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ON DESCRIPTION OF THE MULTISTAGE PROCESSES OF FAILURE OF THE HEREDITARILY ELASTIC BODIES

Abstract

The new formula for the value characterizing the multistage process of failure of hereditarily elastic bodies is suggested.

It is known that at deformation of hereditarily elastic bodies the damage accumulation process happens which finally reduces the body to the failure (to the failure of continuity). The represented in [1] experimental data show that in the polyethylene with high density (HDP), in the polymer material the damages accumulation process is multistage: it begins from some time, happens quickly in the initial stage, then slows down its growth and at the last stage it is quick-action. As it was remarked in [2], multistage of the damages accumulation process is characteristic for many materials.

In [3] the equation characterizing the kinetic process of damages accumulation was given for the hereditarily elastic materials in which this process in dependence on the level of the stresses and temperature begins in some incubation after beginning of the deformation process. This equation we write in the form:

$$\Pi(t)=H(t-t')\left[\zeta_1(\sigma_*(t),T(t))+\int_0^t M(t-\tau)\zeta_2(\sigma_*(\tau),T(\tau))d\tau\right], \qquad (1)$$

where $H(t)$ is the Hevyside unit function, $\Pi(t)$ is the value characterizing the degree of damaging of the macroparticle of the hereditarily elastic body. At that $\Pi(t')=0$ and $\Pi(t_*)=1$, where t' and t_* are correspondingly, the time antecedent damages accumulation and the time before failure of the macroparticle of the hereditarily elastic body at arbitrary change of time t of fields of temperature $T(t,x_i)$ and stresses $\sigma_{ij}(t,x_i)$, determined by some equivalent stress $\sigma_*(t,x_i)$ (in formula (1) and in further formulas the arguments x_i - coordinates of points of the body have been omitted). As the equivalent stress σ_* it can be taken, for instance, the intensity of stresses $\sigma_*=\sigma_+=(3s_{ij}s_{ij}/2)^{1/2}$, where $s_{ij}=\sigma_{ij}-\sigma\delta_{ij}$ are the deviators of tensor of stresses σ_{ij}; $\sigma=\frac{1}{3}\sigma_{ij}\delta_{ij}$ is the mean stress, δ_{ij} is Kroneker's symbol. Other formulas for σ_* were represented in [3]. Moreover, the coming into (1) function $M(t)$ is some function of hereditarily of the material. Functions ζ_1 and ζ_2 are supposed characteristic functions of the material and so in [3] they were determined from the experiments for durable failure in creep conditions at the simplest form of deformation:

$$\zeta_1(\sigma_*,T)=-\frac{\int_0^{t_1(\sigma_*,T)} M(\xi)d\xi}{\int_{t_1(\sigma_*,T)}^{t_0(\sigma_*,T)} M(\xi)d\xi};\quad \zeta_2(\sigma_*,T)=\frac{1}{\int_{t_1(\sigma_*,T)}^{t_0(\sigma_*,T)} M(\xi)d\xi}. \qquad (2)$$

As we see, ζ_1 and ζ_2 are connected with the material functions $t_1(\sigma_*,T)$, $t_0(\sigma_*,T)$ and $M(t)$. Function $t_1(\sigma_*,T)$ corresponds to the times foregoing the damages accumulation in the case of durable failure for different constants $\sigma_* = \sigma_*^{(\nu)} = const$ and $T = T_\mu = const$. Function $t_0(\sigma_*, T)$ corresponds to times before failure for the same data. With respect to function $M(t)$ in [3] the condition was obtained

$$M(t) = \sum_{i=0}^{n-1} M_i' t^{m_i} . \qquad (3)$$

Here the constants M_i' must provide positive $M(t)$ for $t > 0$, $m_i > -1$ $\left(i = \overline{0, n-1}\right)$.

Using (2) and (3) from (1) in [3] the following formula for value $\Pi(t)$ was obtained:

$$\Pi(t) = H(t - t')\left\{ - \frac{M_i t_1^{1+m_i}(\sigma_*(t), T(t))}{M_j \left[t_0^{1+m_j}(\sigma_*(t), T(t)) - t_1^{1+m_j}(\sigma_*(t), T(t)) \right]} + \right.$$

$$\left. + M_k (1 + m_k) \int_0^t \frac{(t-\tau)^{m_k} d\tau}{M_r \left[t_0^{1+m_r}(\sigma_*(\tau), T(\tau)) - t_1^{1+m_r}(\sigma_*(\tau), T(\tau)) \right]} \right\}, \qquad (4)$$

where $M_i = M_i' / (1 + m_i)$.

Here by any repeated index (i, j, k, r) summation has been made from 0 till $n-1$.

Time t' foregoing the accumulation of damages is determined from (4) under condition $\Pi(t') = 0$. We call the remarked condition of damaging. Time t_* before failure (durability) of the macroparticle of the hereditarily elastic body for arbitrary change of time of fields of temperature and stresses is found from the durable strength condition $\Pi(t_*) = 1$ using (4).

In [3] acceptability of formula (4) was shown for description of multistage processes of accumulation of damages. However, it should be noted that for rather right description of the multistage process of accumulation of damages it is required to keep in (4) the most number of the one-stage process of accumulation of damages (the curves of accumulation of damages have not got a point of bend) is described by (4) only keeping one constant m_0, then for description of the three stage process we have to use (4) keeping nine constants $m_i, M_i/M_0$ [3].

Taking into account the influence of the accumulated damages on the mechanical properties of materials not only the conditions determining times t' and t_* have significant value, but also the mathematical representation of the law of change of damages accumulation in the interval of time $t' \le t \le t_*$. So, it has a sense to describe the multistage process of accumulation of damages by the unit functional containing relatively less number of experimentally determined constants.

Let in formula (3) beginning from the second component the constants M_i' and m_i satisfy the conditions:

$$M_{i+1}' = (-1)^i \frac{a^{2i}}{(2i)!} M_1' ; \quad m_{i+1} = m_1 + 2i\delta , \quad i = \overline{0, n-2} ,$$

where a and δ are some constants. In this case in (3) passing to the limit for $n \to \infty$ we obtain:

$$M(t) = M_0' t^{m_0} + M_1' t^{m_1} \cos\left(a t^\delta\right). \qquad (5)$$

Here $m_0, m_1 > -1; M_0' > 0; 0 < \delta < 1$.

As we will be persuaded below, the existence in (5) of function cosine let characterize the considered multistage process of accumulation of damages by the simple functional.

From (2) taking (5) into account we will determine ζ_1 and ζ_2, using which and also (5) and taking $m_1 = \delta - 1$ we will transform the functional (1) to the form

$$\Pi(t) = H(t-t')\left[-\frac{M_0 t_1^{1+m}(\sigma_*(t),T(t)) + M_1 \sin\left[a t_1^{\delta}(\sigma_*(t),T(t))\right]}{\Phi\left[t_0(\sigma_*(t),T(t)),t_1(\sigma_*(t),T(t))\right]} + \right.$$
$$\left. + \int_0^t \frac{M_0(1+m)(t-\tau)^m + M_1 a\delta(t-\tau)^{\delta-1}\cos\left[a(t-\tau)^{\delta}\right]}{\Phi\left[t_0(\sigma_*(\tau),T(\tau)),t_1(\sigma_*(\tau),T(\tau))\right]}d\tau\right]. \qquad (6)$$

Here $m = m_0$, $M_0 = M_0'/(1+m)$, $M_1 = M_1'/a\delta$,

$$\Phi\left[t_0(\sigma_*(t),T(t)),t_1(\sigma_*(t),T(t))\right] = M_0\left[t_0^{1+m}(\sigma_*(t),T(t)) - t_1^{1+m}(\sigma_*(t),T(t))\right] +$$
$$+ M_1\left[\sin\left(a t_0^{\delta}(\sigma_*(t),T(t))\right) - \sin\left(a t_1^{\delta}(\sigma_*(t),T(t))\right)\right]. \qquad (7)$$

Note that under the conditions given above for the material constants it is provided the monotone of $\Pi(t)$, determined by the correlation (6).

The condition $\Pi(t') = 0$ has the form:

$$\frac{M_0 t_1^{1+m}(\sigma_*(t'),T(t')) + M_1 \sin\left[a t_1^{\delta}(\sigma_*(t'),T(t'))\right]}{\Phi\left[t_0(\sigma_*(t'),T(t')),t_1(\sigma_*(t'),T(t'))\right]} =$$
$$= \int_0^{t'} \frac{M_0(1+m)(t'-\tau)^m + M_1 a\delta(t'-\tau)^{\delta-1}\cos\left[a(t'-\tau)^{\delta}\right]}{\Phi\left[t_0(\sigma_*(\tau),T(\tau)),t_1(\sigma_*(\tau),T(\tau))\right]}d\tau. \qquad (8)$$

The condition $\Pi(t_*) = 1$ is represented in the form:

$$\frac{M_0 t_0^{1+m}(\sigma_*(t_*),T(t_*)) + M_1 \sin\left[a t_0^{\delta}(\sigma_*(t_*),T(t_*))\right]}{\Phi\left[t_0(\sigma_*(t_*),T(t_*)),t_1(\sigma_*(t_*),T(t_*))\right]} =$$
$$= \int_0^{t_*} \frac{M_0(1+m)(t_*-\tau)^m + M_1 a\delta(t_*-\tau)^{\delta-1}\cos\left[a(t_*-\tau)^{\delta}\right]}{\Phi\left[t_0(\sigma_*(\tau),T(\tau)),t_1(\sigma_*(\tau),T(\tau))\right]}d\tau. \qquad (9)$$

The condition of damaging (8) using (7) determines the time t' of beginning of the process of accumulation of damages for the given $\sigma_*(t)$, $T(t)$, experimentally determined functions $t_1(\sigma_*,T)$, $t_0(\sigma_*,T)$ and the constants m, a, δ, M_1/M_0. For these data the time t_* - durability of work of the construction is found from the condition of the durable strength (9) taking (7) into account.

The equation (6) for $\sigma_* = \sigma_*^0 = const$, $T = T_0 = const$ passes to the following:

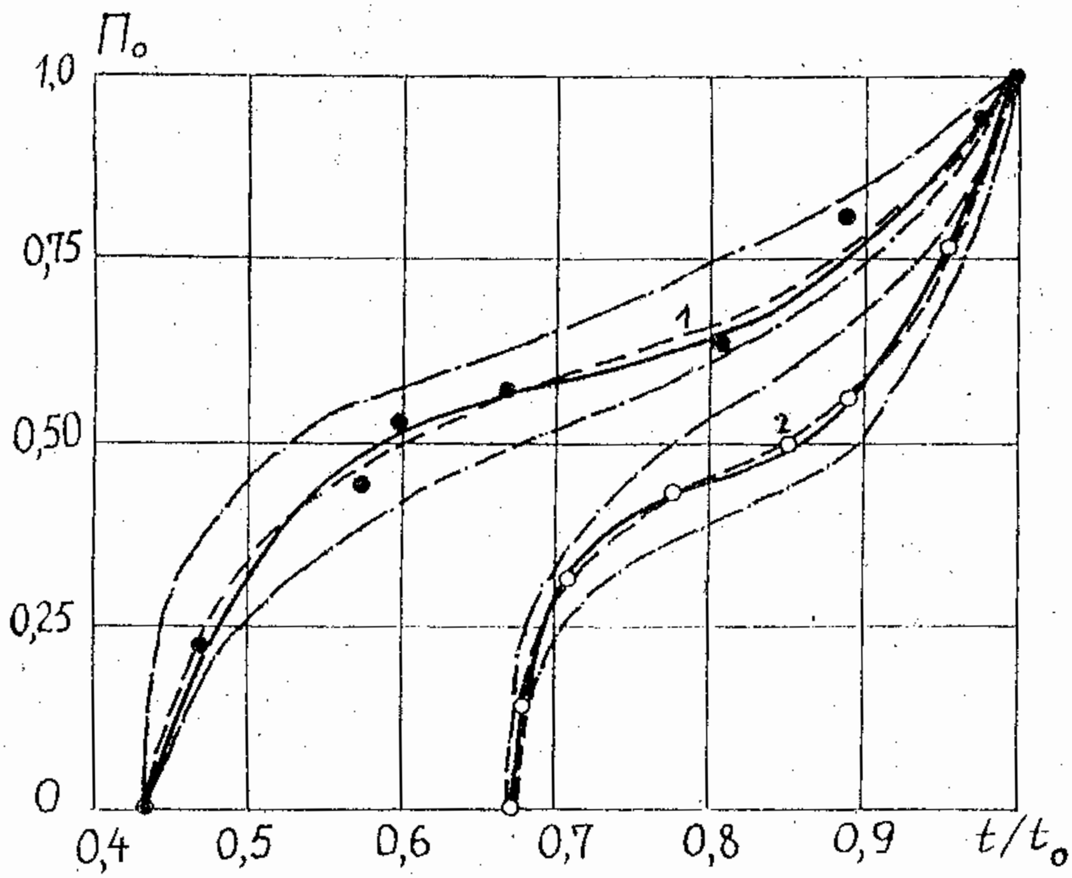
$$\Pi_0(t) = \frac{M_0\left(t^{1+m} - t_1^{1+m}\right) + M_1\left[\sin\left(a t^{\delta}\right) - \sin\left(a t_1^{\delta}\right)\right]}{M_0\left(t_0^{1+m} - t_1^{1+m}\right) + M_1\left[\sin\left(a t_0^{\delta}\right) - \sin\left(a t_1^{\delta}\right)\right]}, \qquad (10)$$

where $t_1 = t_1\left(\sigma_*^0, T_0\right)$, $t_0 = t_0\left(\sigma_*^0, T_0\right)$.

It is clear, that (10) is the equation of the curve of the accumulation of damages and so for the known by the experiment curve of accumulation of damages it serves for determination of the material constants. The results of the experiments carried out in [1] have been worked out. Following [3] the material functions t_0 and t_1 are approximated in the form:

$$t_i = t_{is}\exp\left[\beta_i\left(1 - \sigma_*/\sigma_s\right) + d_i\left(1 - T/T_s\right)\right], \quad (i = 0,1), \qquad (11)$$

where β_0, d_0, β_1, d_1 are the constants which must be determined experimentally, σ_s is the stress, T_s is reducing temperature, t_{0s} and t_{1s} are the time before failure and the time of beginning of accumulation of damages for $\sigma = \sigma_*$, $T = T_s$.



In the represented figure the experimental points, fields (restricted by the dash lines) of their scattering and the averaging are shown taking into account the statistics of the curves of accumulation of damages (the dash lines) HDP in the creep conditions for the stress 3,92 MPa, temperatures 353 K (curve 1) and 343 K (curve 2) [1]. As the value of reducing $\sigma_s = 3{,}92$ MPa, $T_s = 343$K, $t_{1s} = 840$ r, $t_{0s} = 1240$ r were taken. The values of the constants coming into (11) are equal to: $d_0 = 56{,}7$; $\beta_0 = 2{,}76$; $d_1 = 70{,}5$; $\beta_1 = 2{,}76$. As the result of calculation of values of the constants coming into (10) the following were obtained: $m = -0{,}9$; $\delta = 0{,}82$; $at_{0s}^{\delta} = 4\pi$; $M_0 t_{0s}^{1+m} / M_1 = 104{,}5$. In this case the curves $\Pi_0(t)$ constructed according to (10) taking (11) into account approximately coincide with the curves $\Pi_0(t)$ calculated by the corresponding equation obtained from (4) for nine constants m_r, M_1/M_0 (the continuous lines in figure). As we can see the multistage process (in the considered case of the experiment the three stage process) of accumulation of damages HDP for $\sigma_* = \sigma^0 = const$, $T = T_0 = const$ is described by the equation (10) taking (11) into account. It certificates about that the analogous process for $\sigma_* = \sigma_*(t)$, $T = T(t)$ can be described by (6) using (7) and (11).

Let's make the following note. The function of hereditarily of the form (5) can be used also for description of the processes of deformation (creep and relaxation) of hereditarily bodies.

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