BABAEV A.A.

EQUALITY OF CANONICAL MORPHISMS IN BICLOSED CATEGORIES

Abstract

New method for recognition of canonical morphisms in biclosed categories was proved by virtue of methods of theory of proofs.

The present paper helps to find criterions of equality of canonical morphisms in biclosed categories.

According to correspondence between canonical morphisms in closed categories and conclusions in deductive calculations (see [1] and [2]), the problem of equality of canonical morphisms is reduced to the problem of equivalence of corresponding conclusions.

Biclosed category K is determined by the following datas:

I. category $\mathcal{K}_\varnothing$;

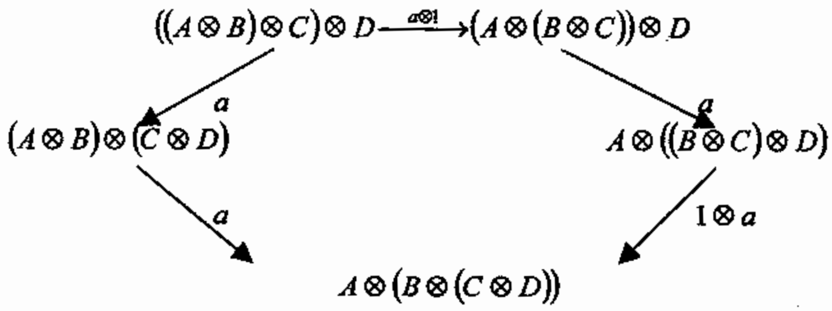
II. object $I \in 0bK$;

III. two-seater functor $\otimes : K \times K \to K$, $\supset : K^{op} \times K \to K$, $\subset : K^{op} \times K \to K$;

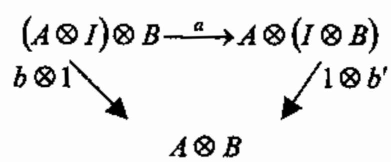
IV. natural isomorphisms
$a_{ABC} : (A \otimes B) \otimes C \to A \otimes (B \otimes C)$,
$b_A : A \otimes I \to A$,
$b'_A : I \otimes A \to A$;

V. natural transformations
$d_{AB} : A \to B \supset A \otimes B$, $d'_{AB} : A \to B \subset B \otimes A$,
$e_{AB} : (A \supset B) \otimes A \to B$, $e'_{AB} : A \otimes (A \subset B) \to B$.

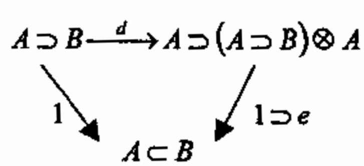
The following diagrams should commutate for that



(1)



(2)



(3)

$$\begin{array}{ccc} A\otimes B & \xrightarrow{d\otimes 1} & (B\supset(A\otimes B))\otimes B \\ & {\searrow}_{1} \qquad {\swarrow}_{e} & \\ & A\otimes B & \end{array} \qquad (4)$$

$$\begin{array}{ccc} A\subset B & \xrightarrow{d'} & A\subset(A\otimes(A\subset B)) \\ & {\searrow}_{1} \qquad {\swarrow}_{1\subset e'} & \\ & A\subset B & \end{array} \qquad (5)$$

$$\begin{array}{ccc} A\otimes B & \xrightarrow{1\otimes d'} & A\otimes(A\subset A\otimes B) \\ & {\searrow}_{1} \qquad {\swarrow}_{e'} & \\ & A\otimes B & \end{array} \qquad (6)$$

The morphisms, which can be constructed from $1, a, a^{-1}, b, b^{-1}, b', b^{-1}, d, d'$, e, e' by the help of $\otimes, \supset, \subset$ and composition are called a canonical morphisms in biclosed categories.

The equality of canonical morphisms is determined: by functority of $\otimes, \supset, \subset$; by the property of unit; by the fact, that a, b, b' are isomorphisms; by naturality of a, b, b', d, d', e, e'; by commutative property of diagrams (1)-(6); by associativity of composition and ordinary rules for equality. Sequent is an expression of the type $\Gamma \to A$, where A is a formula, Γ is a list (maybe, empty) of formulas. Formulas are constructed from variables and constant I by the help of $\otimes, \supset, \subset$.

We will say that subformula enters into formula positively (negatively), if it is situated on even (odd) place of implication sendings. We will call entering essentially positive, if it is not situated in any implication sending. $A \prec B$ means, that A is essentially positively enters to B.

A sequence is said to be balanced if every variable that occurs in it has exactly two occurrences with different signs. It is called I - balanced if it is balanced and, in any subformula of the form $A \supset B$ with B constant (i.e., constructed from the constant I), the premise A is likewise constant.

In paper [3] deductive terms, corresponding to canonical morphisms in BC-categories were defined. Here we will describe this definition.

To the each term we will subscribe as a type sequent. The record $t:\Gamma \to A$, or $t^{\Gamma\to A}$, means, that t is a term of type $\Gamma \to A$. Terms would be constructed from constant II and variables $x^A, y^A, \ldots$ for each formula of A. Term and its type are defined inductively by the following axioms and rules:

(1) x^A is a term of type $A \to A$;

(2) II is a term of type $\to$ I;

(3) if $t:\Gamma A \to B$, then $\lambda x^A t:\Gamma \to A \supset B$;

(4) if $t:\Gamma A \to B$, then $\mu\, x^A t:\Gamma \to A \subset B$;

(5) if $t:\Gamma \to A \supset B$, $s:\Sigma \to A$, then $(t,s):\Gamma\Sigma \to B$;

(6) if $t:\Gamma \to A \subset B$, $s:\Sigma \to A$, then $|t,s|:\Gamma\Sigma \to B$;

(7) if $t:\Gamma\to A$, $s:\Sigma\to B$, then $<t,s>:\Gamma\Sigma\to A\otimes B$;
(8) if $t:\Gamma AB\Sigma\to C$, $s:\Delta\to A\otimes B$, then $t_{x^A x^B}[ls,rs]:\Gamma\Delta\Sigma\to C$;
(9) if $t:\Gamma\Sigma\to A$, $s:\Delta\to I$, then $\{t,s\}:\Gamma\Delta\Sigma\to A$.

We would call these terms by the BC-terms.

Each canonical morphisms $f:A\to B$ associates with terms $\tau(f):A\to B$ and determines equivalence relation $\underset{BC}{\equiv}$ between terms:

$t\underset{BC}{\equiv}\lambda x^A(t,x^A)$, where $t:\Gamma\to A\supset B$ and x^A is new variable;

$t\underset{BC}{\equiv}\mu x^A|t,x^A|$, where $t:\Gamma\to A\subset B$ and x^A is new variable;

$t\underset{BC}{\equiv}<lt,rt>$, where $t:\Gamma\to A\otimes B$;

$t_{x^I}[s]\underset{BC}{\equiv}\{t_{x^I}[\Pi],S\}$, where $s:\Pi\to \mathrm{I}$, x^{I} freely enters into t;

$(\lambda x^A t,s)\underset{BC}{\equiv}t_{x^A}[s]$, $|\mu x^A t,s|\underset{BC}{\equiv}t_{x^A}[s]$, where $t_{x^A}[s]$ means the result of substitution of s instead of free entry of variable x^A into t;

$[l<s_1,s_2>,r<s_1,s_2>]\underset{BC}{\equiv}t_{x^A x^B}[s_1,s_2]$.

Theorem 1. *For any canonical morphisms $f,g:A\to B$ in BC-categories the following equivalence holds: $\tau(f)\underset{BC}{\equiv}\tau(g)$.*

In paper [4] the symmetrical monoidally closed (SMC) categories were considered, SMC-terms were constructed, equivalence relation $\underset{SMC}{\equiv}$ between terms was determined and the following theorem was proved.

Theorem 2. *For any canonical morphisms $f,g:A\to B$ in SMC-categories $f=g$, if and only if $\tau(f)\underset{SMC}{\equiv}\tau(g)$.*

Let $t:\Gamma\to A-BC$ is BC-term. By t^- we will denote the result of substitution in t all μ by λ, $|\,|$ by $(\,)$, and in variables, entered in t, the symbols $\subset$ by $\supset$. Γ^- is the result of substitution of all $\subset$ by $\supset$.

In paper [3] it was proved, that if $t:\Gamma\to A$ is BC-term, then $t^-:\Gamma^-\to A^-$ is SMC-term. Moreover, the following theorem was proved.

Theorem 3. *$t_1\underset{BC}{\equiv}t_2$ if and only if $t_1^-\underset{SMC}{\equiv}t_2^-$.*

Definition. *Let BC-terms be $t,t':\Gamma\to A$ and sequent $\Gamma\to A$ be balanced. We will say that satisfies to "condition of pair", if t and t' doesn't contain subterms $\left[s_1^{\Delta\to B*C},s_2^{\Sigma\to B}\right]$ and $\left[s_1'^{\Delta'\to B'*C'},s_2'^{\Sigma'\to B'}\right]$ correspondingly, where $B*C<\Sigma'$, $B'*C'<\Sigma$, $I<C,C'$, $[\,]$ is $(\,)$, when $*$ is $\supset$, $[\,]$ is $|\,|$, when $*$ is $\subset$.*

In paper [5] for SMC-terms the analogous definition was and was proved

Theorem 4. *If SMC-terms $t_1,t_2:\Gamma\to A$ are normal, sequent $\Gamma\to A$ is balanced, then $t_1\underset{SMC}{\equiv}t_2$ if and only if t_1 and t_2 satisfies to "condition of pair".*

From Theorems 3 and 4 follow

Theorem 5. *If BC-terms* $t_1, t_2 : \Gamma \to A$ *are normal, sequent* $\Gamma \to A$ *is balanced, then* $t_1 \underset{BC}{\equiv} t_2$ *if and only if* t_1 *and* t_2 *satisfies to "condition of pair".*

And from Theorems 1 and 5 follow

Theorem 6. *For any canonical morphisms* $f, g : A \to B$ *in BC- categories* $f = g$ *if and only if* $\tau(f)$ *and* $\tau(g)$ *satisfies to the "condition of pair".*

References

[1]. Lambek J. *Deductive systems and categories II.* Ject.Notes Math., 1969, v. 86, p. 76-122.
[2]. Минц Г.Е. *Теория категорий и теория доказательств.* В кн.: Актуальные вопросы логики и методологии наук, Киев, 1979, с. 252-276.
[3]. Бабаев А.А. *Равенство морфизмов и теорема когерантности для бизамкнутых категорий.* Зап.науч.семинаров, Ленингр.отд.мат.ин-та АН СССР, т. 105, 1981, с. 3-9.
[4]. Бабаев А.А. *равенство канонических морфизмов в замкнутых категориях II.* Изв. АН Азерб., сер.физ.-мат.наук, 1981, №2, с. 3-10.
[5]. Бабаев А.А. *К вопросу равенства морфизмов в замкнутых категориях.* Труды математического ин-та РАН, 1992, т. 193, с. 31-36.

Babaev A.A.
Institute of Mathematics and Mechanics of AS Azerbaijan,
9, F. Agayev str., 370141, Baku, Azerbaijan.
Tel.: 39-47-20.

Received October 10, 1999; Revised December 14, 1999.
Translated by Panarina V.K.