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ON DETERMINATION OF STRESS CONDITION IN THE INFINITE ANISOTROP BODY CONTAINING TWO NEIGHBOURING PERIODICALLY CURVED FIBERS.

Abstract

In [1,2] the method for investigation of the stress state in the infinite body containing a single curved fiber was proposed. In this paper this method is developed for the case where the infinite body contains two neighbouring periodically curved fibers. It is supposed that the materials of the fibers and the matrix are transversal - isotrop and homogeneous. Introducing the small parameter characterizing the curving degree of the fibers the solution of the corresponding problems is reduced to the solution of the subsequental boundary problems for the canonic many-bonded areas. The solution of each of these problems is reduced to the solution of the system of the infinite algebraic equations whose coefficients contain different complex combinations of Bessel's functions. It is proved that the determinants of these infinite systems are the normal type determinants.

1. Formulation of the problem.

We consider an infinite body (matrix) containing two non-intersecting neighbouring periodically curved fibers. The values related to the fibers are denoted by upper index (2)m, where m shows the number of a fiber, and the values related to the matrix by upper index (1). We associate Cartesian system of coordinates $o_m x_{1m} x_{2m} x_{3m}$ and a cylindrical system of coordinates $o_m r_m \theta_m x_{3m}$ with the m-th fiber. We will assume that the fibers lie along the $\theta_m x_{3m}$ axis and $x_{31} = x_{32} = x_3$.

We examine the case where the middle line of the fiber (fig.1) lies on the $q_m x_{2m} x_{3m}$ plane and the materials of the matrix and the fibers are transversal – isotrop whose isotropy axis coincides with $o_m x_{3m}$ axis.

Thus, we write the equilibrium equation, Hook's law and the Cauchy relations within the limits of the fibers and the matrix.

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{r3}}{\partial x_3} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0\ ,\ \ldots,$$

$$\varepsilon_{rr} = \frac{1}{E_1}\sigma_{rr} - \frac{\nu}{E_1}\sigma_{\theta\theta} - \frac{\nu_1}{E_3}\sigma_{33},\ \ldots, \tag{1}$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r};$$

In (1) the conventional notation is used. Moreover in (1) the upper indices which are omitted will be taken into account when we apply these equations.

We assume that there is complete cohesion in the fiber - matrix interfaces S_m :

$$(\sigma_{rr}^{(2)m} n_r^m + \sigma_{r\theta}^{(2)m} n_\theta^m + \sigma_{r3}^{(2)m} n_3^m)\big|_{S_m} = (\sigma_{rr}^{(1)m} n_r^m + \sigma_{r\theta}^{(1)} n_\theta^m + \sigma_{r3}^{(1)} n_3^m)\big|_{S_m},\ \ldots,$$

$$u_r^{(2)m}\big|_{S_m} = u_r^{(1)}\big|_{S_m},\ \ldots, \tag{2}$$

where n_r^m, n_q^m, n_3^m are the components of the unite normal vector to the surface S_m in the cylindrical system of coordinates $q_m r_m q_m x_{3m}$.

The equation of the middle line of the m-th fiber is written as:

$$x_{2m} = A\sin(2\pi x_3|\ell) = \ell \cdot A|\ell\sin(2\pi x_3|\ell) = \ell\varepsilon\sin(2\pi x_3|\ell), \quad x_{1m} = 0 \qquad (3)$$

$\varepsilon = A|\ell$ is the dimensionless parameter, $A < \ell$, $\varepsilon \in [0,1)$.

2. Method of solution.

We will seek the quantities characterizing the stress-strain state of the matrix and fiber in the form of series in positive powers of the small parameter ε:

$$\sigma_{rr}^{(k)m} = \sum_{q=0}^{\infty}\varepsilon^q\sigma_{rr}^{(k)m,q},\ldots,\varepsilon_{rr}^{(k)m} = \sum_{q=0}^{\infty}\varepsilon^q\varepsilon_{rr}^{(k)m,q},\ldots,$$

$$u_r^{(k)m} = \sum_{q=0}^{\infty}\varepsilon^q u_r^{(k)m,q},\ldots. \qquad (4)$$

We suppose that the cross sections of the fiber normal to the middle line of the fiber are circles with constant radius R_m along the entire length of the fiber.

Remaking the operations given in [1,2] for each fiber we obtain the following expressions for coordinates of the surfaces S_m and for orto-normals:

$$r_m = R_m + \sum_{q=1}^{\infty}\varepsilon^q a_{rqm}(R_m,\theta_m,t_3),$$

$$x_{3m} = t_3 + \sum_{q=1}^{\infty}\varepsilon^q a_{3q}(R_m,\theta_m,t_3),$$

$$n_r^m = 1 + \sum_{q=1}^{\infty}\varepsilon^q b_{rq}(R_m,\theta_m,t_3), \qquad (5)$$

$$n_\theta^m = \sum_{q=1}^{\infty}\varepsilon^q b_{\theta q}(R_m,\theta_m,t_3),$$

$$n_3 = \sum_{q=1}^{\infty}\varepsilon^q b_{3q}(R_m,\theta,t_3).$$

According to fig.1 we can write the following relations:

$$x_{31} = x_{32} = x_3, \quad x_{11} = x_{12}, \quad x_{21} = R_{12} + x_{22},$$
$$r_1\cos\theta_1 = r_2\cos\theta_2, r_1\sin\theta_1 = R_{12} + r_2\sin\theta_2. \qquad (6)$$

We assume that the middle line of the fibers lies on the plane $x_{11} = x_{12} = 0$ and consider the following cases:

1) the curving of the fibers is co-phase, i.e. the equation of the middle lines is taken as follows:

$$x_{21} = \varepsilon\delta(x_3), \quad x_{11} = 0 \text{ for the 1-st fiber}$$
$$x_{22} = \varepsilon\delta(x_3), \quad x_{12} = 0 \text{ for the 2-nd fiber} \quad \text{(fig.1)} \qquad (7)$$

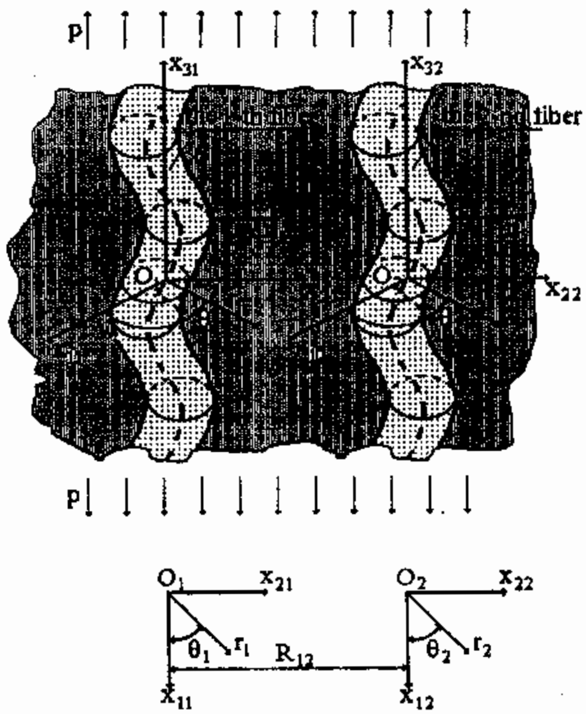

Fig. 1.

2) the curving of the fibers is anti-phase,

$$x_{21} = \varepsilon\delta(x_3), \quad x_{11} = 0 \text{ for the 1-st fiber}$$
$$x_{22} = -\varepsilon\delta(x_3), \quad x_{12} = 0 \text{ for the 2-nd fiber} \quad \text{(fig.2)} \qquad (8)$$

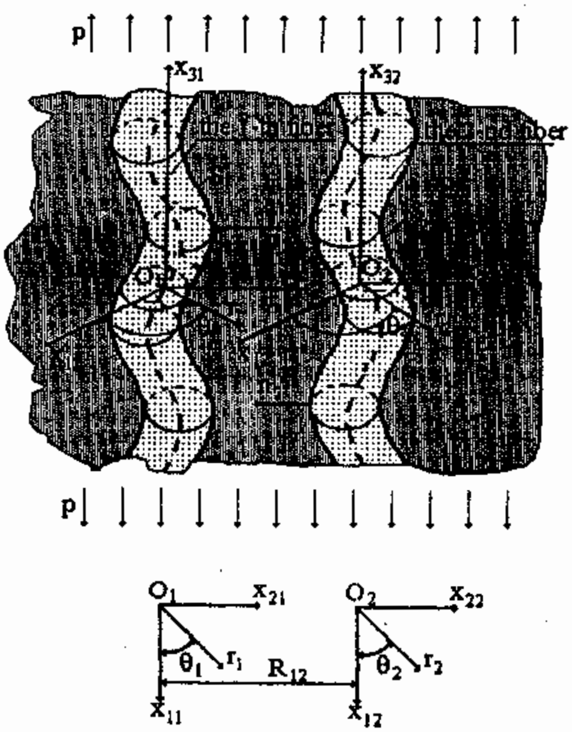

Fig. 2.

We will consider the periodic curving of the fibers and therefore the expressions for the parameter ε and for the function $d(x_3)$ are taken as

$$\varepsilon = \frac{L}{\ell}, \quad \delta(x_3) = \ell \sin\alpha\, x_3, \tag{9}$$

where $L < \ell$, $\alpha = 2\pi/\ell$; L and ℓ are respectively the amplitude and length of the periodic curving form.

Repeating the operations performed in [1,2] we obtain the contact relations for each approximation. Due to linearity of the governing equations we obtain a closed system of equations for each approximation separately.

We consider the determination of the values of the zeroth and the first approximations.

The zeroth approximation. The determination of this approximation is reduced to the investigation of the generalized plane strain state in the infinite plane containing two neighbouring circular inclusions. This investigation must be made in the case where

$$u_3^{(1),0} = u_3^{(2)1,0} = u_3^{(2),2,0} = \varepsilon_{33}^{(1),0} x_3, \quad \varepsilon_{33}^{(1),0} = \frac{p}{E_3^{(1)}}. \tag{10}$$

It should be noted that the method of solution for such problems has been improved by many researches; the list of contributors can be found in [1,2] and elsewhere. It follows from the results of these investigations that the stresses arising under (10) in the matrix (plane) and in the fibers (in each circular inclusion) can be represented as follows:

$$\begin{aligned}&\left\{\sigma_{rr}^{(2)m,0};\sigma_{\theta\theta}^{(2)m,0};\sigma_{r\theta}^{(2)m,0};\sigma_{rr}^{(1),0};\sigma_{\theta\theta}^{(1),0};\sigma_{r\theta}^{(1),0}\right\}=\\&=\left(\nu_1^{(1)}-\nu_1^{(2)}\right)\left\{s_{rr}^{(2)m,0};s_{\theta\theta}^{(2)m,0};s_{r\theta}^{(2)m,0};s_{rr}^{(1),0};s_{\theta\theta}^{(1),0};s_{r\theta}^{(1),0}\right\}.\end{aligned} \tag{11}$$

The relation (11) shows that in the cases where

$$\nu_1^{(1)} = \nu_1^{(2)} \tag{12}$$

the zeroth approximation corresponds to the homogeneous stress state and only the stresses

$$\sigma_{33}^{(1),0} = p, \quad \sigma_{33}^{(2)1,0} = \sigma_{33}^{(2)2,0} = \frac{E_3^{(2)}}{E_3^{(1)}} p \tag{13}$$

remain non-zero.

For simplicity we assume that the relation (12) holds, no great difficulties arise if it does not hold.

The first approximation. We consider only the co-phase curving case (fig. 1) and note that the anti-phase case (fig.2) can be analyzed similarly. Thus, for the first approximation we obtain the following contact conditions.

$$\begin{aligned}&\left(\sigma_{rr}^{(1),1} - \sigma_{rr}^{(2)m,1}\right)\Big|_{(R,\theta_m,t_3)} = 0;\left(\sigma_{r\theta}^{(1),1} - \sigma_{r\theta}^{(2)m,1}\right)\Big|_{(r,\theta_m,t_3)} = 0,\\&\left(\sigma_{r3}^{(1),1} - \sigma_{r3}^{(2)m,1}\right)\Big|_{(r,\theta_m,t_3)} = 2\pi\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)m,0}\right)\cos\alpha t_3 \sin\theta_m,\\&\left(u_{r;\theta;3}^{(1),1} - u_{r;\theta;3}^{(2)m,1}\right)\Big|_{(R,\theta,t_3)} = 0, \quad m = 1,2.\end{aligned} \tag{14}$$

In addition to the conditions (14) we impose the attenuation and limitation conditions

$$\left\{\sigma_{rr}^{(1),1};\ldots;\sigma_{r3}^{(1),1};u_r^{(1),1};\ldots;u_3^{(1),1}\right\}\Big|_{r_1;r_2\to\infty} \to 0,$$

$$\left\{ \left| \sigma_{rr}^{(2)m,1} \right|; \ldots; \left| \sigma_{r3}^{(2)m,1} \right|; \left| u_2^{(2)m,1} \right|; \ldots; \left| u_3^{(2)m,1} \right| \right\} \Big|_{r_m \to 0} < M, \quad M = const. \tag{15}$$

Thus the determination of the first approximation is reduced to the solution of the system of equations (1), (2), (3) satisfied within each fiber and matrix separately in the framework of the conditions (14) and (15). For the solution to this problem we use the representation [5]

$$u_r = \frac{1}{r}\frac{\partial}{\partial \theta}\Psi - \frac{\partial^2}{\partial r \partial x_3}X;$$

$$u_\theta = -\frac{\partial}{\partial r}\Psi - \frac{1}{r}\frac{\partial^2}{\partial \theta \partial x_3}X;$$

$$u_3 = \left(A_{13} + G_1\right)^{-1}\left(A_{11}\Delta_1 + G_1\frac{\partial^2}{\partial x_3^2}\right)X;$$

$$\Delta_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \tag{16}$$

Note that the functions Ψ and χ satisfy the following equations

$$\left.\begin{aligned} &\left(\Delta_1 + \xi_1^2\frac{\partial^2}{\partial x_3^2}\right)\Psi = 0, \\ &\left(\Delta_1 + \xi_2^2\frac{\partial^2}{\partial x_3^2}\right)\left(\Delta_1 + \xi_3^2\frac{\partial^2}{\partial x_3^2}\right)\chi = 0 \end{aligned}\right\} \tag{17}$$

The expressions of the constants ξ_i^2 $(i = 1,2,3)$ were given in [5].

According to the contact conditions (14) we select the solution of the equations in the following form:

$$\left\{\begin{matrix} \sin\alpha\, x_3 \\ \cos\alpha\, x_3 \end{matrix}\right\}\sum_{n=-\infty}^{+\infty} f_n(\alpha r)\exp(in\theta), \quad i = \sqrt{-1}. \tag{18}$$

Substituting (18) into equations (17) we obtain the corresponding Bessel equation to determine the functions $f_n(\alpha r)$. The selection of the solutions of latter equations depends on the sign and character (real or complex) of the constants $\left(\xi_i^{(2)m}\right)^2$ and $\left(\xi_i^{(1)}\right)^2$. We assume that $\xi_i^2 > 0$, $\xi_2^2 \neq \xi_3^2$. Taking this situation and the conditions (15) into account we obtain the following solution of the equation (17).

For fibers:

$$\Psi^{(2)m} = \alpha \sin \alpha x_3 \sum_{n=-\infty}^{+\infty} C_n^{(2)m} I_n\left(\xi_1^{(2)}\alpha r_m\right)\exp\left(in\theta_m\right),$$

$$\chi^{(2)m} = \cos \alpha x_3 \sum_{n=-\infty}^{+\infty}\left[A_n^{(2)m} I_n\left(\xi_2^{(2)}\alpha r_m\right) + B_n^{(2)m} I_n\left(\xi_3^{(2)}\alpha r_m\right)\right]\exp\left(in\theta_m\right). \tag{19}$$

For matrix:

$$\Psi^{(1)} = \alpha \sin \alpha x_3 \sum_{m=1}^{2}\sum_{n=-\infty}^{+\infty} C_n^{(1)m} K_n\left(\xi_1^{(1)}\alpha r_m\right)\exp\left(in\theta_m\right),$$

$$\chi^{(1)} = \cos \alpha x_3 \sum_{m=1}^{2}\sum_{n=-\infty}^{+\infty}\left[A_n^{(1)m} K_n\left(\xi_2^{(1)}\alpha r_m\right) + B_n^{(1)m} K_n\left(\xi_3^{(1)}\alpha r_m\right)\right]\exp\left(in\theta_m\right). \tag{20}$$

In (19) and (20) I_n (x) is a Bessel function of a purely imaginary argument, $K_n(x)$ is a McDonald function, $A_n^{(k)m}, B_n^{(k)m}$ and $C_n^{(k)m}$ $(k=1,2; \quad m=1,2)$ are unknown constants. Note that to obtain the solution (20) for the matrix, it is considered as multi - connected (two-connected) regions and the solution is constructed by summing the solutions to the corresponding single-connected regions.

As it follows from (19), (20), (16) and (1) the selected solution satisfies the conditions (15). Now we attempt to satisfy the contact condition (14). For this purpose we must represent the expressions (19), (20) in the m-th cylindrical coordinate system to satisfy the contact conditions on the m-th fiber-matrix interface S_m. The expressions (19) have already been presented in the m-th cylindrical system of coordinates. To make these operations for the expressions (20) we use the summation theorem [3] for the $K_n(X)$ function, which can be written for the case at hand as follows:

$$
\begin{gathered}
r_m \exp i\theta_m = R_{mn} \exp i\varphi_{mn} + r_n \exp i\theta_n, \\
K_\nu(Cr_n)\exp i\nu\theta_n = \sum_{k=-\infty}^{+\infty}(-1)^\nu I_k(Cr_m)K_{\nu-n}(CR_{mn})\exp[i(\nu-n)\varphi_{mn}]\exp ik\theta_m, \\
mn=12;21, \quad m;n=1,2, \quad C=const, \\
r_m < R_{mn}, \quad R_{12}=R_{21}, \quad \varphi_{12}=\frac{\pi}{2}, \quad \varphi=\frac{3\pi}{2}.
\end{gathered}
\tag{21}
$$

Thus applying the summation theorem (21) to the solutions (20), we obtain the following expressions for each cylindrical system of coordinates.

In the $O_1r_1\theta_1x_{31}$ coordinate system:

$$
\begin{gathered}
\Psi^{(1)} = \alpha\sin\alpha x_3 \sum_{n=-\infty}^{+\infty}\Big\{C_n^{(1)1}K_n\left(\xi_1^{(1)}\alpha r_1\right)+ \\
+I_n\left(\xi_1^{(1)}\alpha r_1\right)\sum_{\nu=-\infty}^{+\infty}(-1)^\nu\exp\left[i(\nu-n)\frac{\pi}{2}\right]C_\nu^{(1)2}K_{\nu-n}\left(\xi_1^{(1)}\alpha R_{12}\right)\Big\}\exp in\theta_1, \\
\chi^{(1)} = \cos\alpha x_3\sum_{n=-\infty}^{+\infty}\Big\{A_n^{(1)1}K_n\left(\xi_2^{(1)}\alpha r_1\right)+B_n^{(1)1}K_n\left(\xi_3^{(1)}\alpha r_1\right)+ \\
+I_n\left(\xi_1^{(1)}\alpha r_1\right)\sum_{\nu=-\infty}^{+\infty}(-1)^\nu\exp\left[i(\nu-n)\frac{\pi}{2}\right] \\
\left[A_\nu^{(1)2}K_{\nu-n}\left(\xi_2^{(1)}\alpha R_{12}\right)+B_\nu^{(1)2}K_{\nu-n}\left(\xi_3^{(1)}\alpha R_{12}\right)\right]\Big\}\exp in\theta_1 .
\end{gathered}
\tag{22}
$$

In the $o_2r_2\theta_2x_{32}$ coordinate system:

$$
\begin{gathered}
\Psi^{(1)} = \alpha\sin\alpha x_3\sum_{n=-\infty}^{+\infty}\Big\{C_n^{(1)2}K_n\left(\xi_1^{(1)}\alpha r_2\right)+I_n\left(\xi_1^{(1)}\alpha r_2\right) \\
\sum_{\nu=-\infty}^{+\infty}(-1)^\nu\exp\left[i(\nu-n)\frac{3\pi}{2}\right]C_\nu^{(1)1}K_{\nu-n}\left(\xi_1^{(1)}\alpha R_{12}\right)\Big\}\exp in\theta_2, \\
\chi^{(1)} = \cos\alpha x_3\sum_{n=-\infty}^{+\infty}\Big\{A_n^{(1)2}K_n\left(\xi_2^{(1)}\alpha r_2\right)+B_n^{(1)2}K_n\left(\xi_3^{(1)}\alpha r_2\right)+ \\
+I_n\left(\xi_1^{(1)}\alpha r_2\right)\sum_{\nu=-\infty}^{+\infty}(-1)^\nu\exp\left[i(\nu-n)\frac{3\pi}{2}\right]\Big[A_\nu^{(1)1}K_{\nu-n}\left(\xi_2^{(1)}\alpha R_{12}\right)+ \\
+B_\nu^{(1)1}K_{\nu-n}\left(\xi_3^{(1)}\alpha R_{12}\right)\Big]\Big\}\exp in\theta_2,
\end{gathered}
$$

$$r_2 < R_{12}. \tag{23}$$

According to the form of the expressions (19) and (20) the unknown constants $A_n^{(k)m}$, $B_n^{(k)m}$ and $C_n^{(k)m}$ $(k=1,2;\ m=1,2)$ must be taken as complex numbers and must satisfy the following relations:

$$\begin{gathered} A_n^{(k)m} = \overline{A_{-n}^{(k)m}},\ B_n^{(k)m} = \overline{B_{-n}^{(k)m}},\ C_n^{(k)m} = \overline{C_{-n}^{(k)m}}, \\ \operatorname{Im} A_0^{(k)m} = 0,\ \operatorname{Im} B_0^{(k)m} = 0,\ \operatorname{Im} C_0^{(k)m} = 0, \end{gathered} \tag{24}$$

where the overline indicates complex conjugate. Thus, using (22), (23), (19) and (14) we obtain an infinite system of algebraic equations with respect to the unknown constants (24). Introducing the notation

$$C_n^{(1)m} K_n\left(\xi_1^{(1)} x\right) = y_{n1}^{(1)m} + i z_{n1}^{(1)m}, \quad A_n^{(1)m} K_n\left(\xi_2^{(1)} x\right) = z_{n2}^{(1)m} + i y_{n2}^{(1)m},$$

$$B_n^{(1)m} K_n\left(\xi_3^{(1)} x\right) = z_{n3}^{(1)m} + i y_{n3}^{(1)m}, \quad C_n^{(2)m} I_n\left(\xi_1^{(2)} x\right) = y_{n1}^{(2)m} + i z_{n1}^{(2)m},$$

$$A_n^{(2)m} I_n\left(\xi_2^{(1)} x\right) = z_{n2}^{(2)m} + i y_{n2}^{(2)m}, \quad B_n^{(2)m} I_n\left(\xi_3^{(1)} x\right) = z_{n3}^{(2)m} + i y_{n3}^{(2)m},$$

$$Z_n^{(k)} = \begin{Vmatrix} z_{n1}^{(k),m} \\ z_{n2}^{(k),m} \\ z_{n3}^{(k),m} \end{Vmatrix},\ Y_n^{(k)m} = \begin{Vmatrix} y_{n1}^{(k),m} \\ y_{n2}^{(k),m} \\ y_{n3}^{(k),m} \end{Vmatrix},\ D_{n\nu}^{(1)m} = \left\| d_{rs}^{(1)m}(n,\nu) \right\|,\ D_n^{(2)m} = \left\| d_{rs}^{(1)m}(n) \right\|, \tag{25}$$

$$F_{n\nu}^{(1)m} = \left\| f_{rs}^{(1)m}(n,\nu) \right\|,\ F_n^{(2)m} = \left\| f_{rs}^{(2)m}(n) \right\|,\ m=1,2;\ r;s=1,2,3,\ \ x = \frac{2\pi R}{\ell}.$$

We can write this infinite system of equations in the following form:

$$\begin{cases} Z_n^{(1)1} + \sum\limits_{\nu=0}^{\infty} D_{n\nu}^{(1)2} Z_\nu^{(1)2} + D_n^{(2)1} Z_n^{(2)1} = 0, \\ Z_n^{(1)2} + \sum\limits_{\nu=0}^{\infty} D_{n\nu}^{(1)1} Z_\nu^{(1)1} + D_n^{(2)2} Z_n^{(2)2} = 0, \end{cases} \tag{26}$$

$$\begin{cases} Y_n^{(1)1} + \sum\limits_{\nu=0}^{\infty} F_{n\nu}^{(1)2} Y_\nu^{(1)2} + F_n^{(2)1} Y_n^{(2)1} = 2\pi\delta_n^3\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)1,0}\right), \\ Y_n^{(1)2} + \sum\limits_{\nu=0}^{\infty} F_{n\nu}^{(1)1} Y_\nu^{(1)1} + F_n^{(2)2} Y_n^{(2)2} = 2\pi\delta_n^3\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)2,0}\right), \end{cases} \tag{27}$$

where

$$n = 0,1,2,\ldots,\infty, \quad \delta_n^3 = \begin{cases} 1 & if\ n=3 \\ 0 & if\ n \neq 3 \end{cases} \tag{28}$$

Note that the equations (26) and (27) are obtained for the co-phase curving of the fibers (fig. 1). For the anti-phase case (fig. 2) the equations (26) remain valid and instead of equations (27) the following ones are obtained:

$$\begin{cases} Y_n^{(1)1} + \sum\limits_{\nu=0}^{\infty} F_{n\nu}^{(1)2} Y_\nu^{(1)2} + F_n^{(2)1} Y_n^{(2)1} = 2\pi\delta_n^3\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)1,0}\right), \\ Y_n^{(1)2} + \sum\limits_{\nu=0}^{\infty} F_{n\nu}^{(1)1} Y_\nu^{(1)1} + F_n^{(2)2} Y_n^{(2)2} = -2\pi\delta_n^3\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)2,0}\right), \\ n = 0,1,2,\ldots,\infty \end{cases} \tag{29}$$

We omit the detailed expressions for $F_{n\nu}^{(1)m}$, $D_n^{(2)m}$, $D_{n\nu}^{(1)m}$ and $F_n^{(2)m}$.

The equation (26) shows that for both co-phase and anti-phase fiber curving

$$z_n^{(k)m} = 0, \quad k = 1,2, \quad m = 1,2. \tag{30}$$

Moreover, as $\sigma_{33}^{(2)1,0} = \sigma_{33}^{(2)2,0}$, then according to the mechanical consideration and according to the equations (27), (28) we write the following relations.
For the co-phase curving

$$Y_n^{(k)1} = Y_n^{(k)2}. \tag{31}$$

For the anti-phase curving

$$Y_n^{(k)1} = -Y_n^{(k)2}. \tag{32}$$

Taking (31) and (32) into account we reduce the equation (27) to the equation

$$Y_n^{(1)1} + \sum_{\nu=0}^{\infty} F_{n\nu}^{(1)2} Y_\nu^{(1)1} + F_n^{(2)1} Y_n^{(2)1} = 2\pi\delta_n^3\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)1,0}\right) \tag{33}$$

and equation (29) to the form:

$$Y_n^{(1)1} - \sum_{\nu=0}^{\infty} F_{n\nu}^{(1)2} Y_\nu^{(1)1} + F_n^{(2)1} Y_n^{(2)1} = 2\pi\delta_n^3\left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2)1,0}\right). \tag{34}$$

3. Foundation of the proposed method.

For numerical investigations the infinite system of algebraic equations (32) and (33) must be substituted by a finite system. To validate such a substitution it must be shown that the determinant of these infinite systems of equations must be normal type [4]; this holds if we can prove the convergence of the series

$$M = \sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left|F_{n\nu}^{(1)2}\right|. \tag{35}$$

For investigation of the series (35) we use the following asymptotic estimates of the functions $I_n(x)$ and $K_n(x)$:

$$I_n(x) < c_1\frac{1}{n!}\left(\frac{|x|}{2}\right)^n, \quad c_1 = const,$$

$$K_n(x) \approx c_2(n-1)!\left(\frac{2}{|x|}\right)^n, \quad c_2 = const. \tag{36}$$

These hold for barge n and fixed x. Moreover we use the following inequality

$$\rho = \frac{R}{R_{12}} > 2, \tag{37}$$

which means that the fibers do not touch.

Note, that elements of $F_{n\nu}^{(1)2}$ have the following characteristic forms

$$\left(const_1\frac{K_{n+1}(c\gamma R)}{K_n(c\gamma R)} + const_2\right);$$

$$const_3 I_{n+1}(c\gamma R) + const_4 I_n(c\gamma R)\left(K_{\nu-n}(c\gamma R_{12}) \pm K_{\nu+n}(c\gamma R_{12})\right)/K_\nu(c\gamma R);$$

$$const_5\frac{I_{n+1}(c\gamma R)}{I_n(c\gamma R)} + const_6. \tag{38}$$

Taking (38) and (36) into account we obtain the following estimate for the series (35)

$$M < c_3 \sum_{n=0}^{\infty} n^{C_4} (\rho - 1)^{-n}, \quad c_3; c_4 = const. \tag{39}$$

As the series on the right-hand side of (39) converges, so does (35). Thus, the determinant of the infinite system of equations (33), (34) is normal and the infinite systems can be replaced by finite system for numerical purposes. The requisite number of equations in these finite systems must be determined from the convergence of the numerical results.

This concludes discussion of the first approximation. Subsequent approximations can be found likewise.

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