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On the solution of thermo viscoelasticity problems with a weakly inhomogeneous temperature field

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Abstract. *In the case of a non-stationary, but uniform temperature field with the aid of the universal function of temperature, the physical time is replaced by a modified one and the solution of the unknown problem is constructed. If the temperature field is also inhomogeneous with respect to the coordinates, then expressing the universal function in terms of a small parameter, the solution of the unknown problem reduces to problems in which the relaxation or creep functions are homogeneous functions of the coordinates and the solution is constructed by the method of successive approximations. As an example, we consider the problem of the bending of a circular plate rigidly fixed along the contour.*

Keywords. modified time · universal function · relaxation · creep · non-stationary · inhomogeneous · fictitious load.

Mathematics Subject Classification (2010): 74D05, 74K20

1 Introduction.

The solution of tasks linearly-viscously elasticity is based on Voltaire's principle and Laplace's transformations. However the situation is that, a case is more difficult if properties of material strongly depend on temperature, i.e. functions of a relaxation of R and creep of the Item depend on temperature. In some cases comes to the rescue so-called "time-temperature" analogy which cornerstone the property which is a simple mathematical consequence of the assumption that all coefficients of viscosity equally depend on temperature is i.e. are proportional to the same general function of temperature $a_T(T)$. If to enter the modified time t' as integral from the physical time divided into general function of temperature.

$$t' = \int_0^t \frac{d\xi}{a_T(\xi)}; \quad \tau' = \int_0^\tau \frac{d\xi}{a_T(\xi)} \tag{1.1}$$

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that dependence between deviator of tension S_{ij} and deformation e_{ij} will take a form [1]:

$$\left.\begin{array}{l} S_{ij}\left(t\right)=\int_0^t R\left(t'-\tau'\right) de_{ij}\left(\tau\right) \\ \sigma\left(t\right)=3K\left(\varepsilon\left(t\right)-\alpha\vartheta\left(t\right)\right) \end{array}\right\} \tag{1.2}$$

where - a ν temperature difference, - α coefficient of temperature expansion to - the elastic module of volume compression.

$$\sigma_{ij}=s_{ij}+\sigma\delta_{ij}\varepsilon_{ij}=e_{ij}+\varepsilon\delta_{ij}\sigma=\frac{1}{3}\delta_{kk}\varepsilon=\frac{1}{3}\varepsilon_{kk}.$$

2 Problem statement

Introduction of the modified time and using the principle of temperature and time analogy to ratios for linear thermo-viscoelasticity, the changing properties at the expense of the non-stationary, but uniform temperature field it is possible to solve a required problem. However these methods are not suitable if the temperature field is also non-uniform on coordinates. We will construct the solution of a required task by a so-called method of small parameter. Let's consider that the equation of inflow of heat is allocated from the general system of the equations of thermo-viscoelasticity and decides separately, i.e. the case not of mutually coherent thermo-viscoelasticity is considered. Solving the heat conductivity equation, we will find temperature as function of coordinates and time $T\left(x_{k,}t\right)$. Therefore general function is known to us. $a_T\left(x_{k,}t\right)$. For further it is convenient to enter function

$$f\left(x_{k,}t\right)=\frac{1}{a_T\left(x_{k,}t\right)}. \tag{2.1}$$

Let now on the basis of function (2.1) some function $f_0\left(t\right)$, time-dependent and independent of coordinates is constructed

$$f_0\left(t\right)\equiv<f\left(x_{k,}t\right)> \tag{2.2}$$

where angular brackets in the right part (2.2) mean averaging somehow of the concluded expression in them on coordinates. For example, it can be made on the volume of V, the occupied considered body or can be taken $f\left(x_{k,}t\right)$ as its value in some fixed point x_k^0. Now the known function $f\left(x_{k,}t\right)$ we will present the sums in the form

$$f\left(x_{k,}t\right)\cong f_0\left(t\right)\left(1+\lambda\frac{f\left(x_{k,}t\right)-f_0\left(t\right)}{f_0\left(t\right)}\right). \tag{2.3}$$

At $\lambda=1$ a ratio (2.3) represents identity. Let's assume that size $\frac{f\left(x_{k,}t\right)-f_0(t)}{f_0(t)}$ in some sense is small in comparison with unit in some area V_0. Then the choice of parameter λ it is possible to achieve $f(x_k,t)\approx f_0\left(t\right)$. In this case small parameters call numerical parameter λ. Since the modified time t' will be expressed by formula (1.1)

$$t'=\int\limits_0^t\frac{d\xi}{a_T\left(\xi\right)}=\int\limits_0^t f_0\left(t\right)dt+\lambda\int\limits_0^t\left[f(x_k,t)-f_0\left(t\right)\right]dt \tag{2.4}$$

hat we will be able to spread out in a row on degrees λ a kernel of a relaxation and creep

$$\left.\begin{array}{l} R\left(t'\right)=R^{(0)}\left(t\right)+\lambda R^{(1)}\left(t,x_k\right)+\lambda^2R^{(2)}(t,x_k)+\cdots \\ \mathit{\Pi}(t')=\mathit{\Pi}^{(0)}\left(t\right)+\lambda\mathit{\Pi}^{(1)}\left(t,x_k\right)+\lambda_2\mathit{\Pi}^{(2)}(t,x_k)+\cdots \end{array}\right\} \tag{2.5}$$

We will look for the solution of an objective U_i for movements in a look

$$U_i\left(x_k,t\right)=U_i^{(0)}\left(t,x_k\right)+\lambda U_i^{(1)}\left(t,x_k\right)+\lambda^2U_i^{(2)}\left(t,x_k\right)+\cdots \tag{2.6}$$

Apparently from (1.2) tension it is also presented in the form

$$\left.\begin{array}{l} S_{ij}\left(x_k,t\right)=S_{ij}^{(0)}\left(x_k,t\right)+\lambda S_{ij}^{(1)}\left(x_k,t\right)+\lambda^2S_{ij}^{(2)}\left(x_k,t\right)+\cdots \\ \sigma\left(x_k,t\right)=\sigma^{(0)}\left(x_k,t\right)+\lambda\sigma^{(1)}\left(x_k,t\right)+\lambda^2\sigma^{(2)}\left(x_k,t\right)+\cdots \end{array}\right\} \tag{2.7}$$

Then the ratio (1.2) in movements will be presented in the form:

$$\left\{\begin{array}{l} S_{ij}=\frac{1}{2}\int\limits_0^t R\left(t'-\tau'\right)d\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial U_k}{\partial x_k}\right) \\ \sigma=k\left(\frac{\partial U_k}{\partial x_k}-3\alpha\nu\right) \end{array}\right. \tag{2.8}$$

Substituting (2.5), (2.6) and (2.7) in (2.8) and equating coefficients at identical degrees λ we will receive

$$\left\{\begin{array}{l} S_{ij}^{(0)}=\frac{1}{2}\int\limits_0^t R\left(t-\tau\right)d\left(\frac{\partial U_i^0}{\partial x_j}+\frac{\partial U_j^0}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial U_k^{(0)}}{\partial x_k}\right) \\ S_{ij}^{(1)}=\frac{1}{2}\int_0^t R^{(0)}\left(t-\tau\right)d\left(\frac{\partial U_i^{(1)}}{\partial x_j}+\frac{\partial U_j^{(1)}}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial U_k^{(1)}}{\partial x_k}\right)-\overline{S}_{ij}^{(1)} \\ ------------------------------ \\ S_{ij}^{(n)}=\frac{1}{2}\int\limits_0^t R^{(0)}\left(t-\tau\right)d\left(\frac{\partial U_i^{(n)}}{\partial x_j}+\frac{\partial U_j^{(n)}}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial U_k^{(n)}}{\partial x_k}\right)-\overline{S}_{ij}^{(n)} \\ \sigma^{(0)}=k\left(\frac{\partial U_k^{(0)}}{\partial x_k}-3\alpha_0\vartheta\right);\sigma^{(1)}=k\frac{\partial U_k^{(1)}}{\partial x_k},....\sigma^{(n)}=k\frac{\partial U_k^{(n)}}{\partial x_k} \end{array}\right. \tag{2.9}$$

where

$$\left\{\begin{array}{l} \overline{S}_{ij}^{(1)}=\frac{1}{2}\int\limits_0^t R^{(1)}\left(x_\alpha,t-\tau\right)d\left(\frac{\partial U_i^{(0)}}{\partial x_j}+\frac{\partial U_j^{(0)}}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial U_k^{(0)}}{\partial x_k}\right) \\ ------------------------------ \\ \overline{S}_{ij}^{(n)}=\frac{1}{2}\sum\limits_{k=1}^{n}\left[\int\limits_0^t R^{(k)}\left(x_\alpha,t-\tau\right)d\left(\frac{\partial U_i^{(n-k)}}{\partial x_j}+\frac{\partial U_j^{(n-k)}}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial U_k^{(n-k)}}{\partial x_k}\right)\right] \end{array}\right. \tag{2.10}$$

3 Solution

Apparently for definition of zero approach it is necessary to solve $U_i^{(0)}\left(x_\alpha,t\right)$ an initial problem, but only the relaxation which is not depending on coordinates of functions $R^{(0)}\left(t\right)$. For definition of the subsequent approximations the initial problem in the absence of the temperature field, but in the presence of the additional fictitious loadings received from the previous approximations is solved.

The required $U_i^{(n)}\left(x_\alpha,t\right)$ decision is defined as the sum of the corresponding approximations that follows from (2.6) at $k>1$ since in this case (2.6) addresses in identity.

As an example we will consider a task about a bend of a round plate under the influence of the uniform distributed loading rigidly fixed on an external contour taking q_0 into account the temperature field. In case of ideal elasticity the balance equation in movements of a bend of W is given to a look [3]:

$$\begin{array}{l} \frac{E}{1-\nu^2}\left(\frac{d^3W}{dr^3}+\frac{1}{r}\frac{d^2W}{dr^2}-\frac{1}{r^2}\frac{dW}{dr}\right)+\frac{d}{dr}\left(\frac{E}{1-\nu^2}\right)\left(\frac{d^2W}{dr^2}+\frac{1}{r}\frac{dW}{dr}\right)+ \\ +\frac{E}{1-\nu}\alpha\frac{d\theta}{dr}+\frac{d}{dr}\left(\frac{E}{1-\nu}\right)\alpha\theta=\frac{6q_0r}{h^3} \end{array} \tag{3.1}$$

where E - the elasticity module, ν - Poisson's coefficient.

Replacing elastic constants in (3.1) corresponding integrated operators in images

$$\frac{E}{1-\nu^2}\to G^*=\frac{1}{2}R^*;\frac{d}{dr}\left(\frac{E}{1-\nu^2}\right)\to G^{*'};\frac{E}{1-\nu}\to^*;\frac{d}{dr}\left(\frac{E}{1-\nu}\right)\to \Pi^{*'};R^*\Pi^*\equiv 1$$

and having entered designations

$$\frac{d^3W}{dr^3}+\frac{1}{r}\frac{d^2W}{dr^2}-\frac{1}{r^2}\frac{dW}{dr}=L_0\left(W\right),\frac{d^2W}{dr^2}+\frac{1}{r}\frac{dW}{dr}=L_1\left(W\right)$$

the ratio (3.1) will register in a look:

$$G^*L_0\left(W\right)+G^{*'}L_1\left(W\right)+\alpha\left({}^*\theta'+{}^{*'}\theta\right)=\frac{6q_0r}{h^3} \tag{3.2}$$

Let's assume that the temperature field is set and depends on the radius and time. Therefore in view of symmetry other than zero a component of movement the deflection will be only. Let also on the set temperature field general function $f(r,t)$ and somehow function be calculated $f_0\left(t\right)$.

Then considering (2.6) - (2.8) ratio (3.2) it will be presented in the form:

$$\begin{array}{l} \left(C_0^*+\lambda C_1^*+\lambda^2C_2^*+\lambda^3C_3^*+...\right)L_0\left(W_0+\lambda W_1+\lambda^2W_2+\lambda^3W_3+...\right)= \\ =-\left(\lambda C_1^{*'}+\lambda^2C_2^{*'}+\lambda^3C_3^{*'}+...\right)L_1\left(W_0+\lambda W_1+\lambda^2W_2+\lambda_3W_3+...\right)- \\ -\left(\Pi_0^*+\lambda\Pi_1^*+\lambda^2]\Pi_2^*+\lambda^3\Pi_3^*+...\right)\alpha\vartheta'- \\ -\left(\lambda\Pi_1^{*'}+\lambda^2\Pi_2^{*'}+\lambda^3\Pi_3^{*'}+...\right)\alpha\vartheta+\frac{6q_0r}{h^3} \end{array} \tag{3.3}$$

Equating coefficients at identical degrees λ (3.3) it will be presented in the form

$$\begin{array}{l} G_0^*L_0\left(W_0\right)=\frac{6q_0r}{h^3}-\Pi_0^*\alpha\vartheta'=\varphi_0 \\ G_0^*L_0\left(W_1\right)=-\left[G_1^*L_0\left(W_0\right)+G_1^{*'}L_1\left(W_0\right)+\alpha\left(\Pi_1^*\vartheta'+\Pi_1^{*'}\vartheta\right)\right]=\varphi_1 \\ \\ G_0^*L_0\left(W_2\right)=-\left[G_1^*L_0\left(W_1\right)+G_2^{*'}L_0\left(W_0\right)+G_1^{*''}L_1\left(W_1\right)+G_2^{*''}L_1\left(W_0\right)+\right. \\ \qquad\qquad\qquad \left.+\alpha\left(\Pi_2^*\vartheta'+\Pi_2^{*'}\vartheta\right)\right]nn=\varphi_2 \\ G_0^*L_0\left(W_3\right)=-\left[\begin{array}{l} G_1^*L_0\left(W_2\right)+G_2^{*'}L_0\left(W_1\right)+G_3^{*'}L_0\left(W_0\right)+G_1^{*''}L_1\left(W_2\right)+ \\ +G_2^{*'}L_1\left(W_1\right)+G_3^{*''}L_1\left(W_0\right)+\alpha\left(\Pi_3^*\vartheta'+\Pi_3^{*'}\vartheta\right) \end{array}\right]=\varphi_3 \\ ------------------------------ \\ G_0^*L_0\left(W_n\right)=-\left[\sum\limits_{k=1}^{n}G_k^*L_0\left(W_{n-k}\right)+\sum\limits_{k=1}^{n}G_k^*L_1\left(W_{n-k}\right)+\alpha\left(\Pi_n^*\vartheta'+\Pi_n^{*'}\vartheta\right)\right]=\varphi_n \end{array}$$

Every time the equation is solved;

$$G_0^*L_0\left(W_n\right)=\varphi_n \tag{3.4}$$

4 Conclusion

Means, that is enough to solve a problem for zero approach $W_0(r,t)$ which can be treated as temperature drop in a task where properties of visco-elastic material do not depend on temperature and they can be considered as the set loads of plate surfaces. Then the subsequent approximations $W_1(r,t), W_2(r,t), ...$ will be found by recurrent formulas (3.4). Then the required decision $W_i^{(n)}(r,t)$ defined as the sum of the corresponding approximations $W_i^{(k)}(r,t)$ that follows from (2.6). Convergence of consecutive approximations it is proved in [4]. Thus, the given method of small parameter reduces an initial task non-uniform thermo - viscously elasticity to tasks in which function a relaxation and creep are some functions of coordinates for a case of the non-uniform temperature field.

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