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ASYMPTOTICS OF THE NATURAL FREQUENCIES OF OSSILATIONS OF THE TRANSTROP HOLLOW CYLINDER

Abstract

On the base of one of the variants of the asymptotic method based on the homogeneous solutions of equations on the dynamic theory of elasticity the free oscillation of the transtrop hollow cylinder is investigated. The asymptotic process for finding of the frequencies of free axial symmetric oscillations of the transtrop hollow cylinder is suggested with any beforehand given precision.

1. Let us consider the axial symmetric dynamic problem of the theory of elasticity for the transtrop hollow cylinder. The cylinder is in the cylindrical coordinate system r, φ, z changing within following limits

$$R_1 \le r \le R_2, \quad 0 \le \varphi \le 2\pi, \quad -l \le z \le l. \tag{1.1}$$

The motion equations in displacements have a view:

$$\begin{gathered} b_{11}\left(\Delta_0 u_\rho - \frac{u_\rho}{\rho^2}\right) + \frac{\partial^2 u_\rho}{\partial \xi^2} + (b_{13}+1)\frac{\partial^2 u_\xi}{\partial \rho \partial \xi} = gR_0^2 G_1^{-1}\frac{\partial^2 u_\rho}{\partial t^2}, \\ (b_{13}+1)\frac{\partial}{\partial \xi}\left(\frac{\partial u_\rho}{\partial \rho} + \frac{u_\rho}{\rho}\right) + \Delta_0 u_\xi + b_{33}\frac{\partial^2 u_\xi}{\partial \xi^2} = gR_0^2 G_1^{-1}\frac{\partial^2 u_\xi}{\partial t^2}. \end{gathered} \tag{1.2}$$

Here $(\rho, \xi, u_\rho, u_\xi) = R_0^{-1}(r, z, u_r, u_z)$, $R_0 = \frac{1}{2}(R_1 + R_2)$ is the radius of the mean surface, g is density of the material; $mb_{11} = 2G_0(1 - \nu_1\nu_2)$; $mb_{13} = 2G_0\nu_1(1+\nu)$; $mb_{33} = 2G_0\nu_1(1-\nu^2)$, $b_{12} = b_{11} - 2G_0$, $E_0 = E^{-1}\cdot E_1$, $G_0 = G\cdot G_1^{-1}$, $\nu_2 = E_0^{-1}\cdot \nu_1$ are the dimensionless quantities; $G, G_1, \nu, \nu_1, \nu_2, E, E_1$ are the elasticity constants; $m = 1 - \nu - 2\nu_1\nu_2$, $\Delta_0 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}$.

It is supposed that the following boundary-valued conditions are fulfilled on the bounds

$$u_\xi = 0, \quad \tau_{\rho\xi} = 0 \ \text{ for } \ \xi = \pm l_0, \tag{1.3}$$

$$\sigma_\rho = 0, \quad \tau_{\rho\xi} = 0 \ \text{ for } \ \rho = \rho_n \ (n = 1,2). \tag{1.4}$$

We construct the solution of the system (1.2)-(1.3) in the form

$$u_\rho = u(\rho)\sin p\xi e^{i\omega t}, u_\xi = w(\rho)\cos p\xi e^{i\omega t} \tag{1.5}$$

$p = \frac{\pi k}{l}$, $l_0 = \frac{l}{R_0}$, ω-vibration frequency.

Substituting (1.5) into (1.2)-(1.4) we obtain

$$\begin{gathered} b_{11}\left(u'' + \frac{1}{\rho}u' - \frac{u}{\rho^2}\right) + (\lambda^2 - p^2)u - (b_{13}+1)pw' = 0, \\ -(b_{13}+1)p^2\left(u' + \frac{u}{\rho}\right) + w'' + \frac{1}{\rho}w' + a_0^2 w = 0, \end{gathered} \tag{1.6}$$

$$\left(b_{11}u' + \frac{b_{12}}{\rho}u + b_{13}w\right)_{\rho=\rho_n} = 0, \qquad (1.7)$$
$$\left(w' - p^2 u\right)_{\rho=\rho_n} = 0$$

$\lambda^2 = gR_0^2 G_1^{-1}\omega^2$ is the frequency parameter.

Let note that in (1.7) the elasticity correlations for the transtrop cylinder have been used.

Not detailing let us give the final solution of equation (1.6)

$$u(\rho) = \left(a_0^2 - \alpha_1^2\right)Z_1(\alpha_1\rho) + \left(a_0^2 - \alpha_2^2\right)Z_1(\alpha_2\rho),$$
$$w(\rho) = (b_{13+l})\left[\alpha_1 Z_0(\alpha_1\rho)\right) + \alpha_2 Z_0(\alpha_2\rho). \qquad (1.8)$$

Here $a_0^2 = \lambda^2 - b_{33}p^2$, $Z_k(\alpha\rho) = c_1 J_k(\alpha\rho) + c_2 Y_k(\alpha\rho)$, the functions $J_k(\alpha\rho)$, $Y_k(\alpha\rho)$ are the linear independent solutions of Bessel's equations; c_1, c_2 are arbitrary constants, $\alpha_n = \sqrt{t_n}$, t_n are the roots of the quadratic equation.

$$t^2 - 2q_1 t + q_2 = 0, \qquad (1.9)$$
$$q_1 = b_{11}^{-1}\left[(b_{11}+1)\lambda^2 - \left(b_{11}b_{33} - b_{13}^2 - 2b_{13}\right)\right]p^2,$$
$$q_2 = b_{11}^{-1}\left(\lambda^2 - p^2\right)a_0^2, \alpha_n = \pm S_n,$$
$$S_n = \sqrt{q_1 - (-1)^n\sqrt{q_1^2 - q_2}}\ .$$

From the condition (1.7) we obtain the frequency equation with respect to λ^2

$$\Delta\left(\lambda^2, p, \rho_1, \rho_2\right) = 8\pi^{-2} l_1 l_2 a_1 a_2 g_1 g_2 +$$
$$+ (a_2 b_1 - a_1 b_2)\{a_1 g_2[l_1 L_{10}(\alpha_2) + l_2 L_{01}(\alpha_2)]L_{11}(\alpha_1) -$$
$$a_2 g_1[l_1 L_{10}(\alpha_1) + l_2 L_{01}(\alpha_1)]L_{11}(\alpha_2)\} -$$
$$-(a_2 b_1 - a_1 b_2)^2(\rho_1 p_2)^{-1} L_{11}(\alpha_1)L_{11}(\alpha_2) + \qquad (1.10)$$
$$+ a_1 a_2 g_1 g_2[L_{10}(\alpha_1)L_{01}(\alpha_2) + L_{01}(\alpha_1)L_{10}(\alpha_2)] -$$
$$- a_2^2 g_1^2 L_{00}(\alpha_1)L_{11}(\alpha_2) - a_1^2 g_2^2 L_{00}(\alpha_2)L_{11}(\alpha_1) = 0,$$

where

$$a_n = -p^2\left(a_0^2 + b_{13}\alpha_n^2\right), \; b_n = -2G_0\left(a_0^2 - \alpha_n^2\right),$$
$$g_n = \alpha_n\left[-B_0 p^2 + b_{11}\left(\lambda^2 - \alpha_n^2\right)\right], \;\; l_n = \left(\alpha_n \rho_n\right)^{-1},$$
$$B_0 = b_{11}b_{33} - b_{13}^2 - b_{13},$$
$$L_{ij}(x) = J_i(x\rho_1)Y_j(x\rho_2) - J_j(x\rho_2)Y_i(x\rho_1), \;\; (i,j = 0,1).$$

2. The left-hand side of equation (1.10) as the whole function of parameter λ^2 has the countable set of zeros with the thickening point at infinity. For effective studying of its roots as in the isotropy case [2] let us have some assumptions with respect to the geometrical parameters of the cylinder. Assume

$$\rho_1 = 1 - \varepsilon, \; \rho_2 = 1 + \varepsilon, \; 2\varepsilon = \frac{R_2 - R_1}{R_0} = \frac{2h}{R_0}\ . \qquad (2.1)$$

We consider that ε is the small parameter. Substituting (2.1) into (1.10) we obtain

$$D\left(\lambda^2, p, \varepsilon\right) = \Delta\left(\lambda^2, p, \rho_1, \rho_2\right) = 0. \qquad (2.2)$$

The case $p=0$ is considered separately. As in the isotropy case [2] with respect to zeros $D(\lambda^2,p,\varepsilon)$ we can formulate the following statement: for any finite $P\left(P=O(\varepsilon^{\beta}),\ \beta\geq 0\ \ for\ \ \varepsilon\to 0\right)$ the function $D(\lambda^2,p,\varepsilon)$ has a finite number of zeros with the following asymptotic properties

$$\Lambda_k=O(\varepsilon^{q}),\ q\geq 0\ \left[\Lambda^2=2^{-1}(1+\nu)^{-1}\lambda^2\right].$$

Let give the scheme of the proof of this statement. For that let us decompose $D(\lambda^2,p,\varepsilon)$ into the series by ε:

$$D(\lambda^2,p,\varepsilon)=A\varepsilon^2\left[b_0D_0(\lambda^2,p)+\frac{1}{3}D_1(\lambda^2,p)\varepsilon^2+\frac{1}{45}D_2(\lambda^2,p)\varepsilon^4+\ldots\right]=0,\quad (2.3)$$

where

$$A=128(1+\nu)^2m^{-1}\pi^{-2}G_0\left(\alpha_1^2-\alpha_2^2\right)^2a_0^2b_0(b_{13}+1)^2;$$

$$b_0=1-\nu_1\nu_2,D_0(\lambda^2,p)=\Lambda^2\left(G_0-b_0\Lambda^2\right)+E_0G_0\left(\Lambda^2-G_0\right)p^2;$$

$$D_1(\lambda^2,p)=-(E_0G_0)^2p^6+2E_0G_0^2\left\{\Lambda^2\left[b_0-E_1G_1^{-1}-\nu_1(1+\nu)\right]-\right.$$

$$\left.-2(1+\nu)G_0(E_0G_0-\nu_1)\right\}p^4-\left\{9b_0E_0G_0^2+\right.$$

$$+2G_0\left[2\nu_1(1+\nu)+\nu\nu_1+4\nu_1^2(1+G_0)+\left(\nu^2-3\right)E_0-2(\nu+3)E_0G_0\right]\Lambda^2+$$

$$\left.+\left[4(1+\nu)b_0E_0G_0\left(2+b_{11}^{-1}\right)+G_0^{-1}(b_0-\nu_1-\nu\nu_1)^2\right]\Lambda^4\right\}p^2+$$

$$+9b_0G_0\Lambda^2+2b_0\left[2(m-2b_0)G_0+b_0^{-1}m^2-4m-2b_0\right]\Lambda^4+$$

$$+2(1+\nu)b_0G_0^{-1}\left[2G_0+1-\nu-2\nu_1\nu_2(1+G_0)\right]\Lambda^6;$$

$$D_2(\lambda^2,p)=-8(1+\nu)b_0^{-1}(E_0G_0-\nu_1)E_0^2G_0^2p^8+\ldots.$$

Let assume that the main terms of asymptotic λ_k and p have the forms:

$$\Lambda_k=\Lambda_{k0}\varepsilon^{q},\ \ p=p_0\varepsilon^{\beta},\ \ \Lambda_{k0}=O(1),\ \ P_0=O(1),\ \ q\geq 0,\ \ \beta\geq 0.\quad (2.4)$$

Substituting (2.4) into (2.3) from the incontradictoriness condition of the constructed asymptotic process we obtain that here only the cases $q=0$ and $q=\beta$ are possible.

Let note that here and further sometimes we will divide the general interval of changing of the parameters q and β into the subintervals so as by virtue are q and β, the zeros in $D(\lambda^2,p,\varepsilon)$ have different asymptotic representations.

In the first case $\left(q=0,p=p_0\varepsilon^{\beta},\beta>0\right)$ we seek $\lambda_k(k=1)$ in the form

$$\Lambda_k=\Lambda_{k0}+\varepsilon^{2\beta}\Lambda_{k2}+\ldots.\quad (2.5)$$

After substitution (2.5) into (2.3) we obtain

$$\Lambda_{k0}^2=G_0b_0^{-1},\ \ \Lambda_{k2}=(2\Lambda_{k0}b_0)^{-1}P_0^2\nu_1\nu_2E_0G_0\ \ (0<\beta<1),$$

$$\Lambda_{k2}=(2\Lambda_{k0}b_0)^{-1}\left[\nu_1\nu_2E_0P_0^2+\frac{3\nu_1\nu_2+4\nu+1}{3b_0}\right]G_0\quad \beta=1,$$

$$\Lambda_{k2}=(3\nu_1\nu_2+4\nu+1)G_0\left(6\Lambda_{k0}b_0^2\right)^{-1}\quad \beta>1.$$

In the second case $(q=\beta)$ we seek λ_k $(k=2)$ in the form

$$\Lambda_k = \varepsilon^{\beta}\left(\Lambda_{k0}+\Lambda_{k2}\varepsilon^{2\beta}+\ldots\right). \tag{2.6}$$

Then from (2.3) we obtain

$$\Lambda_{k0}^2 = E_0 G_0 P_0^2; \quad \Lambda_{k2} = -\left(2\Lambda_{k0}\right)^{-1} v_1 v_2 E_0^2 G_0 P_0^4 .$$

These frequencies are the frequencies of so called extremely-low-frequencies oscillations [3].

Finally, let consider the case when $q=\beta=0$. We seek λ_k in the form

$$\Lambda_k = \Lambda_{k0}+\varepsilon^2\Lambda_{k2}+\ldots \quad (k=1,2). \tag{2.7}$$

Substituting (2.7) into (2.3) we obtain

$$D_0\left(\Lambda_{k0},P_0\right)=0,\ \Lambda_{k2} = -\left[6b_0\Lambda_{k0}\left(E_0G_0P^2+G_0-2b_0\Lambda_{k0}^2\right)\right]^{-1} D_1\left(\Lambda_{k0},P\right).$$

Thus, it has been proved that for the fixed finite P there are two frequencies of natural oscillations. Let consider the case when P is the boundlessly grows for $\varepsilon\to 0$. Here we will consider the following limit cases: $P\varepsilon\to 0$ for $\varepsilon\to 0$; $P\varepsilon\to const$ for $\varepsilon\to 0$; $P\varepsilon\to\infty$ for $\varepsilon\to 0$. Let determine first such λ_k $(k=1,2)$ when $P\varepsilon\to 0$ for $\varepsilon\to 0$. For that we use again the decomposition (2.3). Assume that the main terms of the asymptotics Λ_k and p have the form:

$$\Lambda_k = \Lambda_{k0}\varepsilon^{-q},\, P=P_0\varepsilon^{-\beta},\, \Lambda_{k0}=O(1),\, P_0=O(1) \\ \left(0\le q<1, 0<\beta<1\right). \tag{2.8}$$

It is not difficult to prove that $q\le\beta$. We will consider separately the cases when $q=0$ and $q=\beta$. In the first case from (2.3) we obtain $0<\beta<\frac{1}{2}$. The case $\beta=\frac{1}{2}$ is considered separately. We seek λ_k $(k=1)$ in the forms:

$$\Lambda_k = \Lambda_{k0}+\varepsilon^{2\beta}\Lambda_{k2}+\ldots\left(0<\beta\le\frac{1}{3}\right), \tag{2.9}$$

$$\Lambda_k = \Lambda_{k0}+\varepsilon^{2-4\beta}\Lambda_{k2}+\ldots \quad \left(\frac{1}{3}<\beta<\frac{1}{2}\right). \tag{2.10}$$

After substitution of these decompositions into (2.3) we obtain

$$\Lambda_{k0}^2 = G_0;\ \Lambda_{k2} = -\left(2\Lambda_{k0}E_0\right)^{-1} v_1v_2G_0P_0^{-2} \quad \left(\beta\neq\frac{1}{3}\right),$$

$$\Lambda_{k2} = \frac{G_0}{2\Lambda_{k0}}\left[\frac{E_0}{3b_0}P_0^4-\frac{v_1v_2}{E_0}P_0^{-2}\right] \quad \left(\beta=\frac{1}{3}\right),$$

$$\Lambda_{k0}^2 = G_0;\ \Lambda_{k2} = \frac{G_0E_0}{6b_0\Lambda_{k0}}P_0^4 \quad \left(\frac{1}{3}<\beta<\frac{1}{2}\right).$$

In the case when $q=0,\ \beta=\frac{1}{2}$ we obtain

$$\Lambda_k = \Lambda_{k0}+\varepsilon\Lambda_{k1}+\ldots \quad (k=2), \tag{2.11}$$

$$\Lambda_{k0}^2 = G_0\left[1+\frac{E_0}{3b_0}P_0^4\right],$$

$$\Lambda_{k2}=-(10E_0G_0)^{-1}\{[5b_0(2G_0-1)+10E_1G_0G_1^{-1}-$$
$$-10v_1(1+v)G_0-8(1+v)(E_0G_0-v_1)]\Lambda_{k0}^3+$$
$$+G_0[5+16(1+v)(4-5G_0)(E_0G_0-v_1)-10b_0G_0-10G_0E_1G_1^{-1}+10v_1(1+v)]\Lambda_{k0}+$$
$$+12(1+v)G_0^2(E_0G_0-v_1)\Lambda_{k0}^{-1}\}P_0^{-2}.$$

By analogy in the case $q=\beta$ from (2.3) we obtain

$$\Lambda_k=\Lambda_{k0}\varepsilon^{-\beta}+\Lambda_{k2}\varepsilon^{\beta}+\ldots\ 0<\beta\le\frac{1}{2},$$
$$\Lambda_k=\Lambda_{k0}\varepsilon^{-\beta}+\Lambda_{k2}\varepsilon^{2-3\beta}+\ldots\ \frac{1}{2}<\beta<1,\ \ k=2, \tag{2.12}$$

$$\Lambda_{k0}^2=E_0G_0b_0^{-1}P_0^2,\ \ \Lambda_{k2}=(2\Lambda_{k0}b_0)^{-1}v_1v_2G_0\quad 0<\beta<\frac{1}{2},$$
$$\Lambda_{k2}=(2\Lambda_{k0}b_0)^{-1}(v_1v_2G_0+A_1)\ \beta=\frac{1}{2},$$
$$\Lambda_{k2}=(2\Lambda_{k0}b_0)^{-1}A_1\quad \frac{1}{2}<\beta<1,$$
$$A_1=\left(3b_0^2\right)^{-1}E_0G_0P_0^4\left\{b_0\left[b_0(2G_0-1)+\right.\right.$$
$$\left.+2G_0E_1G_1^{-1}-2v_1(1+v)G_0-4(1+v)E_0G_0\left(1+b_{11}^{-1}\right)\right]+$$
$$\left.+2(1+v)E_0m-G_0^{-1}(b_0-v_1-vv_1)^2\right\}$$

In case $q\neq 0,\ q<\beta$, substituting (2.8) into (2.3) and remaining only the main terms for λ_{k0} we obtain the following limit equations

$$D(\lambda^2,p,\varepsilon)=A)\{E_0G_0\Lambda_{k0}^2b_0P_0^2+$$
$$+O\left[\max\left(\varepsilon^{2\beta-2q},\varepsilon^{2q}\right)\right]+\frac{1}{3}\{-E_0^2G_0^2P_0^6+ \tag{2.13}$$
$$+O\left[\max\left(\varepsilon^{2\beta-2q},\varepsilon^{2-2\beta}\right)\right]\}\varepsilon^{2-6\beta}=0.$$

Hence we obtain $q=2\beta-1$. And from the condition $q>0$ we have $\beta>\frac{1}{2}$. Therefore, $\frac{1}{2}<\beta<1$. Now we seek Λ_k in the form

$$\Lambda_k=\varepsilon^{1-2\beta}\left(\Lambda_{k0}+\Lambda_{k2}\varepsilon^{2-2\beta}+\ldots\right)\ (k=1) \tag{2.14}$$

After substitution in to (2.13) we get

$$\Lambda_{k0}^2=(3b_0)^{-1}E_0G_0P_0^4,\ \ \Lambda_{k2}=B_k\quad \beta\neq 2/3$$
$$\Lambda_{k2}=B_k+(2\Lambda_{k0})^{-1}G_0$$
$$B_k=10^{-1}\Lambda_{k0}^3\left[10b_0E_0^{-1}+10EG_1^{-1}-\right.$$
$$\left.-10v_1(1+v)E_0^{-1}-8(1+v)(E_0G_0-v_1)E_0^{-1}G_0^{-1}-5b_0E_0^{-1}G_0^{-1}\right]P_0^{-2}.$$

Let consider the second case when $P\varepsilon\to const$ for $\varepsilon\to 0$. In principle it is possible the case

$$P = P_0\varepsilon^{-1},\ \lambda = O(1)\ (P >> \lambda)\ \text{for}\ \varepsilon \to 0. \tag{2.15}$$

However, as it was remarked in [4] in dependence on the properties of the material ν, ν_1, ν_2, G_0 and the frequency parameter λ the parameters q_1, q_2 in equation (1.9) admit various values that calls the different writings of the solutions through the Bessel's functions. And it reduces to the different asymptotically representations of Bessel's function.

Let note that in this case

$$q_1 = b_{11}^{-1}\left(b_{11}b_{33} - b_{13}^2 - 2b_{13}\right)P_0^2 = \tilde{q}_1 P_0^2$$

$$q_2 = b_{11}^{-1}b_{33}P_0^4 = \tilde{q}_2 P_0^4.$$

Let consider the following possible cases

a) $\tilde{q}_1 > 0, \tilde{q}_1^2 - \tilde{q}_2 \neq 0\ \alpha_{1,2} = \pm S_1 P_0, \alpha_{3,4} = \pm S_2 P_0,$

$$S_{1,2} = \sqrt{\tilde{q}_1 \pm \sqrt{\tilde{q}_1^2 - \tilde{q}_2}},\ \tilde{q}_1^2 > \tilde{q}_2,$$

$$S_{1,2} = \aleph + i\gamma = \sqrt{\tilde{q}_1 \pm i\sqrt{\tilde{q}_2 - q_1^2}},\ \tilde{q}_1^2 < \tilde{q}_2;$$

b) The roots of the characteristic equation are divisible

$$\alpha_{1,2} = \alpha_{3,4} = \delta P_0, \tilde{q}_1 > 0, \tilde{q}_1^2 - q_2 = 0, \delta = \sqrt{\tilde{q}_1};$$

c) $\tilde{q}_1 < 0,\ \tilde{q}_1^2 - \tilde{q}_2 \neq 0,$

$$\alpha_{1,2} = \pm iS_1 P_0,\ \alpha_{3,4} = \pm iS_2 P_0,$$

$$S_{1,2} = \sqrt{|\tilde{q}_1| \pm \sqrt{\tilde{q}_1^2 - \tilde{q}_2}},\ \tilde{q}_1^2 > \tilde{q}_2,$$

$$S_{1,2} = \sqrt{|\tilde{q}_1| \pm i\sqrt{\tilde{q}_2^2 - \tilde{q}_1}}, \tilde{q}_1^2 < \tilde{q}_2;$$

d) $\tilde{q}_1 < 0\ \ \tilde{q}_1^2 - \tilde{q}_2 = 0,\ \alpha_{1,2} = \alpha_{3,4} = i\delta P_0,\ \delta = \sqrt{|\tilde{q}_1|}.$

In the cases a) and b) after substitution (2.15) into (1.10) and its transform with help of the asymptotic decompositions $J_k(x), Y_k(x)$ for P_0 correspondingly we obtain

$$(S_2 - S_1)\sin(S_1 + S_2)P_0 \pm (S_2 + S_1)\sin(S_2 - S_1)P_0 = 0, \tag{2.16}$$

$$\aleph \sin 2\gamma P_0 \pm \gamma\, sh \aleph P_0 = 0, \tag{2.17}$$

$$\sin 2\delta P_0 \pm 2\delta P_0 = 0. \tag{2.18}$$

What about the cases c) and d) then for them the results are obtained from the cases a) and b) by formal substitution of S_1, S_2, δ by $iS_1, iS_2, i\delta$. As it is known [4,5], these equations have only the complex roots. So the real parameter P_0 can not be the solution of these equations. So in this case the shell can make only forced oscillations and $D(\lambda^2, p, \varepsilon) \neq 0$.

Let consider the case when $P\varepsilon \to const$, $\lambda\varepsilon \to const$ for $\varepsilon \to 0$. We seek λ_n $(n = k-2,\ k = 3,4,\ldots)$ in the form

$$\lambda_n = \varepsilon^{-1}\delta_n + O(\varepsilon),\ P = P_0\varepsilon^{-1}\ (n = 1,2,\ldots). \tag{2.19}$$

After substitution (2.19) into (1.10) and its transform with help of the asymptotic decompositions of functions $J_\nu(x), Y_\nu(x)$ for big values of the argument for δ_n we get the equation

$$\begin{aligned}&\left(H_1\aleph_{21}\sin H_{1k}\cos H_{2k}-H_{2k}\aleph_{12}\sin H_{2k}\cos H_{1k}\right)\times\\&\times\left(H_1\aleph_{21}\cos H_{1k}\sin H_{2k}-H_{2k}\aleph_{12}\sin H_{1k}\cos H_{2k}\right)=0,\end{aligned}\tag{2.20}$$

where

$H_{nk}=\sqrt{\tau_{nk}}$, τ_{nk} are the roots of the quadratic equation

$$\begin{aligned}&b_{11}\tau^2-\left[\left(b_{11}b_{33}-b_{13}^2-2b_{13}\right)P_0^2+\left(b_{11}+1\right)\delta_k\right]\tau+\\&+\left(P_0^2+\delta_k^2\right)\left(b_{33}P_0^2+\delta_k^2\right)=0,\end{aligned}\tag{2.21}$$

$$\aleph_{mj}=\left(b_{11}b_{33}-b_{13}^2\right)\left(H_{mk}+\delta_k^2\right)+b_{11}\delta_k^4-$$
$$-\left(b_{11}H_{jk}+2b_{13}P_0^2+b_{33}P_0^4\right)\delta_k,\quad m,j=1,2\quad m\neq j\ .$$

Under conclusion of the formulas (2.19) it was supposed that the roots of the equation (2.21) are real and simple. As above by analogy the other cases are considered. For given P the transcendent equation (2.20) determines the countable set of roots δ_k. Let note that equation (2.20) in the isotropy case completely passes over the frequency equation by Reley-Lamb [2]. In the case 3) denoting $\varepsilon\lambda$ by X, εP by Y in the first term of the asymptotics we obtain again the equation (2.20) remains valid. Therefore, and in the case $\beta>1$ the equation (2.20) remains valid. In the case $P=0$ the boundary-valued problem is devided into two problems:

$$u_r=a_0(\rho)e^{i\omega t},\ u_z\equiv0\ \left(\tau_{rz}\equiv0\right),\tag{2.22}$$

$$u_r=0,\ u_z=b_0(\rho)e^{i\omega t},\ \left(\sigma_r=\sigma_\varphi=\sigma_z\equiv0\right),\tag{2.23}$$

$$a_0''+\frac{1}{\rho}a_0'+\left(\frac{\lambda^2}{b_{11}}-\frac{1}{\rho^2}\right)a_0=0,\tag{2.24}$$

$$\left(b_{11}a_0'+\frac{b_{12}}{\rho}a_0\right)_{\rho=\rho_n}=0,\tag{2.25}$$

$$b_0''+\frac{1}{\rho}b_0'+\lambda^2b_0=0,\tag{2.26}$$

$$b_0'\big|_{\rho=\rho_n}=0.\tag{2.27}$$

The boundary-valued problems (2.24)-(2.25) and (2.26)-(2.27) describe the thickness oscillations of the shell and with precision up to the constant factor they coincide with the analogous-boundary valued problems in the isotropy case [2]. So we will not consider them in details.

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