + \sum_{i=4}^{16} F_i^* \left(\frac{1}{\rho} \right)' \sin 6\theta \right]. \quad (16)$$

Here $A_i^0(\rho)$, $B_i^0(\rho)$, $M_i^0(\rho)$, $A_i^*(\rho)$, $B_i^*(\rho)$, $D_i^*(\rho)$, $E_i^*(\rho)$, $M_i^*(\rho)$, $N_i^*(\rho)$, F_i^* are the linear and non-linear functions with respect to u_{02} , u_{22} , v_{22} .

For determination of arbitrary constants we use the variational equation [1,5].

$$\int_0^{2\pi} \int_0^{\pi} \int_{r_0}^{\infty} [L_1(\rho, \theta) \delta u_{\rho} + L_2(\rho, \theta) \delta u_{\theta}] r_0^2 \pi^2 \sin \theta d\theta d\rho = 0, \quad (17)$$

where according to (14) for the variations δu_{ρ} and δu_{θ} we have the formulas:

$$\delta u_{\rho} = r_0 \left(\frac{1}{\rho^2} - \frac{1}{\rho^4} \right) \delta u_{02} + \frac{r_0 \rho}{4} \left(\frac{1}{\rho^2} - \frac{1}{\rho^4} \right) \delta u_{22} + \frac{3}{4} r_0 \rho \left(\frac{1}{\rho^2} - \frac{1}{\rho^4} \right) \delta u_{22} \cos 2\theta; \\ \delta u_{\theta} = \frac{3}{2} r_0 \left(\frac{1}{\rho^2} - \frac{1}{\rho^4} \right) \delta v_{22} \sin 2\theta. \quad (18)$$

Substituting (18) into (17) and equating the coefficients of the obtained equation to zero for the variations of arbitrary constants and after integration by ρ and θ we come

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to the system of nonlinear algebraic equations with respect to three unknown constants of the following form:

$$\begin{aligned} \frac{8(1-\nu)}{7(1-2\nu)} u_{02} - \frac{8P(1-\nu)}{21E} &= a_2 \left(2A^* - \frac{2}{3} B^* - \frac{2}{15} D^* - \frac{2}{35} E^* \right) \\ \frac{8(1-\nu)}{21(1-2\nu)} u_{02} + \frac{32(7-8\nu)}{525(1-2\nu)} v_{22} - \frac{4(56-97\nu)}{525(1-2\nu)} u_{22} - \frac{4P(6+19\nu-17\nu^2)}{105E(1-2\nu)} &= \\ &= a_2 \left(\frac{2}{3} A^* - \frac{4}{15} B^* + \frac{38}{105} D^* + \frac{26}{315} E^* \right); \\ \left(\frac{38\nu}{105(1-2\nu)} + \frac{16}{75} \right) u_{22} - \left(\frac{192\nu}{175(1-2\nu)} + \frac{16}{15} \right) v_{22} + \frac{32P\nu(1+\nu)}{105E(1-2\nu)} &= \\ &= a_2 \left(\frac{16}{15} M^* - \frac{32}{105} N^* - \frac{16}{315} F^* \right). \end{aligned} \quad (19)$$

The solution of the formulated problem has been obtained according to the expressed in [5] with use of ECM (IBM) for the solution of the system of the non-linear equations (19) and finding the components of the displacements, deformations and stresses in each of the sequential approximations including the zeroth approximation, too; in the last case we come to the solution of the system of the linear algebraic equations. As the result we obtain the solution of the nonlinear problem in the first approximation. Using the method of the elastic solutions by analogy the other approximations are constructed. The numerical results of the solution of the elastic-plastic problems have been obtained in the fourth approximation (the difference of the two last approximations is not more than 2%).

As a numerical example let's consider the stress condition and determination of the coefficient of the concentration near the space in the material AME-6M with the characteristics $E = 0,67 \cdot 10^{11}$ Па; $\nu = 0,3 \div 0,5$; $a_2 = 0,78 \cdot 10^{12}$ Па under action of the loading $P = P_0 \cdot 10^5$ Па, applied for from the concentration area. The solution of this problem in the non-linear formulation has been obtained for the values of loading $P_0 = 5; 10; \dots, 20$ (for the case of linear problems for $P_0 = 1$) and is represented in the tables:

Table I

θ	0	30	45	60	90
(6)	-0,75	0,047	0,64	1,36	2,07
the aprox. solution	-0,72	0,051	0,68	1,34	2,12

Table II

P	5	10	15	20
θ				
0	-0,73	-0,72	-0,69	-0,66
30	0,044	0,041	0,037	0,034
45	0,62	0,53	0,47	0,41
60	1,32	1,27	1,23	1,19
90	1,94	1,91	1,89	1,83

In the tables the values of the coefficients of concentration of maximal values of stresses ($K = \frac{\sigma_{\theta}}{P}$), which are on the contour of the space ($\rho = \rho_0$).

Their change also the contour is given in several characteristic points ($\theta = 0^0, 30^0, 45^0, 60^0, 90^0$) for the linear (Table I) and nonlinear problems (table 2). For estimation of the results of the solution of the problems in the linear formulation by the suggested method in Table I the theoretic data have been reduced which were obtained by Lourie A.I. [6] for the exact solution of the considered problem.

The analysis of the obtained numerical data shows that the difference of the approximated and exact solutions for the linear problems is not more than 2,5%. It gives a reason to consider that the suggested method is effective and can be used for the solution of more complex problems in the three-dimensional formulation. Comparison of the results of the solutions of the non-elastic and elastic plastic problems let find out the influence of taking into account the non-linear (elastic-plastic) properties of the material on the stress condition in the concentration tone. It was obtained that in this example of calculation of elastic-plastic deformations reduces to the decrease of maximal values of stresses for (11-16)% in dependence on the value of the loading.

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