MECHANICS

AGALAROV D.G.

INVESTIGATION OF THE WAVE DEFORMATIONS IN THE NETS

Abstract

The propagation of deformations in net for automodel motion is investigated. The conditions on the front of strong breakage of the parameters are bound. The analytic expression of the form of the wave front is obtained.

In [1] the equation of the net's motion were obtained for the natural coordinates in the form:

$$\begin{aligned}
&\frac{\partial\sigma_1}{\partial s_1}+\frac{\sigma_1}{tg\gamma}\frac{\partial\gamma_1}{\partial s_1}+\frac{\sigma_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial s_2}=\rho\left(\frac{\partial v_1}{\partial t}+\frac{v_1}{tg\gamma}\frac{\partial\gamma_1}{\partial t}+\frac{v_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial t}\right)\\
&\frac{\partial\sigma_2}{\partial s_2}-\frac{\sigma_2}{tg\gamma}\frac{\partial\gamma_2}{\partial s_2}-\frac{\sigma_1}{\sin\gamma}=\rho\left(\frac{\partial v_2}{\partial t}-\frac{v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial t}-\frac{v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial t}\right)\\
&\frac{\partial\varepsilon_1}{\partial t}+\frac{1+\varepsilon_1}{tg\gamma}\frac{\partial\gamma_1}{\partial t}=\frac{\partial v_1}{\partial s_1}+\frac{v_1}{tg\gamma}\frac{\partial\gamma_1}{\partial s_1}+\frac{v_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial s_1}\\
&\frac{\partial\varepsilon_2}{\partial t}-\frac{1+\varepsilon_2}{tg\gamma}\frac{\partial\gamma_2}{\partial t}=\frac{\partial v_2}{\partial s_2}-\frac{v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial s_2}-\frac{v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial s_2}\\
&\frac{1+\varepsilon_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial t}=\frac{\partial v_2}{\partial s_1}-\frac{v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial s_1}-\frac{v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial s_1}\\
&\frac{1+\varepsilon_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial t}=\frac{\partial v_1}{\partial s_1}+\frac{v_1}{tg\gamma}\frac{\partial\gamma_1}{\partial s_2}+\frac{v_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial s_2},
\end{aligned}\qquad(1)$$

where σ_{10},σ_2 are the stresses, $\varepsilon_1,\varepsilon_2$ are the deformations, γ_1,γ_2 are the angles between the threads and the coordinates, ρ is the density, v_1,v_2 are the coordinates of decomposition of the velocity vector by the directions of the threads; s_1,s_2 are Lagrange coordinates, t is time, γ is the angle between the threads.

With purpose of investigation of the automodel motions of the net in the plane, as it was done in [4], the system (1) can be reduced to the form: (for simplicity the elastic case is considered):

$$\frac{\partial\varepsilon_1}{\partial\xi}+\frac{\varepsilon_1+v_1}{tg\gamma}\frac{\partial v_1}{\partial\xi}+\frac{\xi v_2}{\sin\gamma}\frac{\partial v_2}{\partial\xi}+\frac{\eta v_1}{tg\partial\eta}\frac{\partial\gamma_1}{\partial\eta}+\frac{\varepsilon_2+\eta v_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial\eta}+\xi\frac{\partial v_1}{\partial\xi}+\eta\frac{\partial v_1}{\partial\eta}=0\,,$$

$$\frac{\partial\varepsilon_2}{\partial\eta}-\frac{\varepsilon_2+\eta v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial\eta}-\frac{\eta v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial\eta}-\frac{\xi v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial\xi}-\frac{\varepsilon_1+\xi v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial\xi}+\eta\frac{\partial v_2}{\partial\eta}+\xi\frac{\partial v_2}{\partial\xi}=0\,,$$

$$\xi\frac{\partial\varepsilon_1}{\partial\xi}+\eta\frac{\partial\varepsilon_1}{\partial\eta}+\frac{(1+\varepsilon_1)\xi+v_1}{tg\gamma}\frac{\partial\gamma_1}{\partial\xi}+\frac{v}{\sin\gamma}\frac{\partial\gamma_2}{\partial\xi}+\frac{1+\varepsilon_1}{tg\gamma}\eta\frac{\partial\gamma_1}{\partial\eta}+\frac{\partial v_1}{\partial\eta}=0,$$
$$\xi\frac{\partial\varepsilon_2}{\partial\xi}+\eta\frac{\partial\varepsilon_2}{\partial\eta}-\frac{(1+\varepsilon_2)\xi}{tg\gamma}\frac{\partial\gamma_2}{\partial\xi}-\frac{(1+\varepsilon_2)\eta+v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial\eta}-\frac{v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial\eta}+\frac{\partial v_2}{\partial\eta}=0, \quad (2)$$

$$-\frac{(1+\varepsilon_1)\xi+v_1}{\sin\gamma}\frac{\partial\gamma_1}{\partial\xi}-\frac{v_2}{tg\gamma}\frac{\partial\gamma_2}{\partial\xi}-\frac{(1+\varepsilon_1)\eta}{\sin\gamma}\frac{\partial\gamma_1}{\partial\eta}+\frac{\partial v_2}{\partial\xi}=0,$$
$$\frac{(1+\varepsilon_1)\xi}{\sin\gamma}\frac{\partial\gamma_2}{\partial\xi}+\frac{(1+\varepsilon_2)\eta+v_2}{\sin\gamma}\frac{\partial\gamma_2}{\partial\eta}+\frac{v_1}{tg\gamma}\frac{\partial\gamma_1}{\partial\eta}+\frac{\partial v_1}{\partial\eta}=0,$$

where $\xi=s_1/at;\eta=s_2/at,v_1=\vartheta_1/a,v_2=\vartheta_2/a,a=\sqrt{E/\rho},E$ is Young's modulus.

In [4] the families of the waves front of weak breakage in the plane were obtained. And equations of the parameters in the local coordinates obtained in [1] confirm the possibility of existence of only one of families of lines for beforehand stress of the net and show that under absence of the initial stress the weak breakage wave doesn't propagate. It is naturally to suppose about existence of the strong breakage front. According to [2,3] generalizing for the two-dimensional case we can obtain the conditions on the strong breakage front in differentials of the parameters. For that multiplying the equation (2) by $d\xi d\eta$ and taking into account that for passage through the front

$$\frac{\partial\varepsilon}{\partial\xi}d\xi=d\varepsilon,\ \frac{\partial\varepsilon}{\partial\eta}d\eta=-d\varepsilon,$$

where $d\xi$ and $d\eta$ are the differentials of the coordinates along the front line and by analogy for other parameters we obtain:

$$d\eta d\varepsilon_1+\frac{(\varepsilon_1+\xi v_1)d\eta-\eta v_1 d\xi}{tg\gamma}d\gamma_1-\frac{(\varepsilon_2+\eta v_2)d\xi-\xi v_2 d\eta}{\sin\gamma}d\gamma_2+(\xi d\eta-\eta d\xi)dv_1=0$$
$$-d\xi d\varepsilon_2+\frac{(\varepsilon_2+\eta v_2)d\xi-\xi v_2 d\eta}{tg\gamma}d\gamma_2-\frac{(\varepsilon_1+\xi v_1)d\eta-\eta v_1 d\xi}{\sin\gamma}d\gamma_1+(\xi d\eta-\eta d\xi)dv_2=0 \quad (3)$$
$$(\xi d\eta-\eta d\xi)d\varepsilon_1+\frac{[(1+\varepsilon_1)\xi+v_1]d\eta-(1+\varepsilon_1)\eta d\xi}{tg\gamma}d\gamma_1+\frac{v_2}{\sin\gamma}d\eta d\eta_2+d\eta dv_1=0$$

$$(\xi d\eta-\eta d\xi)d\varepsilon_2+\frac{[(1+\varepsilon_2)\eta+v_2]d\xi-(1+\varepsilon_2)d\eta}{tg\gamma}d\gamma_2+\frac{v_1}{\sin\gamma}d\xi d\gamma_1-d\xi dv_2=0$$
$$-\frac{[(1+\varepsilon_1)\xi+v_1]d\eta-(1+\varepsilon_1)\eta d\xi}{\sin\gamma}d\gamma_1-\frac{v_2}{tg\gamma}d\eta d\gamma_2+d\eta dv_2=0$$
$$-\frac{[(1+\varepsilon_2)\eta+v_2]d\xi-(1+\varepsilon_2)\xi d\eta}{\sin\gamma}d\gamma_2-\frac{v_1}{tg\gamma}d\xi_1 d\gamma_1-d\xi dv_1=0.$$

On the front of strong breakage wave in the fiked point one of the parameters can be various and correspondingly the others by (3) will get increments though passing through the front ξ and η don't change. The system (3) can serve for determination of the parameters of the net on the front. However this system is linear homogeneous with respect to the differentials of the parameters and for elimination of indeterminates the supplementary data are necessary which are taken from the boundary conditions. The condition of existence of the solution of the homogeneous system serves for determination of the front form. If to distort the meshes a little, that is $\gamma \sim \pi / 2$, $\sin\gamma \sim 1$, $tg\gamma \sim \infty$, the system (3) will take the form:

$$d\eta d\varepsilon_1 - \left[\left(\varepsilon_2 + \eta v_2\right)d\xi - \xi v_2 d\eta\right]d\gamma_2 + \left(\xi d\eta - \eta d\xi\right)dv_1 = 0, \tag{4a}$$

$$-d\xi d\varepsilon_2 - \left[\left(\varepsilon_1 + \xi v_1\right)d\eta - \eta v_1 d\xi\right]d\gamma_1 + \left(\xi d\eta - \eta d\xi\right)dv_2 = 0, \tag{4b}$$

$$\left(\xi d\eta - \eta d\xi\right)d\varepsilon_1 + v_2 d\eta dv_2 + d\eta dv_1 = 0, \tag{4c}$$

$$\left(\xi d\eta - \eta d\xi\right)d\varepsilon_2 + v_1 d\xi d\gamma_1 - d\xi dv_2 = 0, \tag{4d}$$

$$-\left\{\left[\left(1+\varepsilon_1\right)\xi + v_1\right]d\eta - \left(1+\varepsilon_1\right)\eta d\xi\right\}d\gamma_1 + d\eta dv_2 = 0, \tag{4e}$$

$$-\left\{\left[\left(1+\varepsilon_2\right)\eta + v_2\right]d\xi - \left(1+\varepsilon_2\right)\xi d\eta\right\}d\gamma_2 - d\xi dv_1 = 0. \tag{4g}$$

Excluding dv_2 from (4b) and (4d) with help of (4e) we will obtain

$$\eta' d\varepsilon_2 - \left\{\left[\left(1+\varepsilon_1\right)\xi^2 - \varepsilon_1\right]\left(\eta'\right)^2 - 2\left(1+\varepsilon_1\right)\xi\eta\eta' + \left(1+\varepsilon_1\right)\eta^2\right\}d\gamma_1 = 0, \tag{5}$$

$$\eta' d\varepsilon_2 - \left(1+\varepsilon_1\right)d\gamma_1 = 0,$$

where $\eta' = d\eta / d\xi$.

From (5) it follows

$$\eta' = \frac{\eta\xi \pm \sqrt{\xi^2 + b^2\left(\eta^2 - 1\right)}}{\xi^2 - b^2}, \tag{6}$$

where $b^2 = \varepsilon / \left(1+\varepsilon\right)$.

This is just the differential equation of the wave front. However, for its integration it is necessary to know $b \equiv b(\varepsilon)$. The quantity b is the velocity of across waves and it is clear that we can take it as $b = c\xi$. The equation (6) will have the form:

$$\eta' = \frac{\eta \pm \sqrt{1 + c^2\left(\eta^2 - 1\right)}}{\left(1 - c^2\right)\xi} \tag{7}$$

or

$$\int \frac{d\eta}{\eta \pm \sqrt{c^2\eta^2 + 1 - c^2}} = \ln\left(D\xi^{\frac{1}{1-c^2}}\right). \tag{7a}$$

The equation (7) for $b = const$ is analogous to the differential equation of the families of the weak breakage fronts [4]. However, the wave picture in [4] confirmed particularly by the equation of characteristics, corresponds completely to the waves of the strong breakage for small but finite disturbations.

The similar picture turned for 180^0 obtained by substitution of the places (ξ and η) is obtained from equations (4a), (4c) and (4f).

The constant D in (7a) can be found if the deformation ε had been given in the point of the front $\eta = 0$ and correspondingly $b = \sqrt{\varepsilon / \left(1+\varepsilon\right)}$ and $\xi = b / c$. The value of C can be approximately determined by the theory of one-dimensional waves [3] whence it follows that $0 < c < 1$. For the given $c^2 = 0{,}2$; $D = 0{,}5$ the approximate calculation of the integral gives

Table 1

$\|\eta\|$	0	0,2	0,4	0,6	0,8	1,0
$\|\xi\|$	0,5	0,408	0,312	0,212	0,108	0

In difference from the results in [4] there is the insignificant convexity from the center.

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Agalarov D.G.
Institute of Mathematics and Mechanics of AS Azerbaijan,
9, F. Agayev str., 370141, Baku, Azerbaijan.
Tel.: 38-65-13.

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