MECHANICS

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BUCKLING OF THE MULTI-LAYER RING AT CREEPING TAKING INTO ACCOUNT THE INSTANT NON-LINEAR-ELASTIC DEFORMATION

Abstract

In this work the buckling process of the non-homogeneous by thickness of the thin-walled cylindrical shell is investigated for nonlinear creep taking into account the instant non-linear-elastic deformation. It is supposed that the shell has a rather big length in comparison with the sizes of the cross-section so that we can neglect the influence of the boundary conditions. The problem is reduced to the investigation of the buckling of the ring with the unit width chosen from this shell.

Development of modern technics is based more and more on achievements of fundamental and applied scientific investigations. The engineering constructions are complexed, so it is impossible to perform their projection without beforehand detail calculation of behavior of these constructions and their elements in those or other conditions. This fact is naturally connected with formation of new and preceing of well-known solution methods. In different constructions as carrying elements the cylindric shells are widely used. Analysis of their behavior under action of the external uniformly distributed pressure for high temperature generally is connected with determination of the time during which they can carry the given loading. Thereupon in this work the buckling process of the non-homogeneous by thickness of this walled cylindric shell at non-linear creep taking into account the instant non-linear elastic deformation. It is considered that the shell has a rather big length in comparison with the sizes of the cross-section, so as the influence of the boundary value condition can be neglected. Therefore, the problem is reduced to the investigation of buckling of the ring of unique width chosen from this shell.

1. Let assume that the circle ring with R and thickness $2h$ consists of s alternating different by thickness coaxial layer with different modulus of elasticity E_{k+1} and limits of proportionality σ^0_{k+1} $[k = 0,1,2,\ldots,s-1]$. We consider that in each layer they depend on longitudinal coordinate z. Denote the thickness of each layer by δ_k, so as $\sum_{k=1}^{s}\delta_k = 2h$. The contact conditions between the layers of the packet are in their rigid cohesion. From that the equality of displacements, stresses on them and absence of the interpressure of the layer follow. Further we will guided by the hypothesis of plain sections within which the above formulated conditions are fulfilled automatically. We take the non-linear law of instant deformation in the form

$$\varepsilon_y = \frac{\sigma}{E_{k+1}(z)}\left\{1 + \left(\frac{\sigma}{\sigma^0_{k+1}(z)}\right)^n\right\}, \quad (n = 0,2,4,\ldots)$$

or after differentiation by time t

$$\dot{\varepsilon}_y=\frac{\dot{\sigma}}{E_{k+1}(z)}\left\{1+(n+1)\left(\frac{\sigma}{\sigma^0_{k+1}(z)}\right)^n\right\},\quad (n=0,2,4,\ldots).$$

Here n is exponent of the non-linearity degree of the material reciving even values. Let write the velocity of the creepage as [1]:

$$\dot{p}=B_{k+1}(z)\sigma^m,\quad (m=1,2,3,\ldots),$$

where B_{k+1} and m are correspondingly the coefficient and the exponent of the creepage. The given law describes sufficiently well the time properties of the material for established creepage. For these assumptions, for the packet in the whole we can write

$$\dot{\varepsilon}=\dot{\varepsilon}_y+\dot{p},\quad -h+\sum_{j=0}^{k}\delta_j\le z\le -h+\sum_{j=0}^{k+1}\delta_j,\quad (\delta_0=0).$$

Let consider now the non-linear behavior of the chosen ring, which is under action of the contour of the compressing pressure q. For solution of the formulated problem we use the Sanders, Mc. Comb , Schletchte variation principle [2]. Then assuming that at the deformation process $q=const$, we write the used functional in the form (the common taken denotations are used everywhere):

$$J=R\int_{-h}^{h}\int_{0}^{2\pi}\left\{\dot{\sigma}\dot{\varepsilon}+\frac{\sigma}{2R^2}\left(\frac{\partial\dot{w}}{\partial\varphi}\right)^2\right\}d\varphi dz-$$

$$-\frac{R}{2}\int_0^{2\pi}\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}\left[\frac{\dot{\sigma}_2}{E_{k+1}(z)}\left\{1+(n+1)\left(\frac{\sigma}{\sigma^0_{k+1}(z)}\right)^n\right\}\right]dzd\varphi-R\int_0^{2\pi}\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}B_{k+1}(z)\dot{\sigma}\sigma^n dzd\varphi \tag{1.1}$$

where $(n=0,2,4,\ldots)$.

In (1.1) for the short form the denotations have been introduced:

$$a_k=-h+\sum_{j=0}^{k}\delta_j,\quad a_{k+1}=-h+\sum_{j=0}^{k+1}\delta_j=a_k+\delta_{k+1}.$$

By the hypothesis of plain sections we write

$$\varepsilon=\varepsilon_0+\chi z.$$

For the case when the tangential displacement can be neglected and considering the inequality $w/R<<1$ valid the quantity ε_0 and curving χ are determined by the formulas of theory of thin shells [3]

$$\varepsilon_0=\frac{wR}{}+\frac{1}{2R^2}\left(\frac{\partial w}{\partial\varphi}\right)^2,\quad \chi=\frac{1}{R^2}\frac{\partial^2 w}{\partial\varphi^2}. \tag{1.2}$$

By virtue of thin-wall we give the linear law of stress distribution along the thickness

$$\sigma=-\frac{qR}{2h}+\frac{3}{2h^3}Mz. \tag{1.3}$$

For finding of the stationary value of functional (1.1) we use the method by Ritts and the approximating functions are given in the form:

$$w=\omega_0(t)+\omega_1(t)\cos\ell\varphi,\quad M=\mu(t)\cos\ell\varphi. \tag{1.4}$$

We take $\ell=2$ that corresponds to buckling process of the ring in the form of "eight". The further step of calculations is in that (1.2)-(1.4) are substituted into (1.1) and the functional J is found as the function ω_0,ω_1,μ and derivatives of these parameters by time. Further J is variated by $\dot{\omega}_0,\dot{\omega}_1$ and $\dot{\mu}$, at which result the system of two equations is obtained (absent of the third equation is connected with that it is satisfied identically) and combination them for condition $\mu(0)=0$, we will have

$$\dot{w}_1\left\{\frac{4}{9}-\frac{9qR^2}{4h^6}\Phi_2-\frac{n+1}{2^{n+1}h^{n+6}}q^{n+1}R^{n+2}\sum_{j=0}^{n}\frac{3^{j+2}\Phi^{v}_{j+2}C_n^j}{h^{2j}}w_1^j\,\frac{1\cdot 3\cdot\ldots\cdot j+1}{2\cdot 4\cdot\ldots\cdot j+2}\right\}-$$

$$-\frac{q^m R^{m+1}}{2^m h^{m+3}}\sum_{i=1}^{m}\frac{3^{i+1}\Phi^{\Pi}_{i+1}(-1)^{n-i}C_m^i}{h^{2i}}w_1\,\frac{1\cdot 3\cdot\ldots\cdot i}{2\cdot 4\cdot\ldots\cdot i+1}, \quad (1.5)$$

where $C_n^j=\dfrac{n!}{j!(n-j)!}$, $j=0,2,4,\ldots$ and $i=1,3,5,\ldots$.

In (1.5) the following denotations have been introduced

$$\Phi_2=\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}\frac{z^2dz}{E_{k+1}(z)},\ \Phi^{v}_{j+2}=\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}\frac{z^{j+2}dz}{E_{k+1}(z)\left[\sigma^0_{k+1}(z)\right]^{j}},\ \Phi^{\Pi}_{i+1}=\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}B_{k+1}(z)z^{i+1}dz. \quad (1.6)$$

Passing in (1.5) to the dimensionless quantities

$$\frac{\omega_1}{h}=a,\quad \frac{q}{E_1}=\omega,\quad \frac{R}{h}=\xi,\quad tE_1^3B_1=\tau,\quad \frac{9E_1}{4h^3}\Phi_2=\varphi_2,\quad \frac{27}{16h^3B_1}\Phi_2^{\Pi}=\varphi_2^{\Pi},$$

$$\frac{729}{64h^5B_1}\Phi_4^{\Pi}=\varphi_4^{\Pi},\quad \frac{27E_1^3}{16h^3}\Phi_2^{v}=\varphi_2^{v},\quad \frac{729E_1^3}{64h^5}\Phi_4^{v}=\varphi_4^{v}$$

and reading off the corresponding conclusions, we give this equation for the case $n=2$ and $m=3$

$$\frac{d\tau}{da}=\frac{3}{a}\frac{4-\xi^3\omega\varphi_2-\xi^5\omega^3\left(\varphi_2^{v}+\varphi_4^{v}a^2\right)}{\xi^5\omega^3\left(3\varphi_2^{\Pi}+\varphi_4^{\Gamma}a^2\right)}. \quad (1.7)$$

The initial condition for this equation we write as:

$$a(0)=a_0+a_{00}. \quad (1.8)$$

The quantity a_0 represents the given amplitude of the initial imperfect, and a_{00} is the supplementary deflection arising immediately after application of loading. Since the instant deformation is non-linear elastic then for determination of a_{00} it is natural to use the variation method [4] giving the some suppose distribution of stress, deflection and the moment, that is, on representing σ, w and M, by formulas (1.3) and (1.4). However here the corresponding quantities depend on q and we will understand as the point the differentiation by this parameter. Now the functional [4] has in our case the form

$$J=R\int_{-h}^{h}\int_{0}^{2\pi}\left\{\dot\sigma\dot\varepsilon+\frac{\sigma}{2R^2}\left(\frac{\partial\dot w}{\partial\varphi}\right)^2\right\}dzd\varphi-$$

$$-\frac{R}{2}\int_0^{2\pi}\sum_{k=0}^{s-1}\int_{a_k}^{a_{k+1}}\left[\frac{\dot\sigma^2}{E_{k+1}(z)}\left\{1+3\left(\frac{\sigma}{\sigma^0_{k+1}(z)}\right)^2\right\}\right]dzd\varphi+R\int_0^{2\pi}\dot w d\varphi.$$

Calculating this functional and making variation it by $\dot\omega_0, \dot\omega_1$ and $\dot\mu$ after the well-known procedure we come to the following differential equation

$$\frac{d\omega}{da}=\frac{1}{a}\frac{4-\xi^3\omega\varphi_2-\xi^5\omega^3\left(\varphi_2^{v}-\varphi_4^{v}a^2\right)}{\xi^3\varphi_2+\xi^5\omega^2\left(3\varphi_2^{v}+\varphi_4^{v}a^2\right)},$$

which it is necessary to integrate for the initial condition $a(0)=a_0$ in order to determine the critic force ω_{kp} which is determined from the condition $d\omega/da=0$. Here it is quite important, so as consideration of the buckling problem at creepage has a when the acting loading ω is less that the critic loading, that is $\omega < \omega_{kp}$. Supposing that on this restriction the solution of the non-linear elastic problem is known (it will be given numerically below for the concrete view of multi-layer), integrating (1.7) for condition (1.8), we find

$$\tau = \ln\left|\left(\frac{a}{a_0 + a_{00}}\right)^{\frac{\eta_1}{\eta_3}} \left\{\frac{\eta_3 + \eta_4 (a_0 + a_{00})^2}{\eta_3 + \eta_4 a^2}\right\}^{\frac{1}{2}\left(\frac{\eta_2}{\eta_4} + \frac{\eta_1}{\eta_3}\right)}\right|, \qquad (1.9)$$

where $\eta_1 = 12 - 3\xi^3 \omega \varphi_2 - 3\xi^5 \omega^3 \varphi_2^v$, $\eta_2 = 3\xi^5 \omega^3 \varphi_4^v$, $\eta_3 = 3\xi^5 \omega^3 \varphi_2^\Pi$, $\eta_4 = 3\xi^5 \omega^3 \varphi_4^\Pi$ and the value of the critic time must be determined from the condition $d\tau/da = 0$. For conformity of integration the following equation has been used

$$\frac{\eta_1 - \eta_2 a^2}{a(\eta_3 + \eta_4 a^2)} = \frac{\eta_1}{\eta_3 a} - \left(\eta_2 + \frac{\eta_1 \eta_4}{\eta_3}\right) \frac{a}{\eta_3 + \eta_4 a^2}.$$

2. Let us take that each layer is homogeneous and as the example let us consider the case of the three-layer ring $(s = 3)$ with the equal thicknesses. Then calculation of integrals (1.6) is great simplified and we can write them in the dimensionless denotations so:

$$\varphi_2 = \frac{1}{18}(26 + \alpha), \quad \varphi_2^v = \frac{1}{24}\lambda^2 (26 + \alpha\beta^2), \quad \varphi_4^v = \frac{3}{160}\lambda^2 (242 + \alpha\beta^2),$$

$$\varphi_2^\Pi = \frac{1}{24}(26 + \gamma), \quad \varphi_4^\Pi = \frac{3}{160}(242 + \gamma),$$

where $\alpha = E_1 E_2^{-1}$, $\beta = \sigma_1^0 \sigma_2^{0^{-1}}$, $\gamma = B_2 B_1^{-1}$, $\lambda = E_1 \sigma_1^{0^{-1}}$.

Table

$\omega \cdot 10^{-5}$	α			
	0,25	1,25	2,25	3,25
3,023	a=0,112	a=0,113	a=0,114	a=0,115
3,895	a=0,120	a=0,121	a=0,123	a=0,124
4,883	a=0,132	a=0,135	a=0,137	a=0,138
6,045	a=0,161	a=0,166	a=0,172	a=0,177
7,028	a=0,229	a=0,247	a=0,267	a=0,297

In the table the numerical solution of the non-linear elastic problem $a(0)$ corresponding to the different values of $\omega < \omega_{kp}$.

In fig. 1 the dependence of the critic time on α is given and in fig.2 the dependence of the critic time on ω is given. Therefore, for the chosen values of the parameters can make the following conclusions:

- with increase of α the critic time of buckling decreases;
- as it should expect with increase of ω τ_{kp} decreases.

In conclusion let us note that by constructing of non-homogeneity one can increase (decrease) the critic time of buckling and by that in some sense to optimize the construction.

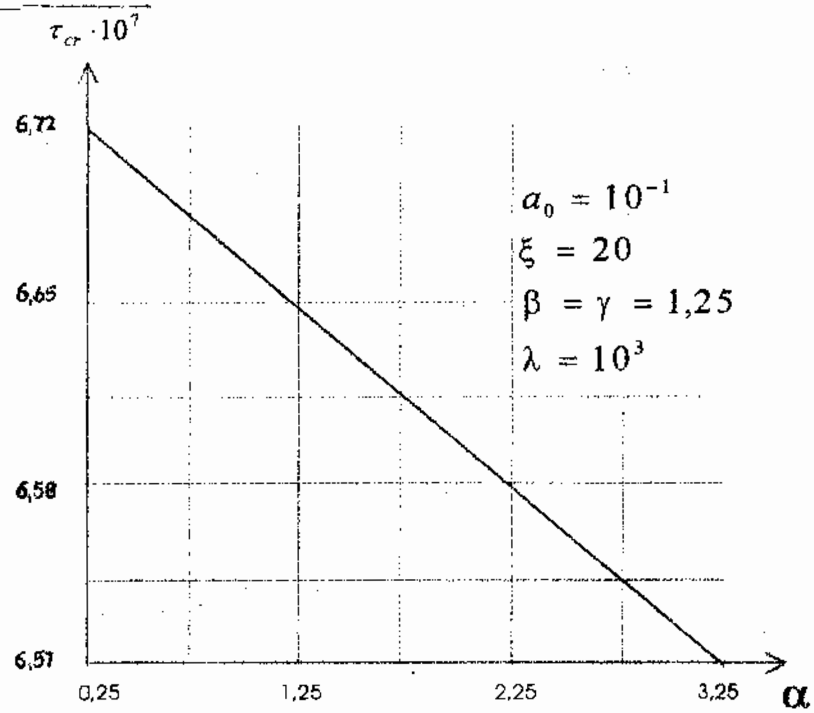

Fig. 1. Dependence of the critic time τ_{cr} on α.

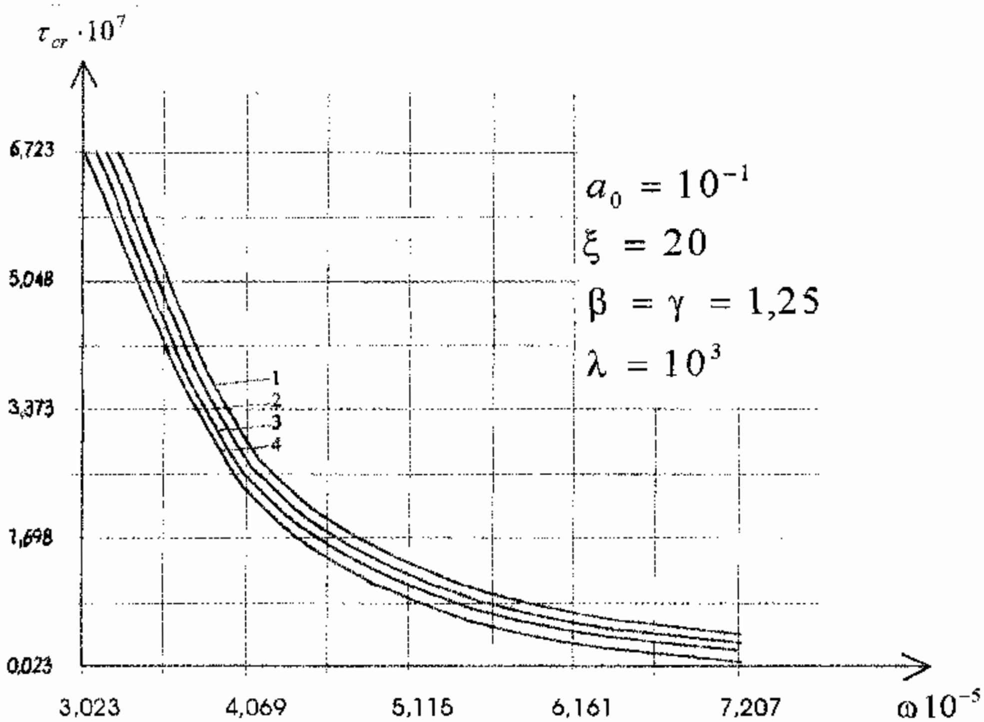

Fig. 2. Dependence of the critic time τ_{cr} on ω for $\alpha = 0,25(1)$, $\alpha = 1,25(2)$, $\alpha = 2,25(3)$, $\alpha = 3,25(4)$.

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