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**FUNDAMENTAL RESULTS AND DEVELOPMENT PERSPECTIVES OF  
MATHEMATICS AND MECHANICS IN AZERBAIJAN**

The XX century will remain in mankind's memory very contradictory in many respects. It was a century of two world and a great number of local wars, a century of creation and crash of totalitarian regime, and at the same time- a century of great democratic reforms. It was a century of space exploration, and obedience of atom energy to mankind, a television and computer century, a century of scientific- technical progress, to which undoubtedly has promoted unprecedented development of scientific investigations. Undoubtedly, along with other scientists from other countries Azerbaijan scientists also have made their contribution to this progress.

Azerbaijan being a part of the high- developed Soviet Science always has differed by its high level of investigations and has made essential contribution to the development of the world science. Today, on the threshold of the XXI century I want to recall the names of outstanding scholars playing an essential role in development and formation of our science. They are M.V. Keldysh, A.N. Bogolyunov, S.N. Bernstein, A.N. Tikhonov, N.I. Muskhelishvili, I.G. Petrovsky, M.A. Lavrentyev, Ya.B. Lopatinskii, O.A. Ladyzhenskaya, O.A. Oleynik, Z.I. Khalilov, Kh.A. Rakhmatulin, A.I. Huseynov, I.I. Ibrahimov, M.A. Rasulov, A.Kh. Mirzajanzadeh, M.G.Gasymov, A. Amenzade, K.A. Kerimov. Today the representatives of the Azerbaijan science successfully work in many scientific centers of the USA, Germany, Russia, Turkey, Iran, and Middle Asia. The results of our scientists are contained in fundamental monographs and manuals and they are known far from our country. Naturally, in a paper it is impossible to stop on all scientific directions represented in Azerbaijan, because it is the theme of separate fundamental work. Here we characterize only the results and development perspectives of mathematical sciences, namely, mathematics and mechanics.

The main trends of the fundamental research in the field of mathematics represented in Azerbaijan are the functional analysis with its different applications, the theory of differential equations and the mathematical physics, the theory of functions and harmonic analysis, non-linear and non-smooth analysis, the algebra, the mathematical logic and the topology, in the field of mechanics- the mechanics of deformable solid and the mechanics of liquid and gases.

The major achievements of the field of functional analysis are the development of the spectral theory of differential operators and the general theory of self-adjoint and non self-adjoint operators.

In these directions are studied the asymptotics of eigen-values and eigen-functions, the expansion by eigen-functions, the research of spectrum, the asymptotical distribution of the eigen-values, the calculation of the regularized traces; the solution of inverse problem for the Sturm-Liouville operators, for Dirac's system and their differential analogies, the completeness of a part of systems of the eigen- and adjoint vectors of the operator bundles and their factorization, the problems of the solvability of the operator- differential equations in the Hilbert and Banach spaces, non- transitivity of different operator's algebras, topological structure of the maximal ideal spaces, the compactness of operators of weighted compositions, the invariant subspaces of the different classes of operators, the structures of numerical regions.

In prospect one of the priority directions of functional analysis and theory of operators should be, by our opinion, the theory of the invariant subspaces.

Generally, the structure of the invariant subspaces of the displacement operator is described by well- known theorem of A. Berling, which first revealed the

relations between the theory of invariant subspaces and classical theory of analytical functions. Its non-commutative analogies should be of the unbounded interest. It needs to note the structure of the invariant subspaces of one-sided and double-sided weighted displacements in the cases that are intermediate between unicellularity and Berlinghessity is investigated insufficiently.

It needs to note that the description of the invariant subspaces of operators is closely related with description of the cyclic subspaces of the transfer operators in the theory of dynamic system control. In general, the problem of existing of the invariant subspaces of the bounded linear operator in the Hilbert space waits its solution. Another prospect direction for our mathematics is, by my opinion, the scattering theory. The aim of this theory is the proof of the existence of the scattering operator and the elucidation of nature of dependence of its properties on the scattering nature.

In the cases when the scattering operator in the sole physically observable characteristics of movement, the main problem is the inverse one: to recover scattering by means of scattering operator. The method of an inverse problem has the important applications for the integrability of many important classes of non-linear differential equations (the equations of Cortoveg- deFriz, non-linear Schrödinger's equations, the equation of Sin-Gordon, etc). The mathematical theory of scattering located at the edge of the classical theory of diffraction, the functional analysis and the theory of differential equations. The modern level of this theory requires such approaches which uncover the deep relations between the scattering theory for self-adjoint problems; and the important class of non-self adjoint operators; in particular, it should allow to apply the method of the functional analysis to the research of the analytical properties of scattering matrix and to the study of the expansions by the poles of the resolution on "non-physical sheet".

The investigations in the field of differential equations and mathematical physics covers both general partial differential equations and quality theory of such equations. In these directions a boundary value problem for different equations and systems, a class of correctness of solutions of the Cauchy problem for on Petrovsky correct high order equations, converging to zero of solutions to the systems correct by Petrovsky have been studied, the radiation principles for the Helmholtz equation and high order elliptic equations in cylindric domains have been studied, the principles of limiting absorption and limiting amplitude for boundary value problems have been proved, the analogies of Sommerfeld conditions have been studied, general boundary value problems of quasielliptic equations on manifolds have been considered, various local and non-local boundary value problems for linear and quasilinear equations have been studied. We are to note the creation of the theory of boundary properties of linear and quasilinear high order elliptic and parabolic equations, the exact description of smoothness properties for solutions of indicated equations, the study of quality properties of solutions to the second order degenerate equations with non- negative characteristic form, the study of behavior of solutions to the second order elliptic and parabolic equations in unrestricted domains. Besides, as perspective directions of development of quality theory for differential equations we indicate the fields as a theory of strongly non- linear high order partial differential equation, the investigation of irregular boundary values for equations with disconnected coefficients, the creation of quality theory of difference equations, and also the equations whose coefficients are generalized functions.

In the field of functions theory and harmonic analysis our scientists have worked out the problem of approximation theory, theory of singular integrals and theory of functional spaces. In these directions the approximation of one or some variable functions by polynomials, rational functions, entire functions, bilinear forms and *sequentials* of linear positive operators on various sets have been studied.

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Density and baseness of various functions systems, interpolation problems covering the investigations of interpolational series and interpolational polynomials have been studied. One-dimensional and multi-dimensional singular integrals of different types in weight and weightless spaces, and potential type operators in anisotropic and isotropic spaces have been studied, smoothness properties of symbols for multi-dimensional singular integrals have been investigated, the oscillating spherical expansions have been studied, equivalent norms of functional spaces on the sphere have been found, singular operators and potentials have been studied in groups, the approximation theory of generalized analytic functions classes have been worked out.

The elaboration of the theory of approximation in manifolds, the construction of functional spaces in manifolds and the study of Fourier multipliers of such spaces, the study of Fourier's integral operators with singular phase functions, the study of Ringe functions and Neural networks by means of exact annihilators, and the continuation of investigations in traditional directions of approximation theory and a wide use of special functions are in perspective.

In the field of non-linear and non-smooth analysis Azerbaijan mathematicians have investigated non-linear integral equations, some equations with monotone operators and non-linear differential equations have been studied, quasilinear hyperbolic type equations have been studied, non-linear problems in connection with the inverse problem have been considered, non-linear operator-differential equations have been investigated, topological characteristics of some non-linear mappings have been studied, behavior of solution to different non-linear equations have been investigated, non-smooth analysis problems related with optimal control and mathematical economics problems have been studied, quasilinear problems of mechanics have been investigated.

In future the following directions of non-linear analysis will be developed: development of topological methods and devices (the obtaining of new fixed point theory, Mourse theory etc.) the study the solvability problems; the elaboration of functional methods (the study of a new class of functional spaces, the generalization of solution, the generalization of coercivity notion, and etc.); the investigation of non-linear problems for all types non-linear differential equations and for the equations of mathematical physics; quality investigations of solutions to non-linear problems; grouped investigation of non-linear problems of mathematical physics, the investigation of existence and properties of attracting sets and attractors; investigation of complex and entropy characteristics of trajectories of evolution problems; investigation of non-linear problems with application of automodel solutions; investigation of non-linear problems with various types of singularities with explicit and implicit degenerations.

The main direction of investigation in algebra, mathematical logics and topology fields in Azerbaijan are: universal algebra, semi-group theory and its  $n$ -th generalization, its application to problem of determinability of mathematical structure: theory proofs and its application to categories with additional structure, theory of Boolean algebras and its application to topology and functional analysis; theoretical-set (total) topology, homologic algebra, algebraic topology, manifolds theory and knot theory. The main obtained results are the followings: the Salomaa problem on fundamental subgroups of symmetric groups was solved; the Knöbel problem on monounar primalization of finite groupoids- was solved; the series of specific properties of lattice of clone's congruence was indicated; the analogous of Birgholf theorem on subdirect expansions for some variants of non-standard expansions was obtained; T. Evans problem on non-expanded in crossection of semigroup manifolds was solved; the technique of simplified recognition of functional completeness for constrains on generated manifold.

The further elaboration of clones theory of finitary functions, variety theory and more general axiomatizable algebra classes and also their application to data base and other sections of informatics and theoretical cybernetics are perspectives in these direction.

The main directions for development of mechanics in Azerbaijan are the mechanics of deformable solids, and fluid and gas mechanics. In these directions our scientists have studied dynamical properties of number of structural materials under loading and unloading, wide investigations have been carried out in a deformable solid dynamics field, in particular, many problems on the wave propagation in elastico-plastic filaments and membranes arranged in resistant medium have been solved, the analytic theory of space motion of the filament have been worked out, a number of one-dimensional and non one-dimensional problems on a cross impact by elastic and inelastic membranes have been solved, the basis for one-dimensional theory of membrane piercing have been worked out, the problems of wave propagation in nets and fabrics have been solved, wave theory in prismatic bodies have been worked out.

Stress-strain state of piece-wise homogeneous, composite elastic media has been studied, the contract problems of elasticity theory, equilibrium problems of plates and shells weakened by the holes and cracks; the problems of complicated loading of elastic-plastic bodies; stability problems of pivots, plates and envelopes differently resisting to expansion and compression with regard to initial stress; stability problems of reinforced constructions; the problems of composite materials mechanics with curved structures have been solved; mathematical theory of deformation and prolonged destruction of hereditary elastic bodies is constructed for materials in which damage accumulation process begins in some incubation time after deformation process beginning; the process of destruction of composite materials with regard to sequential character of the process has been studied beginning from the stage of latent process of defect accumulation formation to the formation of main cracks, its development and full destruction; a method of approximate conversion of Laplace transformation admitting to construct solutions to many applied problems has been suggested; approximate methods to determine the stability under different rheophysical properties of displacing and displaced media have been worked out; filtration module of gas condensate systems with gas microbubbles has been constructed; among perspective investigations we can indicate the followings; the elaboration of the solution method for geometrically non-linear problems of physically linear and non-linear viscoelasticity theory; the development of effective criteria for minor cycle and thermal fatigue of viscoelasticoplastic bodies, simulation of destruction processes of viscoelastic and viscoelasticoplastic constructions; development of effective methods for solving spatial problems of dynamics of filament systems in conformity to textile industry, calculation of braking means of velocity objects; the investigation of problems of stability, linear and nonlinear oscillations and rigidity of elastic and inelastic homogeneous and inhomogeneous one-layer and multi-layer plates and shells interacting with medium with linear and non-linear characteristics; the construction of deformation and destruction theory of hereditary elastic bodies under finite deformations; the elaboration of effective methods for solving dynamic viscoelasticity problems; the study of electrokinetic characteristics of heterogeneous flows and mathematical simulation of heterogeneous systems flow in pipes and porous media.

Finishing the notes on some fields of science close to me, I want to express my confidence that science in Azerbaijan and in the XXI century will be developed rapidly, preserving and strengthening the gained positions, and our scientists will enrich the world science with new remarkable achievements and make contribution to investigations directed to the service of all mankind.