MATHEMATICS

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THE INVESTIGATION OF THE STRONGLY GENERALIZED SOLUTION OF AN ONE-DIMENSIONAL MIXED PROBLEM FOR A CLASS OF POLYLINEAR DIFFERENTIAL EQUATIONS IN ROBOT-TECHNIQUE .II.

Abstract

The existence and uniqueness of the local and global generalized solution of a mixed problem for a class of nonlinear differential equation are proved.

The work is devoted to the studying of existence and uniqueness problems (both in the small and on the whole) of the strongly generalized solution of the following one-dimensional mixed problem:

$$\begin{cases} U_{tt}(t,x)+2\alpha U_{txxx}(t,x)+U_{xxxx}(t,x)= \\ =F(t,x,U(t,x),U_x(t,x),U_{xx}(t,x),U_{xxx}(t,x),U_{xxxx}(t,x), \\ U_t(t,x),U_{tx}(t,x),U_{txx}(t,x)) \qquad (0\le t\le T,\ 0\le x\le \pi), & (1) \\ U(0,x)=\varphi(x)\ (0\le x\le \pi),\ U_t(0,x)=\psi(x)\ (0\le x\le \pi), & (2) \\ U(t,o)=U(t,\pi)=U_{xx}(t,0)=U_{xx}(t,\pi)=0 \quad (0\le t\le T), & (3) \end{cases}$$

where $\alpha>1$ is a fixed number; $0<T<+\infty$; F,φ,ψ are given functions, and $U(t,x)$ is a desired function, and under the strongly generalized solution of the problem (1)-(3) we understand the function $U(t,x)$, possessing the properties:

a) $U(t,x),U_x(t,x),U_{xx}(t,x),U_{xxx}(t,x),U_{xxxx}(t,x),$
$U_t(t,x),U_{tx}(t,x),U_{txx}(t,x)\in C([0,T]\times[0,\pi]);$
$U_{xxxxx}(t,x),U_{txxx}(t,x)\in C([0,T];L_2(0,\pi));$

b) the conditions (2) and (3) are satisfied in an ordinary sense;

c) it is fulfilled the integral identity

$$\int_0^T\int_0^\pi\{U_t(t,x)\cdot V_t(t,x)+2\alpha U_{xxxx}(t,x)\cdot V_t(t,x)-U_{xxxx}(t,x)\cdot V(t,x)+$$
$$+F(U(t,x))\cdot V(t,x)\}dxdt+\int_0^\pi\psi(x)\cdot V(0,x)dx+2\alpha\int_0^\pi\varphi^{(4)}(x)\cdot V(0,x)dx=0 \qquad (4)$$

for any function of $V(t,x)$ possessing the properties

$$V(t,x)\in C([0,T];L_2(0,\pi)),V_t(t,x)\in L((0,T);L_2(0,\pi)),V(T,x)=0 \quad \text{п.в. в}(0,\pi), \quad (5)$$

where

$$F(U(t,x))\equiv F(t,x,U(t,x),U_x(t,x),U_{xx}(t,x),U_{xxx}(t,x), \\ U_{xxxx}(t,x),U_t(t,x),U_{tx}(t,x),U_{txx}(t,x)). \qquad (6)$$

To investigate the problem (1)-(3) consider the set $B^{\alpha_0,\dots,\alpha_l}_{\beta_0,\dots,\beta_l,T}$ consisting of all functions of $U(t,x)$ of the form

$$U(t,x)=\sum_{n=1}^{\infty}U_n(t)\sin nx,$$

considered on $[0,T]\times[0,\pi]$ for which all functions $U_n(t)\in C^{(l)}([0,T])$ and

$$J_T(U)=\sum_{i=0}^{l}\left\{\sum_{n=1}^{\infty}\left(n^{\alpha_i}\cdot\max_{0\le t\le T}\left|U_n^{(i)}(t)\right|\right)^{\beta_i}\right\}^{\frac{1}{\beta_i}}<+\infty, \tag{7}$$

where $l\ge 0$ is an integer, $\alpha_i\ge 0\ (i=\overline{0,l})$, $1\le\beta_i\le 2\ (i=\overline{0,l})$. We define the norm in this set as follows:

$$\|U\|=J_T(U).$$

It is known that (see [1]), all these spaces are Banach ones.

Later on, for the functions $U(t,x)\in B^{\alpha_0,\dots,\alpha_l}_{\beta_0,\dots,\beta_l,T}$ we shall use the denotations:

$$\|U\|_{B^{\alpha_0,\dots,\alpha_l}_{\beta_0,\dots,\beta_l,T}}\equiv\sum_{i=0}^{l}\left\{\sum_{n=1}^{\infty}\left(n^{\alpha_i}\cdot\max_{0\le\tau\le t}\left|U_n^{(i)}(\tau)\right|\right)^{\beta_i}\right\}^{\frac{1}{\beta_i}}\qquad(0\le t\le T). \tag{8}$$

It is obvious that each strongly generalized solution of the problem (1)-(3) has the form: $U(t,x)=\sum_{n=1}^{\infty}U_n(t)\sin nx$, where $U_n(t)=\frac{2}{\pi}\int_0^{\pi}U(t,x)\sin nx dx$. After formal application of the Fourier method scheme the finding of Fourier coefficients $U_n(t)$ $(n=1,2,\dots)$ of the desired strongly generalized solution $U(t,x)$ of the problem (1)-(3) is reduced to the solution of the following denumerable system of nonlinear integral-differential equations:

$$\begin{gathered}U_n(t)=\frac{1}{2n^2\sqrt{\alpha^2n^4-1}}\cdot\left(\lambda_n e^{-\mu_n t}-\mu_n e^{-\lambda_n t}\right)\cdot\varphi_n+\frac{1}{2n^2\sqrt{\alpha^2n^4-1}}\cdot\left(e^{-\mu_n t}-e^{-\lambda_n t}\right)\cdot\psi_n+\\+\frac{1}{\pi n^2\sqrt{\alpha^2n^4-1}}\cdot\int_0^t\int_0^{\pi}F(U(\tau,x))\sin nx\left\{e^{-\mu_n(t-\tau)}-e^{-\lambda_n(t-\tau)}\right\}dxd\tau\ (n=1,2,\dots,t\in[0,T]),\end{gathered} \tag{9}$$

where

$$\lambda_n\equiv\alpha n^4+n^2\sqrt{\alpha^2n^4-1},\qquad\mu_n\equiv\alpha n^4-n^2\sqrt{\alpha^2n^4-1}, \tag{10}$$

$$\varphi_n\equiv\frac{2}{\pi}\int_0^{\pi}\varphi(x)\sin nx dx,\qquad\psi_n\equiv\frac{2}{\pi}\int_0^{\pi}\psi(x)\sin nx dx, \tag{11}$$

the operator F has been defined by the relation (6); besides, it is obtained that

$$0<\mu_n<\lambda_n;\ \lambda_n\to+\infty,\quad\mu_n\to\frac{1}{2\alpha}>0\ \text{for}\ n\to\infty. \tag{12}$$

In this paper along with the system (9) under assumption

$$\left.\begin{aligned}&F(U(t,x))\in C([0,T]\times[0,\pi]),\frac{\partial}{\partial x}\{F(U(t,x))\}\in C([0,T];L_2(0,\pi)),\\&F(U(t,x))|_{x=0}=F(U(t,x))|_{x=\pi}=0\qquad\forall t\in[0,T]\end{aligned}\right\} \tag{13}$$

we use the following system obtained from the system (9) by integrating on parts with respect to x for once at its right hand side,

$$U_n(t)=\frac{1}{2n^2\sqrt{\alpha^2n^4-1}}\cdot\left(\lambda_n e^{-\mu_n t}-\mu_n e^{-\lambda_n t}\right)\cdot\varphi_n+\frac{1}{2n^2\sqrt{\alpha^2n^4-1}}\cdot\left(e^{-\mu_n t}-e^{-\lambda_n t}\right)\cdot\psi_n+$$

$$+\frac{1}{\pi n^3\sqrt{\alpha^2 n^4-1}}\cdot\int_0^t\int_0^\pi\frac{\partial}{\partial x}\{F(U(\tau,x))\}\cdot\cos nx\left\{e^{-\mu_n(t-\tau)}-e^{-\lambda_n(t-\tau)}\right\}dxdt$$
$$(n=1,2,...,t\in[0,T]). \qquad (14)$$

Starting from the definition of the strongly generalized solution of the problem (1)-(3), it is proved the following

Lemma. *If* $U(t,x)=\sum_{n=1}^{\infty}U_n(t)\sin nx$ *is any strongly generalized solution of the problem (1)-(3), the functions* $U_n(t)$ $(n=1,2,...)$ *satisfy the system (9) on* $[0,T]$.

The following uniqueness theorem (on the whole) of the strongly generalized solution of the problem (1)-(3) is proved by means of Bellman's inequality.

Theorem 1. *Let* $F(t,x,U_1,...,U_8)\in C([0,T]\times[0,\pi]\times(-\infty,\infty)^8)$ *and* $\forall R>0$ *in* $[0,T]\times[0,\pi]\times[-R,R]^8$

$$\left|F(t,x,U_1,...,U_8)-F(t,x,\widetilde{U}_1,...,\widetilde{U}_8)\right|\le a_R(t)\cdot\sum_{i=1}^{8}\left|U_i-\widetilde{U}_i\right|,\quad a_R(t)\in L_2(0,T).$$

Then the problem (1)-(3) may have no more that one strongly generalized solution.

Further, by combining the generalized oblite mappings principle with Shauder's fixed point principle, the following theorem on the existence in the small of the strongly generalized solution of the problem (1)-(3), is proved.

Theorem 2. *Let*

1. $\varphi(x)\in C^{(4)}([0,\pi]),\varphi^{(5)}(x)\in L_2(0,\pi)\,and\,\varphi(0)=\varphi(\pi)=\varphi''(0)=\varphi''(\pi)=\varphi^{(4)}(0)=\varphi^{(4)}(\pi)=0$
$\psi(x)\in C^{(2)}([0,\pi]),\psi'''(x)\in L_2(0,\pi)\,and\,\psi(0)=\psi(\pi)=\psi''(0)=\psi''(\pi)=0$.

2. $F(t,\xi_0,\xi_1,...,\xi_8),F_{\xi_i}(t,\xi_0,\xi_1,...,\xi_8)(i=\overline{0,8})\in C([0,T]\times[0,\pi]\times(-\infty,\infty)^8)$.

3. $F(t,0,0,\xi_2,0,\xi_4,0,0,\xi_7,0)=F(t,\pi,0,\xi_2,0,\xi_4,0,0,\xi_7,0)=0$
$\forall t\in[0,T],\quad \xi_2,\xi_4,\xi_7\in(-\infty,\infty)$.

Then the strongly generalized solution of the problem (1)-(3) exists in the small.

Proof. for each fixed $U\in B_{1,1,T}^{4,2}$ we define in $B_{2,2,T}^{5,3}$ the operator (with respect to V) P_U:

$$P_U(V(t,x))=\widetilde{V}(t,x)\equiv\sum_{n=1}^{\infty}\widetilde{V}_n(t)\sin nx, \qquad (15)$$

where

$$\widetilde{V}_n(t)=\frac{1}{2n^2\sqrt{\alpha^2n^4-1}}\cdot\left(\lambda_n e^{-\mu_n t}-\mu_n e^{-\lambda_n t}\right)\cdot\varphi_n+\frac{1}{2n^2\sqrt{\alpha^2n^4-1}}\cdot\left(e^{-\mu_n t}-e^{-\lambda_n t}\right)\cdot\psi_n+$$
$$+\frac{1}{\pi n^3\sqrt{\alpha^2n^4-1}}\cdot\int_0^t\int_0^\pi\Phi_U(V(\tau,x))\cos nx\left\{e^{-\mu_n(t-\tau)}-e^{-\lambda_n(t-\tau)}\right\}dxd\tau\ (n=1,2,...,t\in[0,T]), \qquad (16)$$

$$\Phi_U(V(t,x))=G(U(t,x))+g_1(U(t,x))\cdot V_{xxxxx}(t,x)+g_2(U(t,x))\cdot V_{txxx}(t,x), \qquad (17)$$

$$g_1(U(t,x))\equiv F_{\xi_5}(t,x,U(t,x),U_x(t,x),U_{xx}(t,x),U_{xxx}(t,x),$$
$$U_{xxxx}(t,x),U_t(t,x),U_{tx}(t,x),U_{txx}(t,x)), \qquad (18)$$

$$g_2(U(t,x))\equiv F_{\xi_8}(t,x,U(t,x),U_x(t,x),U_{xx}(t,x),U_{xxx}(t,x),$$
$$U_{xxxx}(t,x),U_t(t,x),U_{tx}(t,x),U_{txx}(t,x)), \qquad (19)$$

$$G(U(t,x))\equiv\frac{\partial}{\partial x}\{F(U(t,x))\}-g_1((t,x))\cdot U_{xxxxx}(t,x)-g_2(U(t,x))\cdot U_{txxx}(t,x), \quad (20)$$

and the operator F has been defined by the relation (6).

Using the relations (15), (16) and Bessel's inequality we can easily obtain that for any fixed $U\in B^{4,2}_{1,1,T}$ $\forall V\in B^{5,3}_{2,2,T}$ and $t\in[0,T]$:

$$\|P_U(V)\|^2_{B^{5,3}_{2,2,t}}\le C_0^2+\frac{3(\sqrt{T}+\sqrt{2\alpha})^2}{2\pi(\alpha^2-1)}\cdot\int_0^t\int_0^\pi\{\Phi_U(V(\tau,x))\}^2dxd\tau, \quad (21)$$

where

$$C_0^2\equiv\frac{6}{\pi}\left\{\left(1+\frac{1}{2\sqrt{\alpha^2-1}}\right)^2\cdot\|\varphi^{(5)}(x)\|^2_{L_2(0,\pi)}+\left(\frac{1}{2\sqrt{\alpha^2-1}}+\frac{\alpha}{\sqrt{\alpha^2-1}}\right)\cdot\|\psi^{(3)}(x)\|^2_{L_2(0,\pi)}\right\}. \quad (22)$$

Further, by virtue of the structure of spaces $B^{4,2}_{1,1,T}$ and $B^{5,3}_{2,2,T}$, $\forall U\in B^{4,2}_{1,1,T}, V\in B^{5,3}_{2,2,T}$ and $t\in[0,T]$ we have:

$$\left\|\frac{\partial^i U(t,x)}{\partial x^i}\right\|_{C(\overline{Q}_T)}\le\sum_{n=1}^{\infty}n^4\cdot\max_{0\le t\le T}\le|U_n(t)|=\|U\|_{B^4_{1,T}}\le\|U\|_{B^{4,2}_{1,1,T}}\quad (i=\overline{0,4}), \quad (23)$$

$$\left\|\frac{\partial^{1+j} U(t,x)}{\partial t\partial x^j}\right\|_{C(\overline{Q}_T)}\le\sum_{n=1}^{\infty}n^2\cdot\max_{0\le t\le T}\le|U'_n(t)|=\|U_t\|_{B^2_{1,T}}\le\|U\|_{B^{4,2}_{1,1,T}}\quad (i=\overline{0,2}), \quad (24)$$

$$\int_0^\pi\{V_{xxxxx}(t,x)\}^2dx=\frac{\pi}{2}\cdot\sum_{n=1}^{\infty}\{n^5V_n(t)\}^2\le\frac{\pi}{2}\|V\|^2_{B^5_{2,t}}\le\frac{\pi}{2}\|V\|^2_{B^{5,3}_{2,2,t}}\le\frac{\pi}{2}\|V\|^2_{B^5_{2,2,T}}, \quad (25)$$

$$\int_0^\pi\{V_{txxx}(t,x)\}^2dx=\frac{\pi}{2}\cdot\sum_{n=1}^{\infty}\{n^3V'_n(t)\}^2\le\frac{\pi}{2}\|V_t\|^2_{B^3_{2,t}}\le\frac{\pi}{2}\|V\|^2_{B^{5,3}_{2,2,t}}\le\frac{\pi}{2}\|V\|^2_{B^3_{2,2,T}}, \quad (26)$$

and we have used the denotations

$$Q_T\equiv(0,T)\times(0,\pi);\ U(t,x)\equiv\sum_{n=1}^{\infty}U_n(t)\sin nx;\ V(t,x)\equiv\sum_{n=1}^{\infty}V_n(t)\sin nx. \quad (27)$$

Using the estimates (23), (24) and obvious forms of $G(U(t,x)), g_1(U(t,x)), g_2(U(t,x))$, we can easily obtain that $\forall U\in B^{4,2}_{1,1,T}$:

$$\|G(U(t,x))\|_{C(\overline{Q}_T)}\le C(U)\cdot\left(1+6\|U\|_{B^{4,2}_{1,1,T}}\right), \quad (28)$$

$$\|g_i(U(t,x))\|_{C(\overline{Q}_T)}\le C(U)\quad (i=1,2), \quad (29)$$

where

$$C(U)\equiv\max_{i=0,8}\left\{\|F_{\xi_i}(t,x,U(t,x),U_x(t,x),U_{xx}(t,x),U_{xxx}(t,x),\right.$$
$$\left.U_{xxxx}(t,x),U_t(t,x),U_{tx}(t,x),U_{txx}(t,x))\|_{C(\overline{Q}_T)}\right\} \quad (30)$$

and $\xi_i\ (i=\overline{0,8})$ are denotations of corresponding arguments of functions $F(t,\xi_0,\xi_1,\ldots,\xi_8)$.

Then using the estimates (28), (29), (25), (26) and relation (17), we obtain from (21) that for each fixed $U\in B^{4,2}_{1,1,T}$ the operator P_U acts on the space $B^{5,3}_{2,2,T}$ restrictedly.

Now, using the relations (15)-(20), denotation (30), and estimates (25), (26) (for $V=V_1-V_2$), similar to (21), by the mathematical induction method we get that for any fixed $U\in B^{4,2}_{1,1,T}$ $\forall V_1,V_2\in B^{5,3}_{2,2,T}$ and $t\in[0,T]$:

$$\left\|P_U^k(V_1)-P_U^k(V_2)\right\|^2_{B^{5,3}_{2,2,t}} \le \left\{\frac{\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{\alpha^2-1}\cdot C^2(U)\right\}^k \cdot \left\|V_1-V_2\right\|^2_{B^{5,3}_{2,2,T}}\cdot\frac{t^k}{k!}, \qquad (31)$$

where k is any positive integer.

Thus, for any fixed $U\in B^{4,2}_{1,1,T}$ $\forall V_1,V_2\in B^{5,3}_{2,2,T}$:

$$\left\|P_U^k(V_1)-P_U^k(V_2)\right\|^2_{B^{5,3}_{2,2,T}} \le q_k(U)\cdot\left\|V_1-V_2\right\|_{B^{5,3}_{2,2,T}}, \qquad (32)$$

where

$$q_k(U)\equiv\frac{1}{\sqrt{k!}}\left\{\frac{\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{\alpha^2-1}\cdot C^2(U)\cdot T\right\}^{\frac{k}{2}}. \qquad (33)$$

It is obvious that under large $k=k(U)$: $q_k(U)<1$. For such k, the operator P_U^k is found to be compressed in the space $B^{5,3}_{2,2,T}$. Then by virtue of the generalized principle of compressed mappings, the unique fixed point V of the operator P_U^k in $B^{5,3}_{2,2,T}$ is also the unique fixed point of the operator P_U in $B^{5,3}_{2,2,T}$:

$$V=P_U(V),\ V\in B^{5,3}_{2,2,T}. \qquad (34)$$

Thus, associated to each $U\in B^{4,2}_{1,1,T}$ the unique fixed point V of the operator P_U in $B^{5,3}_{2,2,T}$. Generate the operator H:

$$H(U)=V=P_U(V) \qquad (35)$$

acting form $B^{4,2}_{1,1,T}$ in $B^{5,3}_{2,2,T}$.

Further we show that the operator H acts from $B^{4,2}_{1,1,T}$ in $B^{5,3}_{2,2,T}$ completely continuously. Besides for any fixed $R>\frac{\pi}{\sqrt{6}}\cdot C_0$, where the number C_0 is defined by the relation (22), the closed ball K_R of radius R and with the center in zero under sufficiently small values of T is into transformed. Thus, by virtue of Shauder's fixed point principle, under sufficiently small values of T, the point H has in $B^{4,2}_{1,1,T}$ at least one fixed point $U: H(U)=U$.

Since $U=H(U)=V=P_U(V)$, then $U=V$ and,

$$U=H(U)=P_U(U),$$

and $U\in B^{5,3}_{2,2,T}$, since as it is said above, the operator H acts from $B^{4,2}_{1,1,T}$ in $B^{5,3}_{2,2,T}$. Then, it is obvious that

$$\Phi_U(U)=\frac{\partial}{\partial x}\{F(U(t,x))$$

and, consequently for the found fixed point

$$U=U(t,x)=\sum_{n=1}^{\infty}U_n(t)\sin nx$$

the functions $U_n(t)\,(n=1,2,\ldots)$ satisfy the (14) on $[0,T]$. And since for the found functions $U(t,x)\in B^{5,3}_{2,2,T}$ the conditions (13) are fulfilled, then it is obvious that the functions $U_n(t)(n=1,2,...)$ satisfy on $[0,T]$ and the system (9). Using this we can easily verify that $U(t,x)\in B^{5,3}_{2,2,T}$ is the strongly generalized solution of the problem (1)-(3). The proof of the theorem is completed.

Remark 1. Since from the condition 2 of theorem 2, the fulfillment of theorem 1 follows, then under the conditions of theorem 2, the strongly generalized solution of the problem (1)-(3) not only exists in the small, but it is also unique on the whole.

Finally, by means of Shauder's strong principle on a fixed point the following theorem on the existence of the strong generalized solution of the problem (1)-(3) on the whole is proved.

Theorem 3. *Let*

1. *All conditions of theorem 2 be fulfilled*
2. $B[0,T]\times[0,\pi]\times(-\infty,\infty)^8$

$$|F_x(t,x,U_1,...,U_8)|\le c(t)\cdot(1+|U_1|+...+|U_8|); \tag{36}$$

$$|F_{U_i}(t,x,U_1,...,U_8)|\le c(t) \qquad (i=\overline{1,8}), \qquad c(t)\in L_2(0,T). \tag{37}$$

Then the problem (1)-(3) has a unique strongly generalized solution.

Proof. Let H be an operator appearing in the proof of theorem 2. As we said above, the operator H acts in the space $B^{4,2}_{1,1,T}$ completely continuous. By definition of the operator H

$$\forall U\in B^{4,2}_{1,1,T} \qquad H(U)=V=P_U(V),$$

where the operator $P_U(V)$ is defined by the relations (15)-(20).

Now in $B^{4,2}_{1,1,T}$ consider the equation

$$U=\lambda H(U),\ 0\le\lambda\le 1, \tag{38}$$

and estimate a priori their all possible solutions U in $B^{4,2}_{1,1,T}$. Since

$$U=\lambda H(U)=\lambda,\ V=\lambda P_U(V), \tag{39}$$

then using the inequalities (36), (37), quite similar to (21), $\forall t\in[0,T]$ we have:

$$\|U\|^2_{B^{5,3}_{2,2,t}}\equiv\|\lambda H(U)\|^2_{B^{5,3}_{2,2,t}}\equiv\|\lambda V\|^2_{B^{5,3}_{2,2,t}}\equiv\|\lambda \mathbf{P}_U(V)\|^2_{B^{5,3}_{2,2,t}}\le$$
$$\le\lambda^2+C_0^2+\lambda^2\cdot\frac{3\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{2\pi\left(\alpha^2-1\right)}\cdot\int_0^t\int_0^\pi\{\Phi_U(V(\tau,x))\}^2dxd\tau\le$$
$$\le C_0^2+\frac{51\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{2\left(\alpha^2-1\right)}\cdot\int_0^T c^2(\tau)d\tau+\frac{51\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{\pi\left(\alpha^2-1\right)}\cdot\int_0^t c^2(\tau)\cdot\int_0^\pi\{U^2(\tau,x)+$$
$$+U_x^2(\tau,x)+U_{xx}^2(\tau,x)+U_{xxx}^2(\tau,x)+U_{xxxx}^2(\tau,x)+U_{xxxxx}^2(\tau,x)+U_\tau^2(\tau,x)+$$
$$+U_{\tau x}^2(\tau,x)+U_{\tau xx}^2(\tau,x)+U_{\tau xxx}^2(\tau,x)\}dxd\tau. \tag{40}$$

By virtue of the structure of the space $B^{5,3}_{2,2,T}$, it is obvious that $\forall t\in[0,T]$:

$$\int_0^\pi\left\{\frac{\partial^i U(t,x)}{\partial x^i}\right\}^2dx=\frac{\pi}{2}\sum_{n=1}^\infty\left(n^iU_n(t)\right)^2\le\frac{\pi}{2}\|U\|^2_{B^i_{2,t}}\le\frac{\pi}{2}\|U\|^2_{B^5_{2,t}}\le\frac{\pi}{2}\|U\|^2_{B^{5,3}_{2,2,t}} \quad \left(j=\overline{0,5}\right), \tag{41}$$

$$\int_0^\pi\left\{\frac{\partial^{1+j} U(t,x)}{\partial t\partial x^j}\right\}^2dx=\frac{\pi}{2}\sum_{n=1}^\infty\left(n^jU_n'(t)\right)^2\le\frac{\pi}{2}\|U_t\|^2_{B^j_{2,t}}\le\frac{\pi}{2}\|U_t\|^2_{B^3_{2,t}}\le\frac{\pi}{2}\|U\|^2_{B^{5,3}_{2,2,t}} \quad \left(j=\overline{0,3}\right). \tag{42}$$

Using the estimates (41) and (42), we get from (40) that $\forall t \in [0,T]$:

$$\|U\|^2_{B^{5,3}_{2,2,t}} \le C_0^2 + \frac{51\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{2\left(\alpha^2-1\right)} \cdot \|c(t)\|^2_{L_2(0,T)} + \frac{255\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{\alpha^2-1} \cdot \int_0^t c^2(\tau)\|U\|^2_{B^{5,3}_{2,2,\tau}} d\tau. \quad (43)$$

Applying Bellman's inequality from (43) we get:

$$\|U\|^2_{B^{5,3}_{2,2,T}} \le \left\{ C_0^2 + \frac{51\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{2\left(\alpha^2-1\right)} \cdot \|c(t)\|^2_{L_2(0,T)} \right\} \cdot \exp\left\{ \frac{255\left(\sqrt{T}+\sqrt{2\alpha}\right)^2}{\alpha^2-1} \cdot \|c(t)\|^2_{L_2(0,T)} \right\} \equiv C^2.$$

So, all possible solutions U of equations (38) in $B^{4,2}_{1,1,T}$ are a priori restricted in $B^{5,3}_{2,2,T}$ and the more in $B^{4,2}_{1,1,T}$, since $\|U\|_{B^{4,2}_{1,1,T}} \le \frac{\pi}{\sqrt{6}}\|U\|_{B^{5,3}_{2,2,T}}$. Then, by virtue of Shauder's strong principle or non-zero rotation principle, then operator H has a fixed point U in $B^{4,2}_{1,1,T}$ that belongs to the space $B^{5,3}_{2,2,T}$ and it is a strong generalized solution of the problem (1)-(3). And the uniqueness of the strong generalized solution of the problem (1)-(3) follows from the above remark 1. The theorem is proved.

Remark 2. We are to note that this paper is the continuation of papers [2]-[4] where the existence and uniqueness problems of the weak generalized solution of the problem (1)-(3) have been studied.

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Received October 26, 1999; Revised December 29, 1999.
Translated by AliyevaE.T.