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THE SLIPPAGE EFFECT BY THE FILTRATION OF GASSY NON-NEWTONIAN FLUID

Abstract

The isothermic flow of the gassy non-newtonian fluid in a capillary, in homogeneous and inhomogeneous porous media in a subcritic domain is considered. The stationary solution for the flow of power (Oswald- Weyl's) fluid in the presence of slippage in a capillary and porous medium is found. It is show that at the filtration of gassy non-newtonian fluids in a subcritic domain it is possible the essential modification of rheological characteristics. The results of calculations are confirmed by the experiments on filtration of gassy pseudoplastic polymer solutions. It is noted that the suggested scheme of the solution may be applied for the analysis of the flow of non-newtonian fluids in the presence of the slippage effect.

Introduction. In paper [1] it has been shown that the slippage effect of the liquid allows to describe the isothermic stationary flow of the gassy fluid with subcritical gas germs. In many papers [2, 3, 4, 5] the viscosity anomaly of some non-newtonian fluids is explained by the slippage effect. At the same time, according to [6] at a gas factor, under normal conditions 0.1-10 surface of capillaries of the porous media is covered by the germs of a gas phase. It is clear that in these conditions at the flow of gassy non-newtonian fluid, the slippage effect may lead to essential modification of rheological characteristics.

Isothermic flow of a gassy non-newtonian fluid in the capillary, in homogeneous and inhomogeneous porous media under the pressure higher than the saturation pressure is considered in the present paper. The most applicable power law is used as a mathematical model for the non-newtonian fluid flow. Stationary solution for the flow of the power fluid in the present of slippage in a capillary and porous media is found.

Flow in a capillary. Consider the flow of the fluid subjected to the power law, in a cylindric capillary in the presence of slippage. As we know the relation between the stress and shear velocity for power fluids are defined by the expression:

$$\tau = k_0 \gamma^n ,$$

where k_0 is a constant quantity, and the velocity of the flow in a capillary for power fluid has the form [7]:

$$\mathrm{v} = -\left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{2k_0 l}\right)^{\frac{1}{n}} r^{\frac{1}{n}+1} + C , \qquad (1)$$

where C is a constant coefficient.

By accepting the boundary condition considering the slippage

$$\mathrm{v}_R = -b\left(\frac{d\mathrm{v}}{dr}\right)_R$$

where b is a slippage coefficient, R is a radius of a capillary; and solving the equation (1) we get:

$$\mathrm{v} = \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{2k_0 l}\right)^{\frac{1}{n}} R^{\frac{1}{n}+1}\left(1 + \frac{b}{R}\frac{n+1}{n}\right) - \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{2k_0 l}\right)^{\frac{1}{n}} r^{\frac{1}{n}+1} .$$

We define the flow rate:

$$Q=2\pi\int_0^R \mathrm{v}rdr=\frac{\pi n}{3n+1}\left(\frac{\Delta P}{2k_0 l}\right)^{\frac{1}{n}}R^{\frac{1}{n}+3}\left(1+\frac{3n+1}{n}\frac{b}{R}\right)$$

and find the slippage correction

$$\frac{Q}{Q_0}=\frac{\mathrm{v}}{\mathrm{v}_0}=\left(1+\frac{3n+1}{n}\frac{b}{R}\right),$$

where Q_0 and v_0 are the consumption and velocity of the flow without slippage.

Flow in a homogeneous porous media. Going over to the flow in a porous media by Kozeni-Karman equation [8], we get

$$\mathrm{v}=-B\left|\frac{dP}{dx}\right|^{\frac{1}{n}-1}\frac{dP}{dx}, \tag{2}$$

here $B=B_0\left(1+\frac{3n+1}{n}\frac{b}{R}\right)$ is a filtration coefficient with slippage, and $B_0=\frac{n}{3n+1}\times \times k_0^{-\frac{1}{n}}2^{\frac{1}{2}\left(3+\frac{1}{n}\right)}m^{-\frac{1}{2}\left(1+\frac{1}{n}\right)}k^{\frac{1}{2}\left(1+\frac{1}{n}\right)}$ (m is the porosity, k is the permeability of a porous media), is a filtration coefficient without slippage, definable by passing from the flow in a capillary to the filtration in a porous media.

According to abovedescribed, we accept B dependence on the pressure by the following law:

$$B(p)=B_0\left(1+\frac{3n+1}{n}\frac{b(p)}{R}\right),$$

where $b(p)$ is the experimental dependence of the slippage coefficient on the pressure reduced at paper [1] (note that the discount of slippage may be realized by the introduction of effective permeability in (2) determined in paper [2]):

$$b(p)=b_0\left(a\frac{P-P_c}{P_S-P_c}\exp\left[-c\left(\frac{P-P_c}{P_S-P_c}\right)\right]\right). \tag{3}$$

Further, we use the denotations accepted in [1].

Consider the one-dimensional stationary filtration in a cylindric sample of a porous media for the case $P_S>P_0>P_e>P_c$ (here P_S is the pressure, under which the slippage begins; P_0, P_e are pressures at the entry and exit of a porous media; P_c is the pressure of the fluid saturation by gas), i.e. the slippage holds at all duration of fluid flow (see the scheme at fig. 1a).

By solving equation (2) together with the equation of continuity for the incompressible fluid:

$$\frac{d\mathrm{v}}{dx}=0 \tag{4}$$

after some transformations for the velocity of filtration with slippage we get:

$$\mathrm{v}=\left(-\frac{1}{l}\int_{P_0}^{P_e}B^n(p)dP\right)^{\frac{1}{n}}, \tag{5}$$

where L is the length of the sample with porous media.

We can write for the velocity of filtration of power fluid

$$v_0 = B_0\left(\frac{\Delta P}{l}\right)^{\frac{1}{n}},$$

and for the relation of consumption:

$$\frac{v}{v_0} = \frac{Q}{Q_0} = \frac{1}{B_0}\left(\frac{-\int\limits_{P_0}^{P_e} B^n(p)dP}{\Delta P}\right)^{\frac{1}{n}}. \tag{6}$$

To determine the influence of slippage on the character of filtration, the calculations were carried out by formulas (5), (6) under different values of n, for a porous medium with permeability 0.1 mkm^2 and porosity 0.25 (integral was calculated numerically, i.e. it is calculated analytically only under even n). k_0 was determined for $\tau = 2$ N/m^2, $\lambda = 1312$ 1/s. the remaining parameters were taken from paper [1]. The calculations were carried out under constant pressure on the entry of the porous media P_0. The results are in figures 2-7. As we see from the figures, for initial pseudoplastic fluids (fig. 2,3) qualitative changes of the flow character doesn't occur, although significant growth of velocity for filtration under steady pressure drop is observed. For initially dilatant fluids, full modification of initial rheology takes place (fig. 4,5). So, for $n = 1.1$ dilatant fluid changes the character of filtration to pseudoplastic one (fig. 4), and for $n = 3$ - to S-shaped one (fig. 5). As we see from fig. 6 and 7, the dependence of consumption relation on the pressure drop and $n < 1$ (fig. 6) and for $n > 1$ (fig. 7) has the S-shaped character.

Now consider the more general case $P_0 > P_S > P_e > P_c$, i.e. when there are zones with slippage and without it in a porour medium (see the scheme in fig. 1b).

By solving equations (2) and (4) we get:

$$B(p)\left(-\frac{dP}{dx}\right)^{\frac{1}{n}} = v_S,$$

where v_S is the filtration velocity at the point x_S.

For the distribution of pressure we have:

$$-\int\limits_{P}^{P_0} B^n(p)dP = v_S^n \int\limits_{x}^{0} dx.$$

For a domain $0 < x < x_S$, where $B(p) = B_0$, i.e. for zones without slippage, we get

$$-\int\limits_{P_S}^{P_0} dP = \left(\frac{v_S}{B_0}\right)^n \int\limits_{x_S}^{0} dx. \tag{7}$$

For zones with slippage $(x_S < x < 1)$

$$-\int\limits_{P_e}^{P_S} B^n(p)dP = v_S^n \int\limits_{l}^{x_S} dx.$$

After some transformations we find:

$$x_S = l\left[1+\frac{\int_{P_e}^{P_S} B^n(p)dP}{B_0^n(P_0 - P_S)}\right]^{-1}.$$

From (7) for the filtration velocity we get:

$$\mathrm{v} = B_0\left(\frac{P_0 - P_S}{x_S}\right)^{\frac{1}{n}}.$$

For the consumption relations we have:

$$\frac{\mathrm{v}}{\mathrm{v}_0} = \frac{Q}{Q_0} = \left(\frac{P_0 - P_S}{P_0 - P_e}\frac{l}{x_S}\right)^{\frac{1}{n}}.$$

Calculations on obtained formulas confirm the above-mentioned results.

The pressure destruction $P(l-x)$, calculated for $P_e = 4$ Mpa, $P_S = 8$ Mpa and $P_0 = 16$ Mpa for initial pseudoplastic $(n = 0.7)$ and dilatant $(n = 3)$ fluids is given in fig. 8.

We see from the figure that the presence of slippage in both cases leads to significant lowering the slope of the curve for the pressure distribution at the interval $P_S - P_e$ stipulating the growth of liquid consumption and modification of rheological characteristics.

The calculation results are confirmed by experiments on filtration of gassy pseudoplastic polymer solutions [9].

Thus by filtration of gassy non-newtonian fluids in a substritical domain, the modification of initial rheology is possible. The suggested scheme may be applied for the analysis of flow of non-newtonian fluids in the presence of slippage effect.

Flow in an inhomogeneous porous media. Consider the simplified scheme of the flow of non-newtonian fluid in a laminar inhomogeneous layer [10]. Really, let the filtration happen in two parallel layers with permeability k_1 and k_2 (under other equal conditions), and $k_1 >> k_2$ correspond to the power law. Then according to the formula (2) for $B = B_0$, we get

$$Q_1/Q_2 = (k_1/k_2)(k_1/k_2)^{(1-n)/2n}.$$

As we see from the obtained expression for non-newtonian fluid $(n = 1)$, the consumption relation equals to the permeability relation; for the dilatant fluid $(n > 1)$, the consumption relation decreases, i.e. the profile of the filtration is equalized, and for pseudoplastic fluid $(n < 1)$ the differences of permeabilities are intensified.

According to obtained results, the joint action of dilatant property and slippage effect must promote to equalize the filtration profile.

Really, let the filtration of gassy non-newtonian fluid in pretransient phase state happen in two parallel layers with permeabilities k_1 and k_2 (under other equal conditions), and $k_1 >> k_2$ correspond to the power law with slippage. Then according to (2) we get

$$\frac{Q_1}{Q_2}=\frac{k_1}{k_2}\left(\frac{k_1}{k_2}\right)^{\frac{1-n}{2n}}\left(1+\frac{3n+1}{n}\frac{b\sqrt{m}}{\sqrt{8k_1}}\right)\left(1+\frac{3n+1}{n}\frac{b\sqrt{m}}{\sqrt{8k_2}}\right)^{-1}$$

As we see from the last expression for dilatant fluid $(n>1)$, slippage intensifies the levelling effect of filtration profile, but for pseudoplastic one $(n<1)$ it neutralizes the negative influence of dilution under shear on the filtration profile.

We must note that the germs isolated on the surface of a porous medium in the filtration process of gassy power fluid prevents the accerleration of flow process. In connection with this fact, the obtained results may be used for improvement the existing influence technologies to the layer in oil extraction.

Conclusion.

1) At the flow of gassy non-newtonian (power) fluids in a substrical domain, the modification of initial rheology is possible.
2) At filtration in an inhomogeneous porous medium for the dilatant fluid, the slippage intensifies the levelling effect of the filtration profile, but for the pseudoplastic one, it neutralizes the negative influence of dilution under the shear on the filtration profile.
3) The suggested solution scheme may be applied for the analysis of non-newtonian fluid flow in the presence of slippage effect.
4) The obtained results may be used for improving the existing influence technologies the layer in oil-extraction.

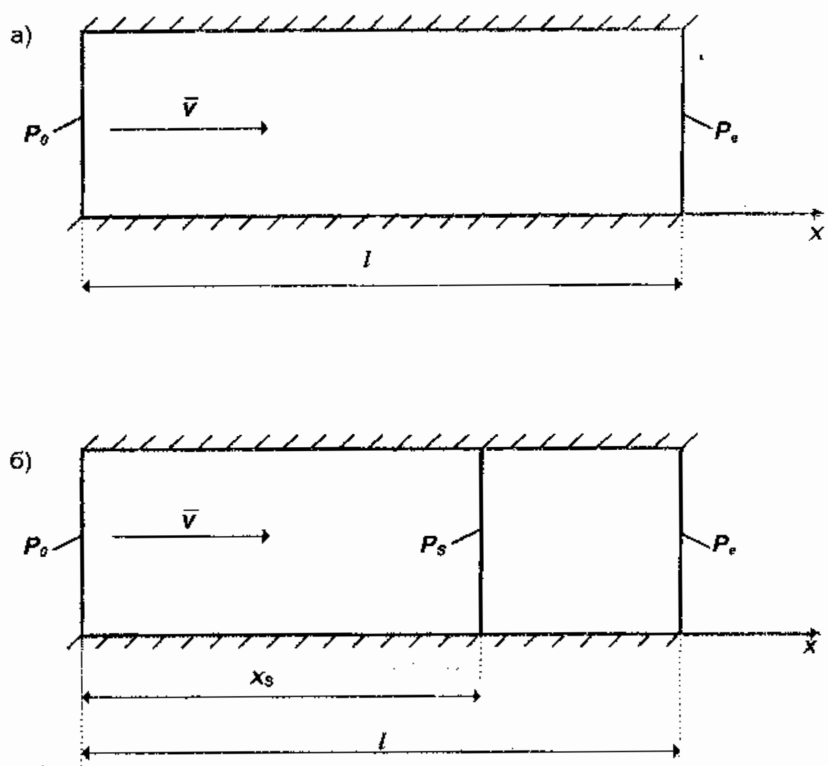

Fig. 1. The scheme of one-dimensional filtrational flow in a cylindric sample of a porous medium (horizontal section):

a) for the case $P_S > P_0 > P_e > P_c$;

b) for the case $P_0 > P_S > P_e > P_c$.

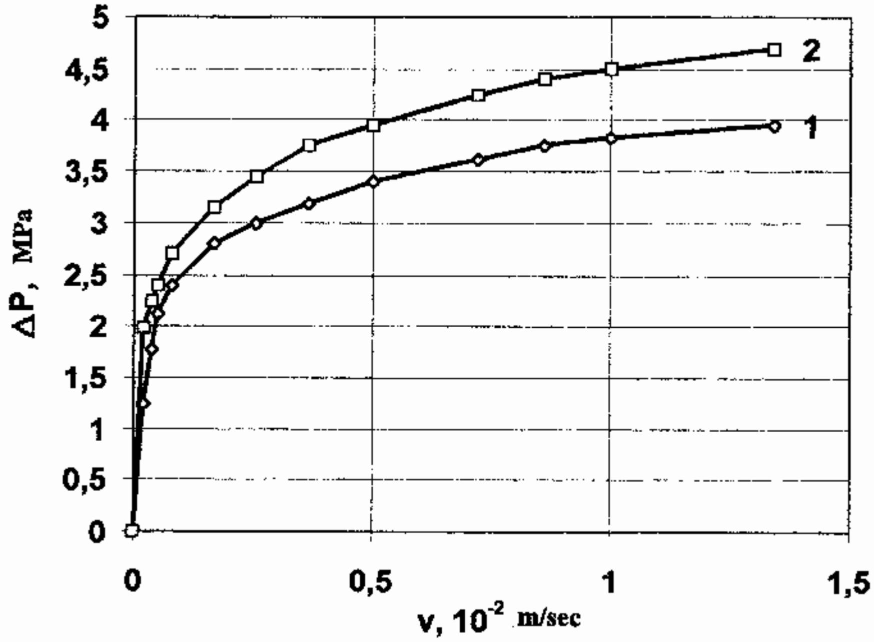

Fig. 2. Calculated curves of the flow for $n = 0.2$:
1- with slippage;
2- without slippage.

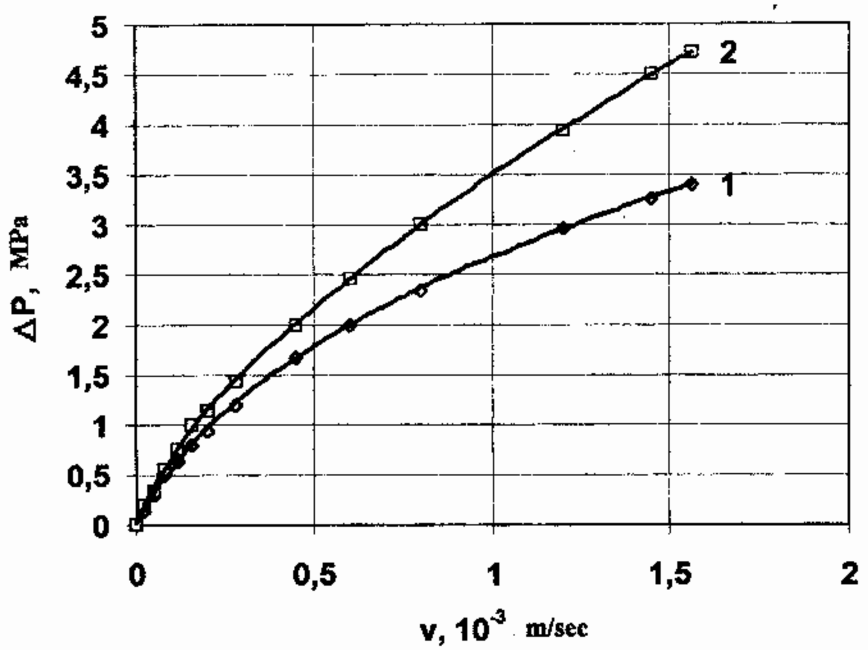

Fig. 3. Calculated curves of the flow for $n = 0.7$:
1- with slippage;
2- without slippage.

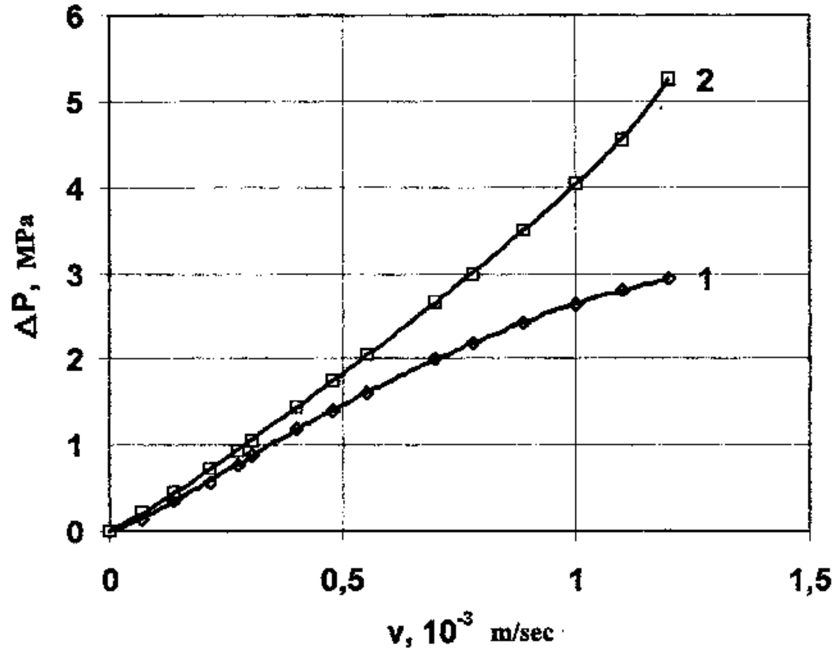

Fig. 4. Calculated curves of the flow for $n=1.1$:
1- with slippage;
2- without slippage.

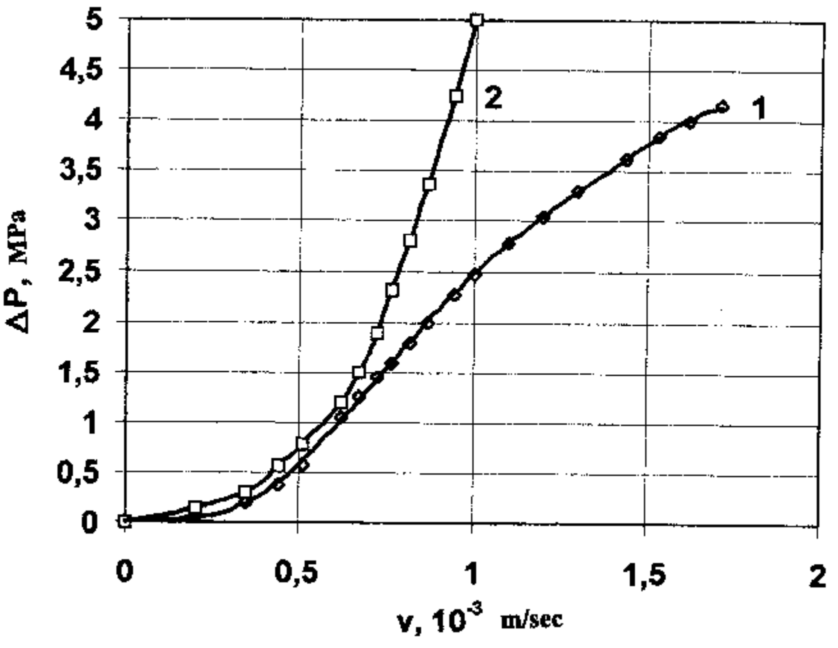

Fig. 5. Calculated curves of the flow for $n=3$:
1- with slippage;
2- without slippage.

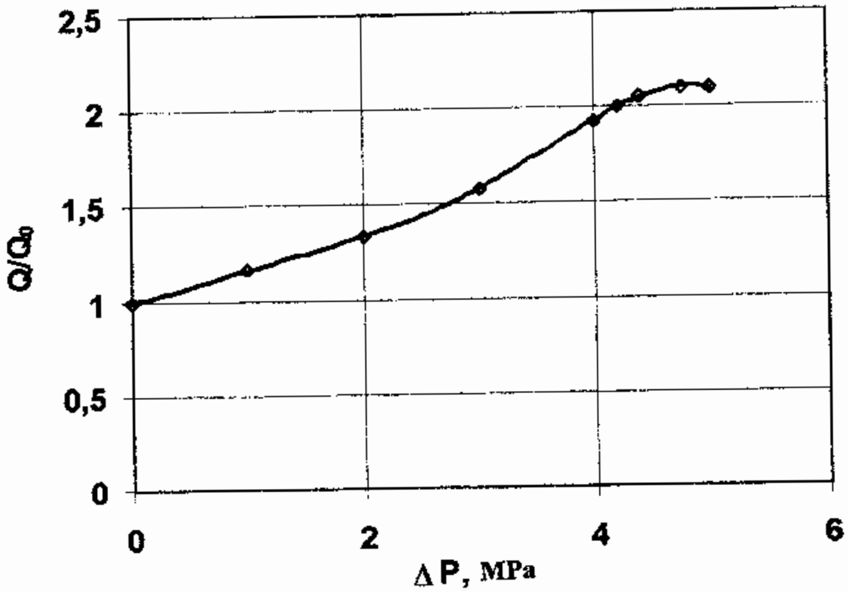

Fig. 6. The dependence of flow rate relation with slippage and without it (Q/Q_0) on the pressure drop for $n = 0.7$.

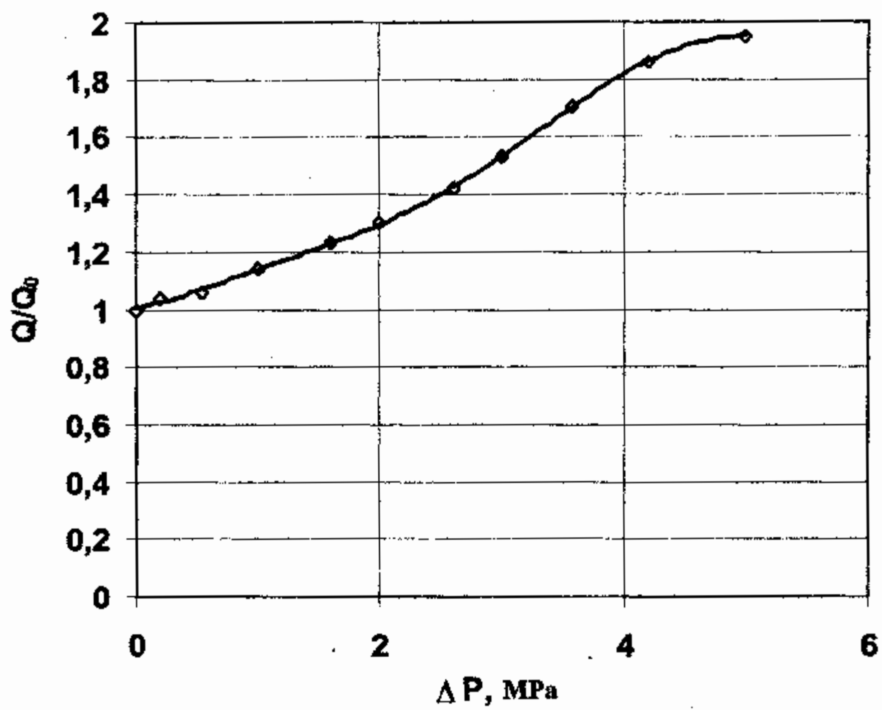

Fig. 7. The dependence of flow rate relation with slipping and without it (Q/Q_0) on the pressure drop for $n = 1.1$.

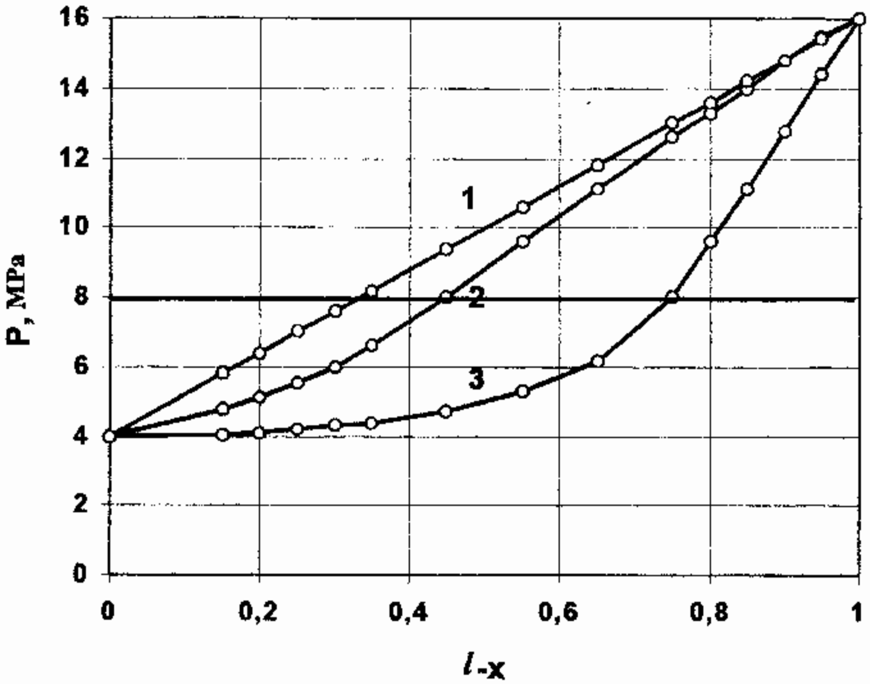

Fig. 8. The pressure distribution for the case $P_0 > P_S > P_e > P_c$:

1- with slippage;

2- with slippage for pseudoplastic fluid $(n = 0.7)$;

3- with slippage for dilatant fluid $(n = 3)$.

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Received October 26, 1999; Revised December 28, 1999.
Translated by Aliyeva E.T.