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Natural vibration of inhomogeneous orthotropic cylindrical shell with regard to visco-elastic resistance

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Abstract. *In the paper, we consider natural vibration of a circular cross-section cylindrical shell inhomogeneous in thickness and resting on a viscous-elastic foundation. The boundary conditions are homogeneous.*
The solution of the problem with respect to the displacement vector component is reduced to the system of three linear equations. The numerical analysis is conducted for the axially-symmetric form of lateral vibration. As a result, we obtain a formula for determining the frequency square.

Keywords. vibration · inhomogeneity · orthotropic property · system of equations · foundation · shell · density · boundary condition.

Mathematics Subject Classification (2010): 74K25 · 74D99

1 Introduction

A circular cross section cylindrical shell is one of the most widespread design models of thin shelled load-carrying structures made of both from traditional and composite materials and used as reservoirs, pressure cylinders, different purpose pipes, body of aircrafts and ships.

Wide use of thin shelled cylindrical shells of a circular cross-section in many fields of engineering is successfully associated with relative simplicity of equations describing cylindrical shells, causes their long-term and intensive study, completing development of effective theories and engineering design methods. Inhomogeneity may be a result of mechanical and thermal processing, inhomogeneity of compositions because of the method of preparation of composite and reinforced materials, etc.

Note that in the case when the constructions are made of continuously inhomogeneous anisotropic materials, the solution of strength, stability and vibration problems is significantly complicated and their not accounting may lead to errors.

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The monograph [1] was devoted to theory of elasticity of inhomogeneous media. The stress-strain stress of structural elements was studied within linear theory. Resistance of external medium was not taken into account.

In [2], theory of homogeneous anisotropic media was developed and numerous problems were solved. The monograph [3] was devoted to the method of homogeneous solutions of anisotropic theory of shells. The considered solutions of specific problems have scientific practical importance.

[4] was devoted to theory of anisotropic shells in shipbuilding. Specific problems were solved.

[5] studies different problems of plates and shells made of glass-reinforced plastics.

In [6], a problem of free vibrations of a continuously inhomogeneous orthotropic rectangular plate lying on an inhomogeneous elastic-plastic foundation is solved.

In [7], an approximate analytic technique used in the problems of natural vibrations of inhomogeneous isotropic rectangular plates is offered. The natural vibrations of an exponentially orthotropic rectilinear plate with regard to two-constant resistance, were studied in [8].

[9] was devoted to linear problem of nonlinear vibration of a cylindrical shell.

[10] deals with the solution of dynamical stability of an inhomogeneous cylindrical shell of circular cross-section. Thermo-elastic problem of nano composite rectangular plates was studied in [11].

In [12] a specific problem of vibration of a cylindrical shell with specific properties is studied.

[13] was devoted to natural vibrations of inhomogeneous rectangular plate lying on a Winkler type foundation.

In [14] the statement of a problem of natural vibration of a shell lying on a visco-elastic foundation was stated.

[15] studies vibrations of inhomogeneous shells lying on two-constant elastic foundation.

Let a cross section cylindrical shell made of a material inhomogeneous in thickness lie on a viscous-elastic foundation of resistance that is connected with flexure $w(r, \varphi, t)$ by the following relation [14]:

$$q = \left(k_1 + k_2 \frac{\partial^2}{\partial t^2}\right) w\left(r, \varphi, t\right) \tag{1.1}$$

Here k_1 and k_2 are the characteristics of the medium, t is time.

We choose the system of coordinates in the following way: the axis φ is directed along the generator, the axis along the arch, the axis Z is normal to the median surface of the shell.

The elasticity and shear module E_1, E_2, G and density ρ are continuous functions, the Poisson's ratio is accepted as constant.

$$E_1 = E_{10} f(z), E_2 = E_{20} f(z), G = G_0 f(z)$$

$$\rho = \rho_0 \psi(z), \nu_1 = const, \nu_2 = const$$

The relation between stresses and deformations is written in the following way [2, 3]:

$$\begin{aligned}
&\sigma_{rr} = \tfrac{E_{10} f(z)}{1-\nu_1 \nu_2} \left[e_r + \nu_1 e_\varphi - z\left(\chi_r + \nu_1 \chi_\varphi\right)\right] \\
&\sigma_{\varphi\varphi} = \tfrac{E_{20} f(z)}{1-\nu_1 \nu_2} \left[e_\varphi + \nu_2 e_r - z\left(\chi_\varphi + \nu_1 \chi_r\right)\right] \\
&\sigma_{r\varphi} = G_0 f(x\left(z\right) \left[e_z - z\chi_z\right]
\end{aligned} \tag{1.2}$$

Here e_0, e_1, e_2 and χ_0, χ_1, χ_2 are small deformations of the curvature of the median surface of the shell, displacement vector components connected by the following relations [3].

$$e_r = \frac{\partial U}{\partial r}, e_\varphi = \frac{\partial V}{\partial \varphi} - \frac{W}{R}, e_z = \frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r};$$
$$\chi_r = \frac{\partial^2 W}{\partial r^2}; \chi_\varphi = \frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}; \chi_z = \frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}. \tag{1.3}$$

R is the shell radius.

Allowing for (3), the relation (2) takes the following form:

$$\sigma_{rr} = \frac{E_{10} f(z)}{1-\nu_1\nu_2}\left[\frac{\partial U}{\partial r} + \nu_1\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) - z\left(\frac{\partial^2 W}{\partial r^2} + \nu_1\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right)\right)\right];$$
$$\sigma_{\varphi\varphi} = \frac{E_{20} f(z)}{1-\nu_1\nu_2}\left[\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) + \nu_2\frac{\partial U}{\partial r} - z\left[\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right) + \nu_2\frac{\partial^2 W}{\partial r^2}\right]\right]; \tag{1.4}$$
$$\sigma_{r\varphi} = G_0 f(z)\left[\left(\frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r}\right) - z\left(\frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}\right)\right].$$

Now we make up expressions of forces T_{ij} and moments M_{ij} .

$$T_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} dz; \quad M_{ij} = \int_{-h/2}^{h/2} z\sigma_{ij} dz; \quad (i, j = 1, 2...)$$

$$T_{rr} = \int_{-h/2}^{h/2} \frac{E_{10} f(z)}{1-\nu_1\nu_2}\left[\frac{\partial U}{\partial r} + \nu_1\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) - z\left(\frac{\partial^2 W}{\partial r^2} + \nu_1\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right)\right)\right] dz;$$
$$T_{\varphi\varphi} = \int_{-h/2}^{h/2} \frac{E_{20} f(z)}{1-\nu_1\nu_2}\left[\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) + \nu_2\frac{\partial U}{\partial r} - z\left[\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right) + \nu_2\frac{\partial^2 W}{\partial r^2}\right]\right] dz;$$
$$T_{r\varphi} = \int_{-h/2}^{h/2} G_0 f(z)\left[\left(\frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r}\right) - z\left(\frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}\right)\right] dz.$$

Introducing the denotation

$$A_1 = \int_{-h/2}^{h/2} f(z) dz; A_2 = \int_{-h/2}^{h/2} z f(z) dz$$

we finally have

$$T_{rr} = \frac{E_{10}}{1-\nu_1\nu_2}\left[A_1\left(\frac{\partial U}{\partial r} + \nu_1\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right)\right) - A_2\left(\frac{\partial^2 W}{\partial r^2} + \nu_1\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right)\right)\right];$$
$$T_{\varphi\varphi} = \frac{E_{20}}{1-\nu_1\nu_2}\left[A_1\left(\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) + \nu_2\frac{\partial U}{\partial r}\right) - A_2\left[\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right) + \nu_2\frac{\partial^2 W}{\partial r^2}\right]\right]; \tag{1.5}$$
$$T_{r\varphi} = G_0\left[A_1\left(\frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r}\right) - A_1\left(\frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}\right)\right].$$

In the same way we can obtain the expression for the moments

$$M_{rr} = \frac{E_{10}}{1-\nu_1\nu_2}\left[A_2\left(\frac{\partial U}{\partial r} + \nu_1\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right)\right) - A_3\left(\frac{\partial^2 W}{\partial r^2} + \nu_1\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right)\right)\right];$$

$$M_{\varphi\varphi} = \frac{E_{20}}{1-\nu_1\nu_2}\left[A_2\left(\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) + \nu_2\frac{\partial U}{\partial r}\right) - A_3\left[\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right) + \nu_2\frac{\partial^2 W}{\partial r^2}\right]\right]; \quad (1.6)$$

$$M_{r\varphi} = G_0\left[A_2\left(\frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r}\right) - A_3\left(\frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}\right)\right].$$

$$A_3 = \int_{-h/2}^{h/2} z^2 f(z) dz.$$

We introduce the following denotation:

$$B_1 = \frac{E_{10}}{1-\nu_1\nu_2}A_1, B_1 = \frac{E_{10}}{1-\nu_1\nu_2}A_2, B_3 = G_0 A_1, B_4 = G_0 A_2$$

$$\alpha = \frac{E_{20}}{E_{10}}; B^* = \frac{E_{10}}{1-\nu_1\nu_2}A_3$$

Then (1.5) and (1.6) take the following form:

$$T_{rr} = B_1\left(\frac{\partial U}{\partial r} + \nu_1\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right)\right) - B_2\left(\frac{\partial^2 W}{\partial r^2} + \nu_1\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right)\right);$$

$$T_{\varphi\varphi} = \alpha\left[B_1\left(\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) + \nu_2\frac{\partial U}{\partial r}\right) - B_2\left[\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right) + \nu_2\frac{\partial^2 W}{\partial r^2}\right]\right]; \quad (1.7)$$

$$T_{r\varphi} = B_3\left(\frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r}\right) - B_4\left(\frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}\right)$$

$$M_{rr} = B_2\left[\frac{\partial U}{\partial r} + \nu_1\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) - B^*\left(\frac{\partial^2 W}{\partial r^2} + \nu_1\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right)\right)\right];$$

$$M_{\varphi\varphi} = \alpha\left[B_2\left(\left(\frac{\partial V}{\partial \varphi} - \frac{W}{R}\right) + \nu_2\frac{\partial U}{\partial r}\right) - B^*\left[\left(\frac{1}{R}\frac{\partial V}{\partial \varphi} + \frac{\partial^2 W}{\partial \varphi^2}\right) + \nu_2\frac{\partial^2 W}{\partial r^2}\right]\right]; \quad (1.8)$$

$$M_{r\varphi} = B_3\left(\frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial r}\right) - B_4\left(\frac{1}{2R}\frac{\partial V}{\partial r} + \frac{\partial^2 W}{\partial r \partial \varphi}\right).$$

In Donnellys equation of motion [5]

$$\begin{cases} \frac{\partial T_{rr}}{\partial r} + \frac{\partial T_{r\varphi}}{\partial \varphi} - \rho\frac{\partial^2 U}{\partial t^2} = 0, \\ \frac{\partial T_{r\varphi}}{\partial r} + \frac{\partial T_{\varphi\varphi}}{\partial \varphi} - \rho\frac{\partial^2 V}{\partial t^2} = 0, \end{cases}$$

we take into account the relations:

$$\rho\frac{\partial^2 U}{\partial t^2} \to 0; \rho\frac{\partial^2 V}{\partial t^2} \to 0$$

and get

$$\begin{cases} \frac{\partial T_{rr}}{\partial r} + \frac{\partial T_{r\varphi}}{\partial \varphi} = 0, \\ \frac{\partial T_{r\varphi}}{\partial r} + \frac{\partial T_{\varphi\varphi}}{\partial \varphi} = 0, \end{cases} \quad (1.9)$$

$$\frac{\partial^2 M_{rr}}{\partial r^2}+2\frac{\partial^2 M_{r\varphi}}{\partial r\partial\varphi}+\frac{\partial^2 M_{\varphi\varphi}}{\partial\varphi^2}+\frac{W}{R}+K_1W+K_2\frac{\partial^2 W}{\partial t^2}+\overline{\rho_0}\frac{\partial^2 W}{\partial t^2}=0$$

$$\overline{\rho_0}=\rho_0 h\int\limits_{-h/2}^{-h/2}\psi(z)dz$$

Substituting (1.7) in (1.9) and (1.8) in (1.10) we finally get the following system of homogeneous equations with respect to displacement vector components:

$$\begin{cases} L_1(U)+L_2(V)+L_3(W)=0\\ L_4(U)+L_5(V)+L_6(W)=0\\ L_7(U)+L_8(V)+L_9(W)=0\end{cases}\qquad(1.10)$$

where $L_1(U),L_2(V),L_3(W),L_4(U),L_5(V),L_6(W),L_7(U),L_8(V),L_9(W)$

$$L_1(U)=B_1\frac{\partial^2 U}{\partial r^2}+B_3\frac{\partial^2 U}{\partial\varphi^2},$$

$$L_2(V)=B_1\nu_1\frac{\partial^2 V}{\partial r\partial\varphi}-B_2\nu_1\frac{1}{R}\frac{\partial^2 V}{\partial r\partial\varphi}-\frac{1}{R^2}\frac{\partial V}{\partial\varphi}+B_3\frac{\partial^2 V}{\partial\varphi\partial r}-B_4\frac{1}{2R}\frac{\partial^2 V}{\partial\varphi\partial r},$$

$$L_3(W)=B_1\nu_1\left(-\frac{1}{R}\frac{\partial W}{\partial r}+\frac{W}{R^2}\right)-B_2\left(\frac{\partial^3 W}{\partial r^3}+\nu_1\frac{\partial^3 W}{\partial r\partial\varphi^2}\right)-B_4\frac{\partial^3 W}{\partial r\partial\varphi^2},$$

$$L_4(U)=B_3\frac{\partial^2 U}{\partial r\partial\varphi}+\alpha B_1\frac{\partial^2 U}{\partial r\partial\varphi},$$

$$L_5(V)=B_3\frac{\partial^2 V}{\partial r^2}-B_4\left(\frac{1}{2R}\frac{\partial^2 V}{\partial r^2}-\frac{1}{2R^2}\frac{\partial V}{\partial r}\right)+\alpha B_1\nu_1\frac{\partial^2 V}{\partial\varphi^2}-\alpha B_2\nu_1\frac{1}{R}\frac{\partial^2 V}{\partial\varphi^2}$$

$$L_6(W)=B_4\frac{\partial^3 W}{\partial r^2\partial\varphi}+\alpha\left[-\frac{1}{R}\frac{\partial W}{\partial\varphi}B_1\nu_1-B_2\left(\frac{\partial^3 W}{\partial r^2\partial\varphi}+\nu_1\frac{\partial^3 W}{\partial\varphi^3}\right)\right].$$

$$L_7(U)=B_2\frac{\partial^3 U}{\partial r^3}+\alpha B_2\nu_2\frac{\partial^3 U}{\partial\varphi^2\partial r}+B_3\frac{\partial^3 U}{\partial\varphi^2\partial r}$$

$$L_8(V)=B_2\left[\nu_1\frac{\partial^3 V}{\partial\varphi\partial r^2}-B^*\nu_1\left(\frac{2}{R^3}\frac{\partial V}{\partial\varphi}-\frac{2}{R^2}\frac{\partial^2 V}{\partial\varphi\partial r}+\frac{1}{R}\frac{\partial^3 V}{\partial\varphi\partial r^2}\right)\right]+$$

$$+\alpha\left[B_2\frac{\partial^3 V}{\partial\varphi^3}-B^*\frac{1}{R}\frac{\partial^3 V}{\partial\varphi^3}\right]+B_3\frac{\partial^3 V}{\partial\varphi\partial r^2}-B_4\left(\frac{1}{2R}\frac{\partial^3 V}{\partial\varphi\partial r^2}-\frac{1}{2R^2}\frac{\partial^2 V}{\partial r\partial\varphi}\right)$$

$$L_9(w)=B_2\left(-\frac{2W}{R^3}-\frac{1}{R}\frac{\partial^2 W}{\partial r^2}-B^*\left(\frac{\partial^4 W}{\partial r^4}-\nu_1\frac{\partial^3 W}{\partial r\partial\varphi^2}\right)\right)-$$

$$-\alpha\left[B_2\frac{1}{R}\frac{\partial^2 W}{\partial\varphi^2}+B^*\left(\frac{\partial^4 W}{\partial\varphi^4}+\nu_2\frac{\partial^4 W}{\partial r^2\partial\varphi^2}\right)\right]-B_4\frac{\partial^3 W}{\partial r^2\partial\varphi}+K_1W+K_2\frac{\partial^2 W}{\partial t^2}+\overline{\rho}\frac{\partial^2 W}{\partial t^2}.$$

2 The solution method

Under axially symmetric form of vibration, the solution of the problem is simplified and the motion equation takes the following form with respects to W :

$$c_1 \frac{\partial^4 W}{\partial r^4} + c_2 \frac{\partial^2 W}{\partial r^2} + c_3 W + (K_2 + \rho) \frac{\partial^2 W}{\partial t^2} = 0 \tag{2.1}$$

Here we adopt the following denotation:

$$c_1 = \frac{E_{10}}{1-\nu_1\nu_2}\left(A_2^2 - A_1\right); c_2 = -\frac{E_{20}}{1-\nu_1\nu_2}\frac{A_2\nu_1}{R}\left(A_1 - 1\right);$$

$$c_3 = -\frac{E_{10}}{1-\nu_1\nu_2}\frac{A_1^2\nu_1 - A_1\nu_2}{R^2} + K_1$$

Assume that the following boundary conditions are fulfilled:

$$T_1 = 0; W = 0; M_1 = 0$$

We will look for the solution (2.1) in the following form:

$$W = W_0 \sin\frac{m\pi}{l} x e^{i\omega t} \tag{2.2}$$

There ω is frequency, l is shells length, m is the number of half waves in the shells length. Substituting (2.2) in (2.1), we have

$$c_1\left(\frac{m\pi}{l}\right)^4 - c_2\left(\frac{m\pi}{l}\right)^2 + c_3 - \omega^2\left(\overline{K_2} + \overline{\rho}\right) = 0$$

For finding minimum value, we use the extremum condition with respect to $\lambda_m = \frac{m\pi}{l}$. Note that,

$$2c_1\lambda_m^2 - c_2 = 0$$

i.e.

$$\lambda_m^2 = \frac{1}{2}\frac{c_2}{c_1}$$

Substituting the values of λ_m^2 in (2.3), make some transformations.

$$\left(\frac{\omega}{\omega_v}\right)^2 = \frac{1}{\mu+1}; \mu = K_1\overline{\rho^{-1}}; K_2 = 0$$

The results of the calculation are given in the form of a table and fig.1

3 Conclusions

In the paper, vibration of an orthotropic inhomogeneous cylindrical shell on a viscous-elastic foundation is studied by the numerical-analytical method.

μ	$\overline{\omega^{*2}}$
0	1
0,2	0,833333
0,4	0,714286
0,6	0,625
0,8	0,555556
1	0,5

Fig 1. Calculation scheme

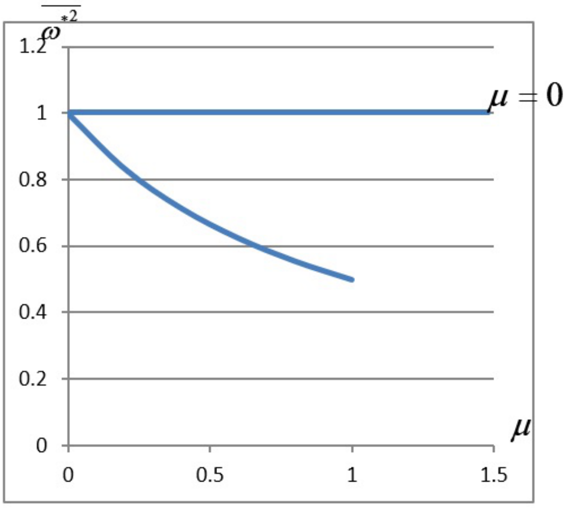

Fig.1. the graph of dependence of pure quantity of frequency on inhomogeneity density and characteristics of foundation.

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