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ON UNIFICATION OF RHEOLOGICAL AND HYDRAULIC CALCULATIONS

Abstract

The article is devoted to an engineering method of rheological stationary fluids flow calculation in pipes and channels. By means of a complex variable method the universal formulas for velocities and flow rates distribution have been received. Characteristic linear size for single-bonded areas selection methods have been offered. The quasi Newtonian approach method has been suggested for the first time.

The «real» rheological equation of condition construction has been pointed out and nonlinear viscosity plastic fluids differential equations have been worked out. The resistance laws generalized for the rheological stationary fluids have been received.

Introduction. In many fields of industry (oil, chemistry, power, light industry, food industry and others) the technologic processes deal with the fluid motion, with different rheologic properties in the pipes and channels with arbitrary cross-section [1,3-6]. Complexity of working out of rheologic hydraulic calculations in the pointed out processes is caused generally by the following reasons:

- the existing exact formulas allows determine the channel capacity and hydraulic losses for the channels with simple geometry;
- the working fluid within one and the same process in dependence on external conditions can behave both Newtonian and non Newtonian;
- there absents the unique methodic for determination of rheoconstans of anomaly fluids which are invariant with respect to the geometry of the flow;
- adequacy of the taken rheologic models is often provided only for the limited diapason of the working gradients of the velocity;
- there absents the unique approach for the choice of the characteristic linear size coming into Reynolds number which let determine the hydraulic losses independently on the geometry of the cross-section.

In this work the results of investigations of recent time on rheology and hydrodynamics of fluids carried out in Azerbaijan State Oil Academy which are the base of the complex approach to calculation of hydrosystems relating to different fields of production.

Invariant representation of the existing formulas

The differential equation of the laminar flow of the viscous fluid in the channel with arbitrary cross-section for constant gradient of pressure has the form [3]:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-\frac{\Delta p}{\mu l} \tag{1}$$

with the boundary condition $u=0$ on the contour.

The precise solution of equation (1) was obtained for the channels with the simple sections [1,3]. For invariant representation of the existing dependencies $Q-\Delta p$ as the force factor the mean by the perimeter the tangential stress $\bar{\tau}_w$ is suggested and as the kinematics characteristic the equalent gradient of the shear velocity $\dot{\gamma}_w$ is suggested. The value of $\bar{\tau}_w$ is determined from the equilibrium of pressure and function forces

$$\bar{\tau}_w = \frac{\Delta p r_H}{l}. \qquad (2)$$

The equivalent gradient of the shear velocity is determined by analogy to the fluid flow in the circular pipe, i.e. according to Hagen-Poiseuille formula

$$\gamma_w = 4\frac{\bar{u}}{r_e}, \qquad (3)$$

where $r_e = 2r_H / \xi$.

Thus, the correlation

$$\bar{\tau}_w = 4\mu\frac{\bar{u}}{r_e} \qquad (4)$$

is universal for the channels with arbitrary cross-section.

Unification of velocity for the viscous fluid

For construction of the field velocities in the channel with arbitrary cross-section the complex variable method [7] is used.

Introducing function

$$\psi = u + \frac{\Delta p}{4\mu l}\left(x^2 + y^2\right) \qquad (5)$$

equation (1) is transformed to Laplace equation with the boundary condition $\psi = \Delta p\left(x^2 + y^2\right)/4\mu l$ on the contour.

That substitution has the following hydradynamic interpretation: the problem of determination of the field of velocities for the flow of the viscous fluid in the channel with arbitrary section has been reduced to the problem of determination of flow of the ideal-fluid in the prism with the same section rotating with the angular velocity $\omega = \Delta p / 2\mu l$. In order to find the function $\psi(x, y)$ for arbitrary area, at first the interior of this area in z-plane $(z = x + iy)$ is reflected into interior of the unique circle in ζ-plane $\left(\zeta = \xi + i\eta = \rho e^{i\theta}\right)$ with help of the power series

$$z = a_0 + a_1\zeta + a_2\zeta^2 + \ldots + a_n\zeta^n + \ldots . \qquad (6)$$

Then using Schwarz's integral the function $f(\zeta)$ is determined in the unique circle by the given real part on the contour. Then taking into account the boundary condition $f(\zeta)$ is determined in the unique circle by the given real part on the contour. Then taking into account the boundary condition $\psi = \frac{1}{2}\omega z \cdot \bar{z}$ for flow function we obtain

$$\psi = \omega\left[\frac{1}{2}\sum_{k=0}^{\infty} a_k^2 + \sum_{k=1}^{\infty}\rho^k \cos k\theta \sum_{s=0}^{\infty} a_s a_{s+k}\right]. \qquad (7)$$

Following (5) we find the velocities distribution in ζ-plane

$$u = \frac{\Delta p}{2\mu l}\left[\frac{1}{2}\sum_{k=0}^{\infty} a_k^2\left(1 - \rho^{2k}\right) + \sum_{k=1}^{\infty}\rho^k \cos k\theta \sum_{s=1}^{\infty} a_s a_{s+k}\left(1 - \rho^{2s}\right)\right]. \qquad (8)$$

Distribution of velocities $u(x, y)$ in z-plane is constructed according to $x = \sum_{k=0}^{\infty} a_k \rho^k \cos k\theta,\ y = \sum_{k=1}^{\infty} a_k \rho^k \sin k\theta$.

Composition of the results of numerical calculations by (8) for the channels with the cross-section in the form of ellipse, the square and infinite strip with the velocity diagram constructed by the exact formulas was given in [13] which certificates of validity of the suggested method.

The generalized formula of the volume expense of the viscous fluid.

Taking into account (5), passing to the polar coordinates in ζ-plane, for fluid discharge we obtain

$$Q=\iint_D u dx dy=\int_0^1\int_{-\pi}^{\pi}\psi\left|\frac{dz}{d\zeta}\right|\rho d\rho d\theta-\frac{1}{2}\omega\int_0^1\int_{-\pi}^{\pi}\left|z\frac{dz}{d\zeta}\right|^2\rho d\rho d\theta. \qquad (9)$$

Substituting the value of $\psi(\rho,\theta)$ from (7) and taking into account (6) from the last we have

$$Q=\frac{\pi\Delta p}{2\mu l}\left[\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}a_j a_{j+k}\sum_{s=0}^{\infty}s a_s a_{s+k}+\frac{1}{2}\sum_{k=0}^{\infty}a_k^2\sum_{k=0}^{\infty}(k+1)a_{k+1}^2-\frac{1}{8}\sum_{k=0}^{\infty}k\left(\sum_{s=0}^{\infty}a_s a_{k-s}\right)^2\right]. \qquad (10)$$

Therefore, for the known representation function $z=z(\zeta)$ by (10) we can determine the discharge of the viscous fluid in the channel with arbitrary single-bounded cross-section. Particularly, for the circle $(a_1=R,\ a_0=a_2=a_3=...=0)$ it coincides with Hagen-Poiseuille formula. The validity of (10) for some channels was checked with results were given in [13].

It should be noted that unification of the discharge characteristic for different channels can be realized also with use of the hydrodynamic analogy by Boussinesq [1]. The generalized formula for discharge of the viscous fluid will be represented in the form:

$$Q=\frac{\Delta p\cdot S^4}{16\pi^2\mu J_0 l}. \qquad (11)$$

Representing (11) in the form of (4) for the approximate value of the equalent radius we obtain:

$$r_e'=S^3/4\pi^2 J_0 r_H=2r_H/\xi', \qquad (12)$$

where $\xi'=8\pi^2 J_0/S\chi^2$ is the approximate value of the coefficient of the form.

In the cases of the circle and the elliptic channels the equation (11) coincides with the exact formulas. For the right n-angle

$$\xi'=\frac{\pi^2}{3n^2}\left(1+3ctg^2\frac{180^\circ}{n}\right).$$

Selection of the characteristic linear size

From the above-represented methods of generalization it follows that the characteristics linear size of the cross-section of the channel can be determined by three ways:

1) by the exact solution of equation (1);
2) on the base of Boussinesq's analogy;
3) with help of the conform representation method.

The generalized formula (10) gives reason to determine the discharge of the viscous fluid in the channel with arbitrary section as in the case of flow in the circular pipe whose radius is equal to the conform radius $\dot{r}$ of the section of channel [9]. In table 1

Table 1

section form	r_e	r'_e	$\dot{r}$	ξ	Note
circle with radius R	R	R	R	1	
ellipse with semiaxes a and b	$\frac{4\alpha E(\varepsilon)}{\pi(1+\alpha^2)}a$	$\frac{4\alpha E(\varepsilon)}{\pi(1+\alpha^2)}a$	$\frac{1+\alpha}{2\sum_{n=0}^{\infty} q^{n(n+1)}\left(1+2\sum_{n=1}^{\infty} q^{n^2}\right)}a$	$\frac{\pi^2(1+\alpha^2)}{8E^2(\varepsilon)}$	$\alpha = b/a, \varepsilon = \sqrt{1-\alpha^2}, q = \left(\frac{1-\alpha}{1+\alpha}\right)^2$ $E(\varepsilon) = \frac{\pi}{2}\left[1 - \frac{1}{2}\frac{\varepsilon^2}{1} - \left(\frac{1\cdot 3}{2\cdot 4}\right)\frac{\varepsilon^4}{3} - ...\right]$
semi-circle with radius R	0,6201R	0,6391R	0,6006R	$\frac{0{,}42\pi^3}{(\pi+2)^2}$	
rectangle with the sides $2a$ and $2b$	$\frac{(1+\alpha)f(\alpha)}{4}a$	$\frac{12\alpha(1+\alpha)}{\pi^2(1+\alpha^2)}a$	$\frac{4\left(1+2\sum_{n=1}^{\infty} q^{n^2}\right)^{-2}}{\pi}b$	$\frac{8}{(1+\alpha)^2 f(\alpha)}$	$\alpha = b/a, q = e^{-\pi/\alpha}$ $f(\alpha) = \frac{16}{3} - \frac{1024}{\pi^5}\left(th\frac{\pi}{2\alpha} + \frac{1}{3^3}th\frac{3\pi}{2\alpha} + ...\right)$
square with side $2a$	$9a/8$	$12a/\pi^2$	1,0787a	0,888	
narrow rectangle with height $2b$	$4b/3$	$12b/\pi^2$	$4b/\pi$	1,5	
equilateral triangle with the side a	$\sqrt{3}a/5$	$9\sqrt{3}a/4\pi^2$	0,3268a	5/12	
isosceles right-angled triangle with cathetusa	0,355a	$\frac{9(2+\sqrt{2})a}{8\pi^2}$	0,3346a	0,822	
right hexagon with side a	0,9208a	0,9476a	0,8985a	0,9406	
concentric annulus with the radius a and b	$\frac{1+\alpha^2+\frac{1-\alpha^2}{\ln\alpha}}{1-\alpha}a$	–	–	$\frac{(1-\alpha)^2}{1+\alpha^2+\frac{1-\alpha^2}{\ln\alpha}}$	$\alpha = b/a$

the values of the equivalent and conform radiuses and also of the coefficients of the forms for some areas have been given.

Quasi Newtonian approach to the calculation of flow of the anomaly fluids

The rheologic behaviour of most of fluids, polymer and paintwork materials, clay and cement mixtures, paraffin-resin oils and oilproducts, confectionery and food products, pastes and suspensions of nuclear fuel, pharmaceutical masses and blood and etc. differ from behaviour of ordinary liquids and relate to the non Newtonian systems [1,4,5,6]. The existing of rheology-hydraulic calculations methods are restricted or with much calculation works. The suggested method of the quasi-Newtonian approach gives an opportunity to forecast the flow of any rheostabil fluid in the channel with arbitrary section on the base of data of the capillary rheometry. The crux of the quasi Newtonian approach is in the following: the equivalent viscosity μ_e of the non Newtonian fluid is substituted by the viscosity of the ordinary Newtonian fluid which origins during flowing as the resistance as the given non Newtonian fluid. Value of μ_e for the given velocity gradient is determined by the consistent curving constructed in the coordinates $\bar{\tau}_w - \dot{\gamma}_w$, which are named the consistent variables. Consequently, the correlation

$$\bar{\tau}_w = \mu_e \cdot \dot{\gamma}_w \tag{13}$$

is considered universe as for the Newtonian, as for the non Newtonian, that is, flow of any rheostabil fluid in the channel with arbitrary section, in the integral approach can be represented as the flow of the Newtonian fluid with equivalent viscosity in the circular pipe whose radius is equal to the equivalent radius of the non-circular channel. In Table 2 the formulas for the equivalent viscosity and the mean velocity for the particular rheologic models are reduced.

Validity of quasi Newtonian approach had been tested by working-out of a lot of data which there are in [12,15] and by own tests which results are given in fig. 1,2.

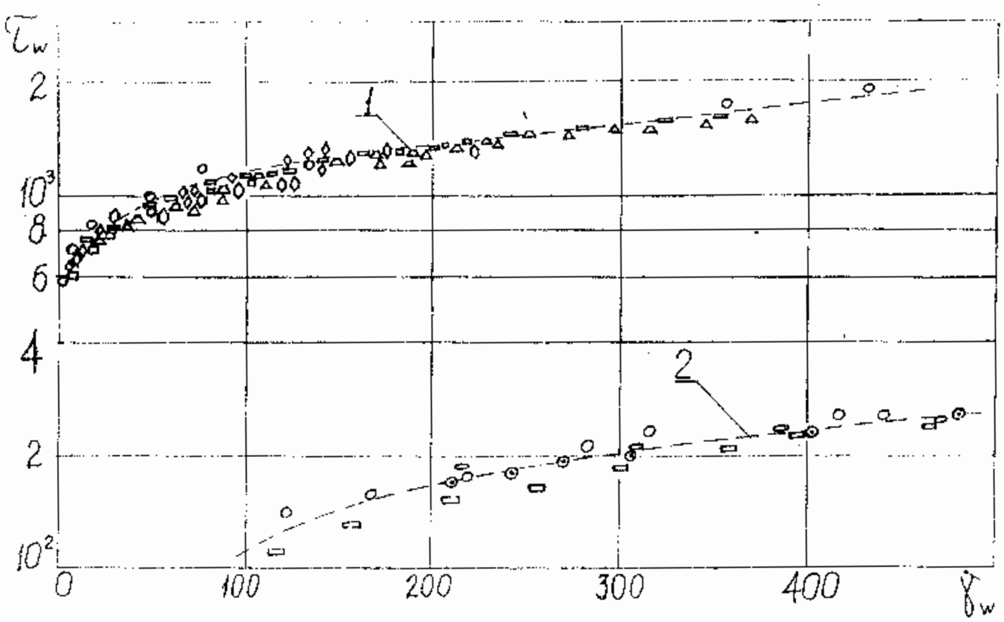

Fig. 1. Data of a capillary rheometry for channels with different cross sections; 1- for solid oil "C" under t=30°C [12], 2- for 7%th solution of CCM under t=20°C [15] (the denotations in the figure corresponds to cross sections forms).

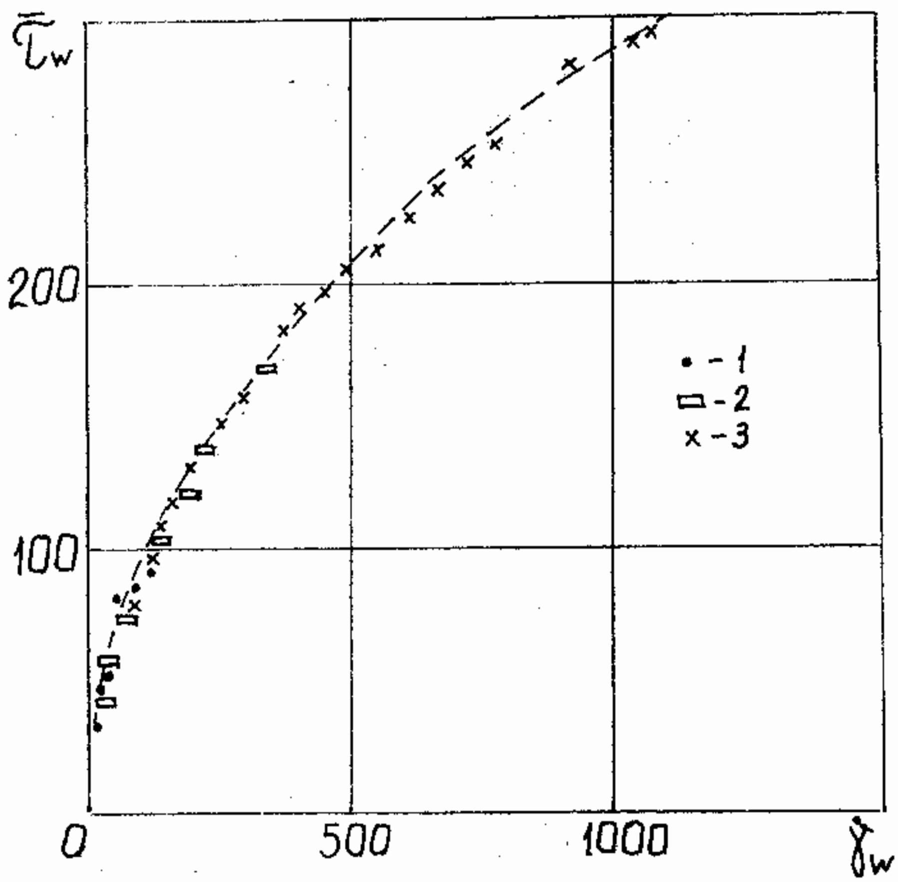

Fig. 2. Rheological curve of oil for well №735 of Kyurovdag deposit under t=18^{0}C; 1- data of a porous medium with air permeability of 16,5 darcy; 2-data for a plane slot of 1,2mm height; 3-data of a capillary viscosimeter d=1,5mm.

Table 2.

Rheomodels	μ_e	$\bar{u}$
Bingham-Schwedoff	$\eta'\left(1+\frac{1}{8}Sen\right)$	$\bar{\tau}_w r_e\left(1-T_0'\right)/4\eta'$
Ostwald-de Waele	$k'\left(4\bar{u}/r_e\right)^{n'-1}$	$\frac{1}{4}r_e\left(\bar{\tau}_w/k'\right)^{1/n'}$
Kesson-Shulman	$\eta'\left[1+\left(\frac{1}{8}Sen\right)^{1/n'}\right]^{n'}$	$\frac{\bar{\tau}_w r_e}{4\eta'}\left[1+\left(T_0'\right)^{1/n'}\right]^{n'}$

Determination of «real» rheological equation of condition.

In order to use the rheometric data for determination of the local characteristics of the flow and also for solving of the problems of convective heat mass transfer it is necessary to construct the «real» rheological equation of condition of the anomaly *fluid.*

The real values of the velocities gradient on the wall of the channel are determined by Mooney- Rabinowitsch formula [6]

$$\dot{\gamma}_{st} = \frac{1}{\bar{\tau}_w^2} \frac{d}{d\bar{\tau}_w} \left[\bar{\tau}_w^3 \cdot F(\bar{\tau}_w) \right]. \tag{14}$$

The dependence $\dot{\gamma}_w = F(\bar{\tau}_w)$ is found by the way of working out of the data of capillary rheometry. The selected model must be differed by adequacy and simplicity. The model is considered adequately if $F_{expl} \leq F_{table}$ [10], where F is Fischer's variance ratio. As the result of working out of data of 26 tests it was ascertained that at 20 tests ($\approx 77\%$) Kesson-Shulman's model was adequately

$$\tau^{1/n} = \tau_0^{1/n} + (\eta\dot{\gamma})^{1/n}. \tag{15}$$

Taking into account the rheomodel (15) represented in the consistent variables from (14) we find

$$\dot{\gamma}_{st} = \frac{1}{4}\dot{\gamma}_w \left(3 + \frac{1}{1 - (T_0')^{1/n}} \right). \tag{16}$$

For every $\bar{\tau}_w$ by (16) the value of $\dot{\gamma}_{st}$ is constructed which is the «real» rheological equation of condition and the found constants τ_0, η are the «real» rheoconstants of the model (15). The results of the calculations had shown that passing from the consistent variables $(\bar{\tau}_w, \dot{\gamma}_w)$ to the model's $(\tau, \dot{\gamma})$ the values of the limit shear stress differ much. Moreover, the difference of values of the structure viscosity is neglect small for practice.

The differential equations of motion of the non-linear visco-plastic systems

Taking into account that the deformation condition of Genky's plastic body after passage through the flowing condition and the rheological behavior of Kesson-Shulman's flluid almost coincide, then the differential equations of motion for the last one can be completed on the base of the deformation theory of plasticity by Genky [2]. By this theory between the intensity of the tangential stress T and the intensity of the shear deformation ε the correlation exists

$$T = \frac{1}{2\varphi}\varepsilon. \tag{17}$$

The value φ in (17) is named Genky's coefficient what in the general case is the function ε. The view of this function is determined by tests. For Kesson- Shulman fluid it will be:

$$\frac{1}{2\varphi} = \left[(\tau_0/\varepsilon)^{1/n} + (\eta^{1/n}) \right]^n. \tag{18}$$

Taking into account (17) and (18) in the equation of motion of the continual medium

$$\rho\frac{D\bar{U}}{Dt} = \rho\bar{F} + divT \tag{19}$$

after some transforms we obtain

$$\rho\frac{D\bar{U}}{Dt} = \rho\bar{F} - gradP + \left(\eta^{1/n} + \tau_0^{1/n}M\right)^n \nabla^2 u + n\tau_0^{1/n}\left(\tau_0^{1/n}M + \eta^{1/n}\right)^{n-1} \varepsilon\ gradP. \tag{20}$$

Correlation (20) is the vector form of the differential equations of motion of nonlinear visco-plastic fluids. For $\tau_0 = 0$ from (20) we obtain Navier-Stokes equation of the Bingnam and Shvedov's [4], for $n = 2$ - the equation of motion of Kesson's fluid.

Now we will complete the differential equations of motion of the low-compressible non-linear visco-plastic fluid in the elastic porous media [11]. The consistent variables are represented by the correlations:

$$\tau_n = \frac{\Delta P\sqrt{K}}{l}, \quad \gamma_n = \frac{u_f}{\sqrt{K}}. \tag{21}$$

Then the filtration process is described by the following system:

- equation of continuity

$$div(\rho\bar{u}_f) + \partial(\rho m)/\partial t = 0; \tag{22}$$

- equation of condition of the system fluid-porous media

$$\rho m \approx \rho_0 m_0\left(1 + \frac{P - P_0}{\beta}\right); \tag{23}$$

- equation of motion

$$u_f = -\frac{K}{\eta}\left(\left|gradP\right|^{1/n} - G_0^{1/n}\right)^n \frac{gradP}{\left|gradP\right|}. \tag{24}$$

Taking into account (23) and (24) in (22) we obtain the differential equation of motion of the low-compressible non-linear visco-plastic fluid in the elastic porous media

$$\frac{\partial P}{\partial t} = \chi div\left(\left|gradP\right|^{1/n} - G_0^{1/n}\right)^n \frac{gradP}{\left|gradP\right|}. \tag{25}$$

For $\tau_0 = 0$ the well-known equation by Shelkachev V.N. [11] is obtained.

The generalized low of resistance rheostabil fluids

Let's represent the formula by Darcy- Weisbach in the form

$$\tau_w = \frac{1}{8}\lambda\rho\bar{u}^2. \tag{26}$$

Comparing (26) with the rheological equation (15) we obtain

$$\lambda = \frac{64}{2\rho\bar{u}r_e/\mu_e} = \frac{64}{\mathrm{Re}^*}. \tag{27}$$

This formula is the generalized low of resistance of the laminar regime for all rheostabil fluids. For the filtration case in (27) the mean velocity is substituted by the equalent radius is substituted by the values $4\sqrt{K}$.

The turbulent motion of rheostabil fluids was studied on the base of the model of academician Millionshikov M.D. [8] that was founded on the superposition principle of the molecular and turbulent viscosities. The following physical reasons were accepted:

- the turbulent motion of the rheostationary fluids happens by the two layer scheme by Prandtle- Taylor;
- the thickness of the laminar sublayer is too small in comparison with the characteristic transverse size of the channel;
- the viscosity of the laminar sublayer in the case of the anomaly fluid is equal to the value of the equivalent viscosity on the wall of the channel;
- in the turbulent kernel area the structure of turbulence is the same for the Newtonian and the non Newtonian fluids.

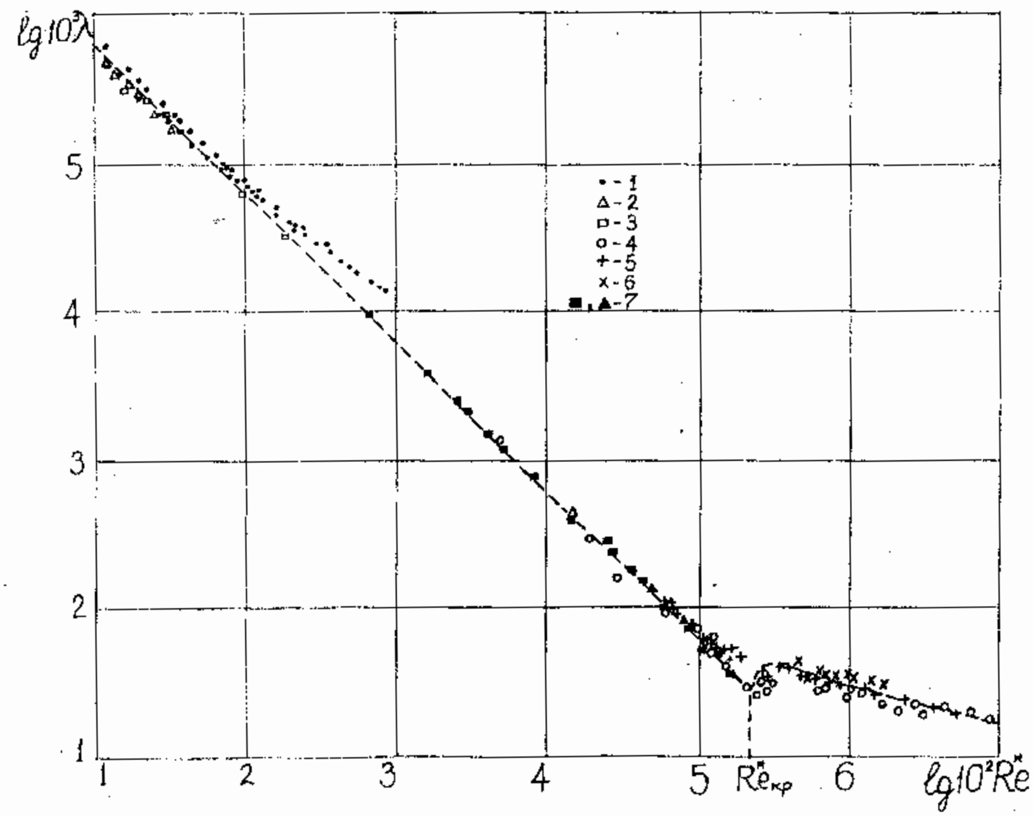

Fig. 3. Comparison of the generalized resistance law (dotted lines) with the experimental data on the flow; 1- of kerosene in a porous medium under K=2000 darcy [14]; 2- of the solid oil in a triangular channels (b=4,4mm, h=3,8mm) [12]; 3- of the resin oil in a plane channel (δ=1,24mm) and in a circle capillar (d=1,5mm); 4- of the 14,3% water mixture of clay in a circle channel [16]; 5- of water in a rectangular channel (7,9×27,8mm) [17]; 6- of the paraffin oil in the pipe-line Grozny-Gudermes (l=41,65km, d=196 mm) [18]; 7- of the 1%-th CCM in rectangular (a/b=1; 2,5; 6,25; 16,67) and triangular ($\alpha=60^0$; a=49mm) pipe-lines, correspondingly [19].

The turbulent motion equation is written in the following form:

$$\tilde{\tau}_w(1-\bar{y})=(\mu_{st}+\mu_t)\frac{du}{dy}. \qquad (28)$$

For distribution of velocities we obtain

$$u/u_*=\eta \quad for \quad 0<\eta<\delta,$$
$$u/u_*=\frac{1}{a}\ln[1+a(\bar{\eta}-\delta)]+\delta, \quad \text{for} \quad \eta\geq\delta. \qquad (29)$$

The mean velocity is determined by

$$\bar{u}=2\int_0^1(1-\bar{y})udy. \qquad (30)$$

Taking into account (29) from the last we find

$$\frac{u}{u_*}=\frac{1}{2}\mathrm{Re}^*\beta=\frac{1}{\beta}\left(\bar{\delta}^2-\bar{\delta}^3+\bar{\delta}^4/4\right)+$$
$$+\frac{\beta^2}{a^3}\left[a^2(\ln\alpha-1{,}5)+2\alpha-0{,}5\right]+\delta. \quad (31)$$

The formula (31) with the formula by Darcy-Weisbach represented in the form

$$\lambda=\frac{8}{\left(\mathrm{Re}^*\cdot\beta/2\right)^2} \quad (32)$$

gives the generalized low of the turbulent motion of the rheostabil fluids in the parametric form where the parameter is β.

In fig. 3 the universal laws of resistance (27), (31) and (32) and also the data on hydroresistance of some rheostabil fluids [12,14, 16-18] are represented.

Denotations

u is the velocity of the fluid in the given point; $\bar{u}$ is the mean velocity of the flow; Q is the volume discharge; ΔP is pressure differential; R,l are the radius and the length of the channel, correspondingly; μ is the dynamic viscosity of the Newtonian fluid; $\bar{\tau}_w$ is mean by the perimeter tangential stress; γ_w is the equivalent gradient of velocity on the wall of the channel; r_e is the equivalent radius o the channel; r_H is the hydraulic radius; ξ is coefficient of the form of the cross-section; J_0 is the polar moment of resistance of cross-section; μ_e is the equivalent viscosity of the non Newtonian fluid; τ_0 is the limit shear stress; η is the plastic viscosity; k is the coefficient of consistence, n is the index of flow or parameter of non-linearity; τ_0',η',k',n' are the rheoconstants determined from the consistent curving; $T_0'=\tau_0'/\bar{\tau}_w$ is the dimensionless limit stress of shear; $Sen=2\tau_0 r_e/\eta\bar{u}$ is Saint- Venant and Ilyushin parameter; λ is the coefficient of the hydraulic resistance; $\mathrm{Re}^*=2\rho\bar{u}r_e/\mu_e$ is the generalized Reynold's number; $\bar{y}$ is the dimensionless distance from the wall of the channel; μ_t is the turbulent viscosity; $\mu_{st},\dot{\gamma}_{st}$ are the values of the equivalent viscosity and gradient of the velocity on the wall of the channel, correspondingly; $u_*=\sqrt{\bar{\tau}_w/\rho}$ is the dynamic distance; $\delta=u_*\rho\delta_0/\mu_{st}$ is the dimensionless thickness of the laminar sublayer; $\beta=\mu_{st}/r_e u_*\rho;\delta_0$ is the thickness of the laminar sublayer; $\bar{\delta}=\delta_0/r_e=\delta\beta$; $\alpha=1+\frac{a}{\beta}\left(1-\bar{\delta}\right)$; a is Prandtle-Karman constant; $M=\varepsilon^{-1/n}$; m, K is the porosity and penetrability ratio of the porous media; u_f is the filtration velocity; $G_0=\tau_0/\sqrt{K}$ is the initial pressure gradient in the porous media; $c=\sqrt{\beta/\rho_0 m_0}$ is the velocity of distribution in the system porous media-fluid; m_0,ρ_0 are the porosity and density of the fluid under pressure P_0; $\beta=\dfrac{\beta_f}{1+\beta_f/m_0\beta_m}$ volume elasticity; β_f,β_m are mediums of the volume elasticity of the fluid and the porous media, correspondingly; $\chi=\rho kc^2/\eta$ is the piezoconductance coefficient.

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