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THE STRENGTH CRITERION OF THE ORTOGONAL ARMOURED COMPOSITE MATERIAL

Abstract

In this work the strength criterion of the orthogonal armoured composite material is given based on the conception on specific defects accumulation by each of the natural states of the material. The system of the independent material constants is determined.

In connection with increase of demands of the constructions calculation precision and estimation of their durability in the exploitation conditions the usual calculation methods on the base of one dimensional models are not satisfy. For composites it is special important, because for them arising of the one axial stress state is little probable. For example, in the layer composite even for the one axial loading the complex stress state realized. So the singularity of the strength criterion for composite materials in addition to consideration of the structurity factor is without doubt the consideration of existence of the complex stress state.

Composite materials represent the important class of anisotrop materials. The failure conditions of anisotrop materials were constructed as usual by analogy with the similar conditions for isotrop materials. So, for example, it was supposed as for an isotrop body in stress space there exists some limit surface districting the field of safe states. In [12] it was suggested to describe this surface by the sum of two joint invariants of the stress tensors and the strength. In [13] as the equations of the limit surface the homogeneous stress function was suggested which is represented by the some invariants. The works have been developed in which the construction of the strength criterion of anisotrop bodies is based on the corresponding plasticity conditions [3,14,15]. In [16] the strength criterion of anisotrop materials are suggested which are based on introduction of some functions of joint invariants of stress tensor and vectors or tensors giving the geometrical symmetry of the body. The similar way was demonstrated for the transversal isotrop body in [17].

Some investigations on construction of strength conditions or plasticity for anisotrop bodies were directed to seeking of more natural analogies with respect to isotrop bodies. In [18] the correlations between the stresses and plastic deformations of an anisotrop medium were obtained on the base of the theory of small elastic-plastic deformations by Ilyushin A.A. by the substitution of the anisotrop body by some isotrop body whose deformed state is adequate to the deformed state of the anisotrop body and the stress state is the function of the stress state of the body and some anisotropy parameters. In [19] the analogies of deviators components and ball parts of the stress and deformations tensors are introduced and also the generalized intensivities of stresses and deformations so that to have a possibility to write three main postulates of the deformation theory of plasticity by Genky in the tradionary for an isotrop body form. The works [20,21] allowed to have a look differently at some attempts. It turned out that in order to pass the state conditions worked out for an elastic isotrop body by the natural way to anisotrop bodies there is not any necessity to model the last ones because of any reasons as the isotrop bodies. For that it is sufficient to substitute decomposition of elastic deformation energy by the finite number of inter independent elastic energies of so-called natural states similarly to that as for the isotrop body the elastic energy is representable in the form of sum of the volume change energy and the form change energy.

In construction of the limitedness criterion of composite materials the direction based on the structure approach [26,22,23,25] is alternative. This approach let concretize more exactly the given composite material, but it gives up to the approach based on consideration of the composite material as a homogen anisotrop material because of opportunities of the general mathematics investigation of constructions behaviour. The work [20] is an example of syntez of the pointed out two approaches so as the general mathematical representation of the governing correlation, given in form of natural states, differs the composite materials respecting to one and the same anisotropy type but which have different structures. The last predefined the reasons of use of the results in the suggested in [11] correlations of deformation and of the scattering failure of hereditarily elastic composites [20]. In [11] corresponding governing equations and strength criterion were suggested for the anisotrop material. In [27] the various aspects of its using are discussed and the experimental confirmation of the written-out correlations is given on model. It was shown that all parameters of the material can be determined from the experiments for the one axial loading.

In this work the strength criterion of the orthogonal armoured composite material is determined which is a kind of ortotrop anisotropy. According to [11,27] the strength criterion for the material with anisotropy of general type has a form:

$$\sum_{\alpha=1}^{n}\chi_\alpha\left[\left(\tilde{\sigma}_\alpha+M_\alpha^*\tilde{\sigma}_\alpha\right)\left(\tilde{\sigma}_\alpha+M_\alpha^*\tilde{\sigma}_\alpha\right)\right]^{1/2}=\sigma_0, \tag{1}$$

where χ_α are empirical constants, $n\le 6$ is the number of natural states of the material, $\tilde{\sigma}_\alpha$ is the stress tensor of α natural state, M_α^* is the damaging operator of this natural state governing the accumulation process of the corresponding specific defects:

$$M_\alpha^*\tilde{\sigma}_\alpha=\sum_{k=1}^{N}\Phi_\alpha\left(t_k^+\right)\int_{t_k^-}^{t_k^+}M_\alpha\left(t_k^+-\tau\right)\tilde{\sigma}_\alpha(\tau)d\tau+\int_{t_{N+1}^-}^{t}M_\alpha(t-\tau)\tilde{\sigma}_\alpha(\tau)d\tau\,. \tag{2}$$

Here $\Phi_\alpha\left(t_k^+\right)$ is the defects healing function depending on the limit on the foregoing active part of damage accumulation process by α - natural state of the stress or strain state level of the same natural state. Moreover the active parts are determined by the condition:

$$\dot{\sigma}_\alpha\ge 0;\quad \sigma_\alpha=\left(\tilde{\sigma}_\alpha\cdot\tilde{\sigma}_\alpha\right)^{1/2}, \tag{3}$$

where the upper joint means the sign of differentiation by time, and σ_α is the norm of stress tensor $\tilde{\sigma}_\alpha$.

Analysis of the strength criterion (1) for one directed armoured composite was given in [28]. For the plain model for the generalized plain stress state the norms of the stress state the norms of the stress tensors of four natural states are represented in the form:

$$\begin{aligned}&\sigma_1=2^{-1/2}\left(p\sin\aleph+\sqrt{3}\sigma_{\bar{k}}\sin(\aleph-\aleph_0)\right),\\&\sigma_2=2^{-1/2}\left(p\cos\aleph-\sqrt{3}\sigma_{\bar{k}}\cos(\aleph-\aleph_0)\right),\\&\sigma_3=2^{1/2}\tau_{\bar{m},\bar{k}},\sigma_4=2^{-1/2}\sigma_{\bar{m}},\end{aligned} \tag{4}$$

$\sigma_{33}=\sigma=const)$ loading of the plain model (fig.2); when $\vec{n}(1,0,0)$; $\vec{m}(0,\sin\theta,\cos\theta)$; $\vec{k}(0,-\cos\theta,\sin\theta)$ we have:

$$\sigma_{\vec{n}}=\tau_{\vec{n}\vec{m}}=\tau_{\vec{k},\vec{n}}=0;\quad \sigma_{\vec{m}}=\sigma\cos^2\theta\quad \sigma_{\vec{k}}=\sigma\sin^2\theta;\quad \tau_{\vec{k},\vec{m}}=\sigma\cos\theta\sin\theta .$$

Then

$$\begin{cases}\sigma_{(N)ij}=\left(p_N n_i n_j+q_N m_i m_j+r_N k_i k_j\right)\left(q_N\sigma_{\vec{m}}+r\sigma_{\vec{k}}\right)\quad N=I,II,III\\ \sigma_{(IV)11}=\sigma_{(IV)12}=\sigma_{(IV)13}=0;\quad \sigma_{(IV)23}=-\tau_{\vec{k},\vec{m}}\cos2\theta;\\ \sigma_{(IV)22}=\sigma_{(IV)33}=\tau_{\vec{k},\vec{m}}\sin2\theta\end{cases}\tag{9}$$

The determined at the experiments elasticity modulus $E(\vec{s})$ in the loading direction $\vec{s}$, Poisson's coefficient $\nu(\vec{s},\vec{r})$, governing the deformation degree in direction $\vec{r}$ at the loading, at the loading in the orthogonal direction $\vec{s}$ and also the shear modulus $G(\vec{s},\vec{r})$ are found by the formulas:

$$\begin{cases}\dfrac{1}{E(\vec{s})}=\sum\limits_{N=1}^{III}\mu_N\left(q_N\cos^2\theta+r_N\sin^2\theta\right)+\dfrac{1}{2}\mu_N\sin^2 2\theta\\ -\dfrac{\nu(\vec{s},\vec{r})}{E(\vec{s})}=\sum\limits_{N=I}^{III}\mu_N\left[\dfrac{1}{4}\left(q_N-r_N\right)^2\sin^2 2\theta+q_N r_N\right]-\dfrac{1}{2}\mu_{IV}\sin^2 2\theta\\ \dfrac{1}{G(\vec{s},\vec{r})}=\sum\limits_{N=1}^{III}\mu_N\left(q_N-r_N\right)^2\sin^2 2\theta+2\mu_{IV}\cos^2 2\theta\end{cases}\tag{10}$$

The method of determination of moduluses $E(\vec{s})$, $\nu(\vec{s},\vec{r})$ demonstrated for the one-directed armoured composite [27] is acceptable also for the orthogonal armoured composite. Further by them the six material constants μ_N are determined and also three independent parameters connecting nine constants P_N, q_N, r_N, $N=I;II;III$. After that the stress tensors of the natural states of the orthogonal armoured composites (8) are quite determined and they govern by that the final form of the strength criterion (1).

As these three independent parameters in [20] the following are suggested:

$$\aleph_N=p_N+q_N+r_N .\tag{11}$$

However, it can be persuaded that these three parameters are not independent. Let show it. Let's take in the space the orthogonal frame $\{\vec{e}_i\}$ and in this frame let's consider three vectors determined as:

$$\vec{\vartheta}_N=p_N\vec{e}_1+q_N\vec{e}_2+r_N\vec{e}_3 .\tag{12}$$

It is easy possible to show that the vectors $\vec{\vartheta}_N$ represent the three of ortonormalized vectors:

$$\vec{\vartheta}_N\cdot\vec{\vartheta}_M=\delta_{NM} .\tag{13}$$

Here δ_{NM} are Cronecker's symbols. Really, on the base of the fundamental property of the basis tensors

$$\left(\omega_N\right)_{ij}\cdot\left(\omega_M\right)_{ij}=\delta_{NM}\tag{14}$$

from the first three formulas (5) we obtain

$$\begin{cases} \omega_{(N)ij} \cdot \omega_{(N)ij} = p_N^2 + q_N^2 + r_N^2 = \left|\vec{\vartheta}_N\right|^2 = 1 \\ \omega_{(N)ij} \cdot \omega_{(M)ij} = p_N \cdot p_M + q_N \cdot q_M + r_N \cdot r_M = \\ = \vec{\vartheta}_N \cdot \vec{\vartheta}_M = 0 \end{cases} \quad (15)$$

Let's take in the frame of the vector $\vec{a}(1,1,1)$ and $\vec{b}(\aleph_1, \aleph_2, \aleph_3)$ (fig.3). It is not difficult to be persuaded to that in the frame $\{\vec{V}_i\}$ determined by (12), vector $\vec{a}$ will take the position corresponding to the position in the frame $\{\vec{e}_i\}$ of the vector $\vec{b}$ (fig.4), that is it will have the projections. Really:

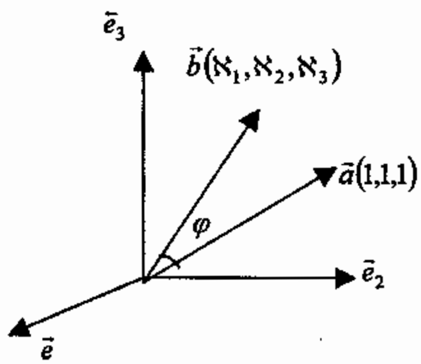

Fig. 3.

$$\Pi P_{\vec{\vartheta}_N} \vec{a} = \left(\vec{a}, \vec{\vartheta}_N\right) = p_N + q_N + r_N = \aleph_N \quad N = I, II, III. \quad (16)$$

But by the known property of projections of the vector for the length of the vector we have:

$$|\vec{a}|^2 = \sum_{N=I}^{III} \left(\Pi P_{\vec{\vartheta}} \vec{a}\right)^2 = \aleph_1^2 + \aleph_2^2 + \aleph_3^2 = 3. \quad (17)$$

Existence of correlation (17) connecting three parameters $\aleph_N$, shows that only two of them are independent. Further, let us consider the choosing of the third independent parameter.

From the foregoing reasonings it follows that if to make a turn around the axis $\vec{c}$ which is perpendicular to the plane of the vectors $\vec{a}$ and $\vec{b}$ for angle φ between them until vector $\vec{b}$ coincides with vector $\vec{a}$, then frame $\{\vec{e}_i\}$ will pass to the frame $\{\vec{\vartheta}_i\}$ but with precision upto the turn angle around the vector $\vec{a}$ for some angle γ, or rotation of the frame $\{\vec{e}_i\}$ around the axis $\vec{a}$ doesn't change anything. Therefore, for coincide of the frame $\{\vec{e}_i\}$ with the frame $\{\vec{\vartheta}_i\}$ it is necessary first to make a turn around the axis $\vec{c}$ which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$ until the vector $\vec{b}$ coincides with the vector $\vec{a}$ and then to make a turn around the axis $\vec{a}$ for angle γ. Such one-to-one translation of $\{\vec{e}_i\}$ into $\{\vec{\vartheta}_i\}$ chooses the parameter γ which can be taken as the third independent parameter as for example $\aleph_1$ and $\aleph_2$.

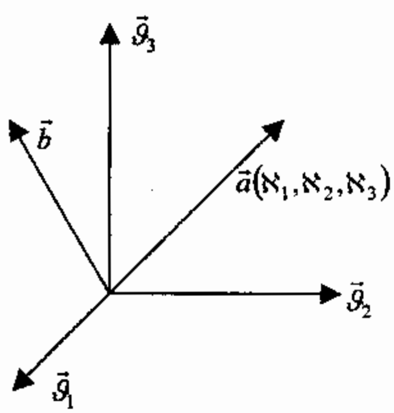

Fig. 4.

The reasonings are constructive and done let obtain the formulas of dependence of the nine quantities p_N, q_N, r_N $(N = I, II, III)$ on three independent parameters $\aleph_1, \aleph_2, \gamma$.

$$\begin{cases}\omega_{(N)ij}\cdot\omega_{(N)ij}=p_N^2+q_N^2+r_N^2=\left|\vec{\vartheta}_N\right|^2=1\\ \omega_{(N)ij}\cdot\omega_{(M)ij}=p_N\cdot p_M+q_N\cdot q_M+r_N\cdot r_M=\\ =\vec{\vartheta}_N\cdot\vec{\vartheta}_M=0\end{cases}\qquad(15)$$

Let's take in the frame of the vector $\vec{a}(1,1,1)$ and $\vec{b}(\aleph_1,\aleph_2,\aleph_3)$ (fig.3). It is not difficult to be persuaded to that in the frame $\{\vec{V}_i\}$ determined by (12), vector $\vec{a}$ will take the position corresponding to the position in the frame $\{\vec{e}_i\}$ of the vector $\vec{b}$ (fig.4), that is it will have the projections.
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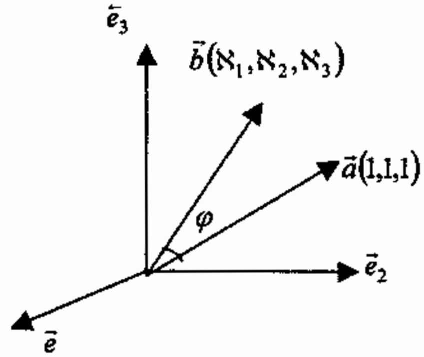

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$$\Pi P_{\vec{\vartheta}_N}\vec{a}=\left(\vec{a},\vec{\vartheta}_N\right)=p_N+q_N+r_N=\aleph_N\quad N=I,II,III.\qquad(16)$$

But by the known property of projections of the vector for the length of the vector we have:

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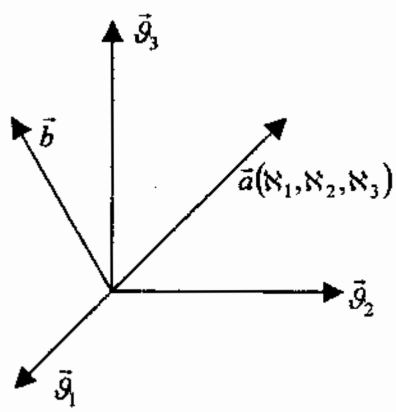
Existence of correlation (17) connecting three parameters $\aleph_N$, shows that only two of them are independent. Further, let us consider the choosing of the third independent parameter.

From the foregoing reasonings it follows that if to make a turn around the axis $\vec{c}$ which is perpendicular to the plane of the vectors $\vec{a}$ and $\vec{b}$ for angle φ between them until vector $\vec{b}$ coincides with vector $\vec{a}$, then frame $\{\vec{e}_i\}$ will pass to the frame $\{\vec{\vartheta}_i\}$ but with precision upto the turn angle around the vector $\vec{a}$ for some angle γ ,or rotation of the frame $\{\vec{e}_i\}$ around the axis $\vec{a}$ doesn't change anything. Therefore, for coincide of the frame $\{\vec{e}_i\}$ with the frame $\{\vec{\vartheta}_i\}$ it is necessary first to make a turn around the axis $\vec{c}$ which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$ until the vector $\vec{b}$ coincides with the vector $\vec{a}$ and then to make a turn around the axis $\vec{a}$ for angle γ. Such one-to-one translation of $\{\vec{e}_i\}$ into $\{\vec{\vartheta}_i\}$ chooses the parameter γ which can be taken as the third independent parameter as for example $\aleph_1$ and $\aleph_2$.



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The reasonings are constructive and done let obtain the formulas of dependence of the nine quantities p_N,q_N,r_N $(N=I,II,III)$ on three independent parameters $\aleph_1,\aleph_2,\gamma$.

Let us denote by R the transform of the turn around the axis $\vec{c}$, which is perpendicular to the plane of vectors, $\vec{a}$ and $\vec{b}$ for angle φ between them until the direction of vector $\vec{b}$ coincide with vector $\vec{a}$. Then we will have:

$$R_{ij} = \delta_{ij}\cos\varphi + (1-\cos\varphi)c_i c_j - \sum_{k=1}^{3} c_k \varepsilon_{jk}\sin\varphi, \tag{18}$$

where δ_{ij} is Kronecker's symbol, ε_{ijk} is the tensor by Lavy-Chavit and

$$\vec{c} = \frac{\left|\vec{b}\times\vec{a}\right|}{\left|\left[\vec{b}\times\vec{a}\right]\right|} \tag{19}$$

and

$$\begin{cases} \cos\varphi = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \dfrac{\aleph_1+\aleph_2+\aleph_3}{\sqrt{3}\sqrt{\aleph_1^2+\aleph_2^2+\aleph_3^2}} = \dfrac{1}{3}(\aleph_1+\aleph_2+\aleph_3) \\ \vec{c}\sin\varphi = \dfrac{1}{3}\left\{(\aleph_2-\aleph_3)\vec{e}_1 + (\aleph_3-\aleph_1)\vec{e}_2 + (\aleph_1-\aleph_2)\vec{e}_3\right\} \end{cases} \tag{20}$$

Let denote by Q the transform of the turn around the vector $\vec{a}$ for angle γ. The components of the matrix giving this transform will be

$$Q_{ij} = \delta_{ij}\cos\gamma + \frac{1}{3}(1-\cos\gamma) - \frac{1}{3}\sin\gamma\sum_{k=1}^{3}\varepsilon_{ijk}. \tag{21}$$

Now the abovementioned reasonings about transform of the frame $\{\vec{e}_i\}$ into the frame $\{\vec{V}_i\}$ form of the following formula:

$$QR\{\vec{e}_i\} = \{\vec{\vartheta}_i\}. \tag{22}$$

Writing the transform (22) in projections for each system of the translation $\vec{e}_i \sim \vec{\vartheta}_i$, $i=1,2,3$, taking into account (18), (20), (21) we obtain

$$p_1 = \frac{1}{9}\left\{(1+2\cos\gamma)\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_2-\aleph_3)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right] + \left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\times\right.$$

$$\left.\times\left[\frac{(\aleph_1-\aleph_2)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_1-\aleph_3\right] + \left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_1-\aleph_2)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_1-\aleph_3\right]\right\};$$

$$q_1 = \frac{1}{9}\left\{\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_2-\aleph_3)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]+\right.$$

$$+(1+2\cos\gamma)\left[\frac{(\aleph_3-\aleph_1)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_1-\aleph_2\right]+$$

$$\left.+\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_1-\aleph_2)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_1-\aleph_3\right]\right\};$$

$$r_1 = \frac{1}{9}\left\{\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_2-\aleph_3)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right] + \left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\times\right.$$

$$\times\left[\frac{(\aleph_3-\aleph_1)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_1-\aleph_2\right]+(1+2\cos\gamma)\left[\frac{(\aleph_1-\aleph_2)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_1-\aleph_3\right]\Bigg\};$$

$$p_2=\frac{1}{9}\Bigg\{(1+2\cos\gamma)\left[\frac{(\aleph_3-\aleph_1)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_2-\aleph_1\right]+\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\times$$

$$\times\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_3-\aleph_1)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]+\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_1-\aleph_2)(\aleph_3-\aleph_1)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_2-\aleph_1\right]\Bigg\};$$

$$q_2=\frac{1}{9}\Bigg\{\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_2-\aleph_3)(\aleph_3-\aleph_1)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_2-\aleph_1\right]+(1+2\cos\gamma)\times$$

$$\times\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_3-\aleph_1)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]+\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_1-\aleph_2)(\aleph_3-\aleph_1)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_2-\aleph_3\right]\Bigg\};$$

$$r_2=\frac{1}{9}\Bigg\{\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_2-\aleph_3)(\aleph_3-\aleph_1)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_2-\aleph_1\right]+\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\times$$

$$\times\left[\aleph_1+\aleph_2+\aleph_{3+}\frac{(\aleph_3-\aleph_1)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]+(1+2\cos\gamma)\left[\frac{(\aleph_1-\aleph_2)(\aleph_3-\aleph_1)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_2-\aleph_3\right]\Bigg\};$$

$$p_3=\frac{1}{9}\Bigg\{\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_1-\aleph_2)(\aleph_3-\aleph_1)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_3-\aleph_1\right]+\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\times$$

$$\times\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_1-\aleph_2)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]+(1+2\cos\gamma)\left[\frac{(\aleph_1-\aleph_2)(\aleph_2-\aleph_3)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_3-\aleph_1\right]\Bigg\};$$

$$q_3=\frac{1}{9}\Bigg\{\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_2-\aleph_3)(\aleph_1-\aleph_2)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_3-\aleph_1\right]+(1+2\cos\gamma)\times$$

$$\times\left[\frac{(\aleph_3-\aleph_1)(\aleph_1-\aleph_2)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_3-\aleph_1\right]+\left(1-\cos\gamma+\sqrt{3}\sin\gamma\right)\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_1-\aleph_2)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]\Bigg\};$$

$$r_3=\frac{1}{9}\Bigg\{\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)\left[\frac{(\aleph_2-\aleph_3)(\aleph_1-\aleph_2)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_3-\aleph_1\right]+\left(1-\cos\gamma-\sqrt{3}\sin\gamma\right)+$$

$$\times\left[\frac{(\aleph_3-\aleph_1)(\aleph_1-\aleph_2)}{3+\aleph_1+\aleph_2+\aleph_3}+\aleph_3-\aleph_1\right]+(1+\cos\gamma)\left[\aleph_1+\aleph_2+\aleph_3+\frac{(\aleph_1-\aleph_2)^2}{3+\aleph_1+\aleph_2+\aleph_3}\right]\Bigg\}\qquad(23)$$

By the written-out correlations taking into account (17), the quantities p_N, q_N, r_N are expressed by three independent parameters $\aleph_1, \aleph_2$ and γ.

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