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Elastic parameters of materials at high and ultrahigh deformations in case of their description by harmonic potential

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Abstract. *The relationships between the parameters of the velocity of propagation of elastic waves and the level of deformations in solid deformable media are determined in the case of all-round compression in this paper. These relationships are determined using the ratios of the non-classically-linearized approach (NLA) using the harmonic type of the elastic potential. The results are compared with similar results obtained earlier for potentials of the Murnaghan type and a quadratic potential.Specific numerical results are obtained for materials: plexiglas, steel* $092s$*, and plagiogranite.*

Keywords. high pressure · instability · velocities of elastic waves.

Mathematics Subject Classification (2010): 74J30

1 Introduction.

It is known that the main geospheres of the Earth - the Crust, the Mantle and the Earth's Core are considered to be deformable solids. [Dziewonski and Anderson 1981, Anderson 2007].

Usually, in order to determine the mechanical properties of the medium, their chemical composition and physical processes occurring at different depths, seismological data are processed. The seismic velocities are compared and matched with the laboratory experimental data of geological materials (of relevant rocks and minerals), taking into account the Birch law. [5]

The empirical relation is knownas Birch's law, established by Francis Birch, indicates a linear relationship between wave velocity V_p and density ρ of rocks and minerals:

$$V_p^2 = (K/\rho)(3-3\sigma)/(1+\sigma);\ V_s^2 = \frac{K_S}{\rho}\frac{3-6\sigma}{2+2\sigma};\ V_p^2 \text{ and } V_s^2 \sim \frac{1}{\rho}$$

Here, Vp is the velocity of compression waves,V_S - is the velocity of rotation and shear waves σ is the Poisson coefficient, ρ is density, K is incompressibility, and K_S is adiabatic incompressibility.

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This relationship may break with increasing depth in the mantle. The ratios cease to be fair with increasing pressures, which begins with the transition zone in the Earth's mantle [5].

The materials begin to exhibit inelastic propertiesat high pressures and temperatures. The validity of linear approximation in such conditions becomes unreasonable. Experiments show that different thermobaric effects affect the nature of changes in physico-mechanical parameters at different levels of deformation. It is not possible to determine any single trend of this influence for different materials.

The character of the propagation of elastic waves can be influenced by factors of different nature: gravitational, thermo-baric (convection), nuclear reactions, electromagnetic, porosity, saturation, etc. Due to the huge number of these vary factors that contribute to one degree or another to the current picture of the state of matter in the depths of the Earth, it is almost impossible to take them all into account. One of the possible ways out of this situation is the use of elastic potentials. It might be chosen the most suitable type of potential that can better describe the effects of a particular number of specific factors for these conditions. The results obtained for potentials can also be compared, each of which describes better one material or another. The propagation of elastic waves in deformed bodies is studied in detail in the following papers: [Truesdell 1975, Biot 1965, Guz 1986, 2004, Litasov, K.D. and Shatskiy, A.F. 2016, H.H. Guliyev et al. 2017,2018]. This approach is widely used in geophysics [Guliev and Jabbarov 1998, 2000, Alexandrov et al. 2001, Akbarov 2015, Teachavorasinskun and Pongvithayapanu 2016]

2 Main part.

Analytical formulae are obtained to calculate the parameters of pressure and velocity of elastic pressure and shear waves in homogeneous deformable isotropic media within non-classically linearized approach (NLA) in the work [H.H. Guliyev 2018]. Elastic potentials of the Murnaghan type and quadratic elastic potential, materials: plexiglass, steel and plagiogranite have been reviewed. We emphasize that under the term "parameter" of velocity is meant the velocity of propagation of elastic waves in a medium that is under deformation.

In this article, these questions are investigated in the case of a harmonic potential, and a comparison and analysis of identical results is carried out.

To determine the "true" propagation velocities of shearand pressure waves, the following formulas have been used [Guz A.N. 1986, 2004]:

$$\rho C_{lx_1}^2 = \lambda_1^4 A_{11} + \lambda_1^2 S_{11}^0;\ \rho C_{Sx_2}^2 = \lambda_1^2\lambda_2^2\mu_{12} + \lambda_1^2 S_{11}^0;\ \rho C_{Sx_2}^2 = \lambda_1^2\lambda_3^2\mu_{13} + \lambda_1^2 S_{11}^0 \quad (2.1)$$

Here λ_1, λ_2 are elongation coefficients. S_{11}^0 components of the stress tensor referred to the unit area in the natural state, ρ is the density of the medium in the natural state. Values A_{11},$\mu_{12}, \mu_{13}, S_{11}^0$ algebraic expressions, the form of which is specified depending on the version of the problem statement.

In the case of the theory of large initial strains and all-round compression, the following relation is true:

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_0 = \sqrt{1 + 2\varepsilon_0} \quad (2.2)$$

Here ε_0 is a parameter of comprehensive deformation.

For harmonic potential [Guz A.N. 1986, 1991.]:

$$\begin{aligned} A_{i,\beta} &= \lambda(\lambda_i\lambda_\beta)^{-1} + \delta_{i,\beta}[2, \mu - \lambda\,(\lambda_1 + \lambda_2 + \lambda_3 - 3)]\lambda_i^{-3} \\ \mu_{ij} &= [2\mu - \lambda(\lambda_1 + \lambda_2 + \lambda_3 - 3)](\lambda_i\lambda_j)^{-1}(\lambda_i + \lambda_j)^{-1} \\ S_{\beta\beta}^0 &= [\lambda\,(\lambda_1 + \lambda_2 + \lambda_3 - 3) + 2\mu(\lambda_\beta - 1)]\lambda_\beta^{-1} \end{aligned} \quad (2.3)$$

Here λ, μ are Lame's parameters.

Substituting the expression (2.2) and (2.3) in (2.1), we obtain the following correlations:

$$\frac{C_{l_{x_1}}}{C_{l_0}} = (1+2\varepsilon_0)^{\frac{5}{4}} \tag{2.4}$$

$$\frac{C_{S_{x2}}}{C_{S_0}} = \frac{C_{S_{x3}}}{C_{S_0}} = \sqrt{(1+2\varepsilon_0)^{\frac{3}{2}}\left[(1+2\varepsilon_0)\left(\frac{2-\nu}{1-2\nu}\right) - \sqrt{1+2\varepsilon_0}\left(\frac{1+\nu}{1-2\nu}\right)\right]} \tag{2.5}$$

$$\frac{P_0}{\mu} = \frac{3\lambda+2\mu}{\mu}\left(\sqrt{1+2\varepsilon_0}-1\right) = \frac{2+2\nu}{1-2\nu}\left(\sqrt{1+2\varepsilon_0}-1\right) \tag{2.6}$$

Parameter $\frac{P_0}{\mu}$– characterizes the pressure.

3 Numerical results.

Figures 1-3 show the results of calculations for parameters: $\frac{C_{lx1}}{Cl_0}$; $\frac{C_{Sx2}}{Cs_0}$; $\frac{C_{Sx3}}{Cs_0}$ and $\frac{P_0}{\mu}$.
The calculations used data from Table 1. These data are taken from[H.H. Guliyev 2017,2018].

Table 1

Parameters of the media	$10^{-3}a$, MPa	$10^{-3}b$, MPa	$10^{-3}c$, MPa	$10^{-3}\lambda$, MPa	$10^{-3}\mu$, MPa	ν
Plexiglas	$\frac{-3{,}99}{0{,}268}$	$\frac{-7{,}16}{-3{,}12}$	$\frac{-14{,}4}{-6{,}77}$	4,04	1,9	0,3401
Steel	$\frac{-325}{-269}$	$\frac{-309}{-214}$	$\frac{-799}{-483}$	94,4	79,0	0,2722
Plagiogranite	$\frac{-3{,}87}{-}$	$\frac{-1{,}99}{-}$	$\frac{-6{,}24}{-}$	35,67	39,95	0,235

* The numerator contains data related to t.o.l.i.d., in denominator – t.s.v.o.t.o.s.i.d.

** Dashes in the columns for Plagiogranite indicate the lack of data for this case.

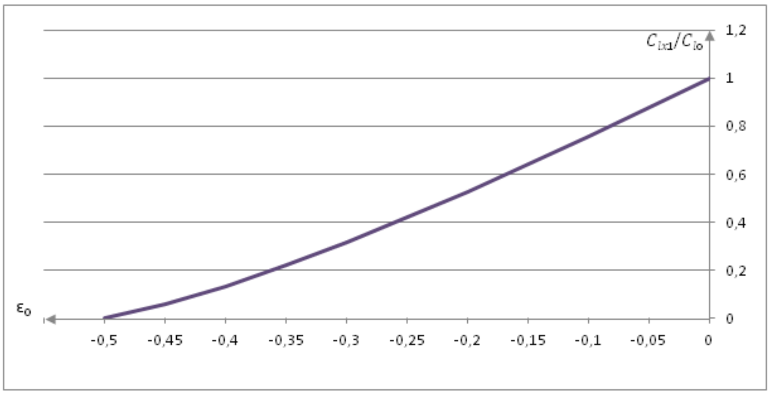

Fig. 1. The relationship between the parameter of velocity of propagation of elastic pressure waves in the medium and the deformation parameter. Formula (4).

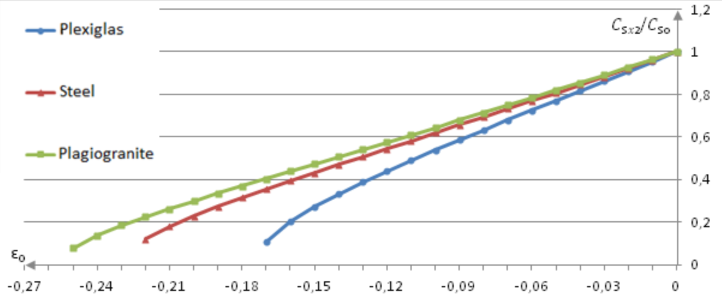

Fig. 2. The relationship between the parameters of the velocity of propagation of share elastic waves in the medium and the deformation parameter. Formula (5).

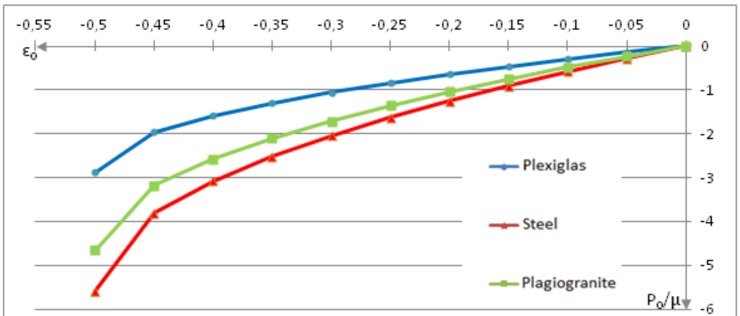

Fig. 3. The relationship between the value, characterizing the pressure in the medium, and the deformation parameter. Formula (6).

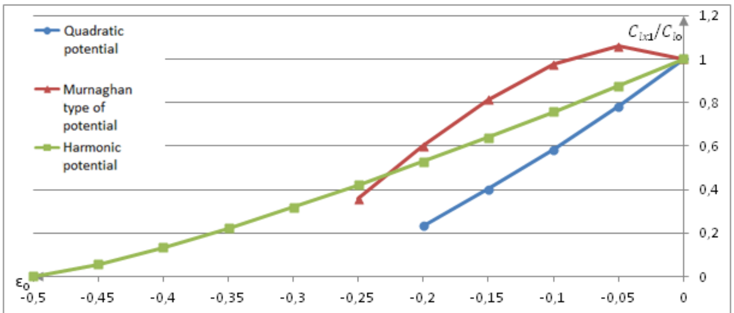

Fig. 4. Comparison of the velocity parameters of elastic pressure waves C_{lx1}/C_{lo} on ε_0, for plexiglas in cases of quadratic, harmonic, and Murnaghan type potentials.

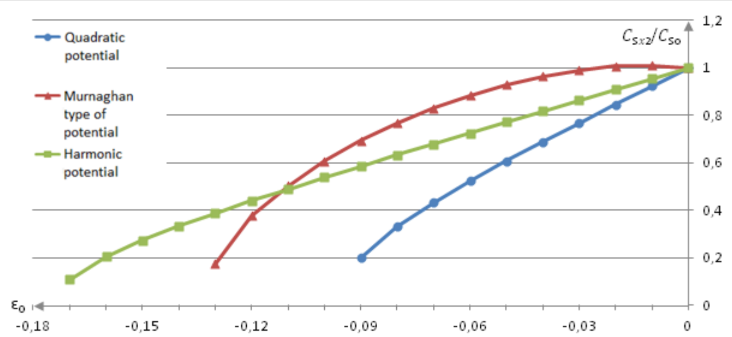

Fig. 5. Comparison of the velocity parameters of elastic shear waves C_{sx2}/C_{so} on ε_0, for plexiglas in cases of quadratic, harmonic and Murnaghan type potentials.

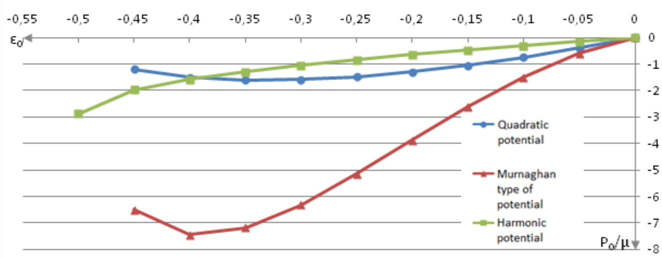

Fig. 6. Comparison of parameters P_0/μ on ε_0, for plexiglas in cases of quadratic, harmonic, and Murnaghan type potentials.

Table 2. Critical values of parameters of plexiglas

Potentials	$\frac{C_{lx1}}{Cl_0}$	ε	$\frac{C_{Sx2}}{Cs_0}$	ε	$\frac{P_0}{\mu}$	ε
Quadratic potential	0,232498002	-0,2	0,200043194	-0,09	-1,19262	-0,45
Murnaghan type of potential	0,356146365	-0,25	0,174119481	-0,13	-6,5079529	-0,45
Harmonic potential	0	-0,5	0,107023799	-0,17	-2,87581	-0,5

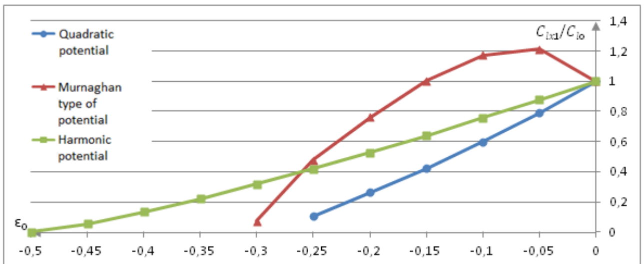

Fig. 7. Comparison of the velocity parameters of elastic pressure waves C_{lx1}/C_{lo} on ε_0, for steel in cases of quadratic, harmonic, and Murnaghan type potentials

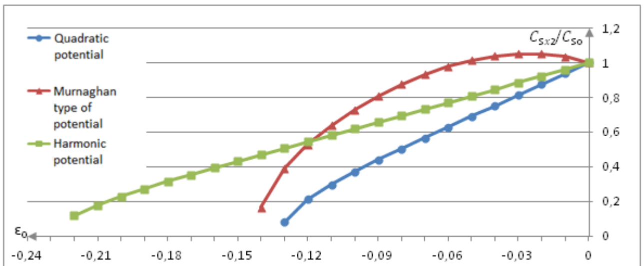

Fig. 8. Comparison of the velocity parameters of elastic shear waves C_{Sx2}/C_{So} on ε_0, for steel in cases of quadratic, harmonic and Murnaghan type potentials.

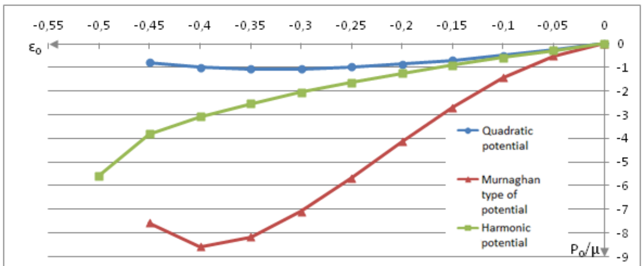

Fig. 9. Comparison of parameters P_0/μ on ε_0, for steel in cases of quadratic, harmonic, and Murnaghan type potentials.

Table 3. Critical values of parameters of steel

Potentials	$\frac{C_{lx1}}{Cl_0}$	ε	$\frac{C_{Sx2}}{Cs_0}$	ε	$\frac{P_0}{\mu}$	ε
Quadratic potential	0,10553	-0,25	0,081168155	-0,13	-0,79472	-0,45
Murnaghan type of potential	0,071779	-0,3	0,167101082	-0,14	-7,57039	-0,45
Harmonic potential	0	-0,5	0,11956	-0,22	-5,58472	-0,5

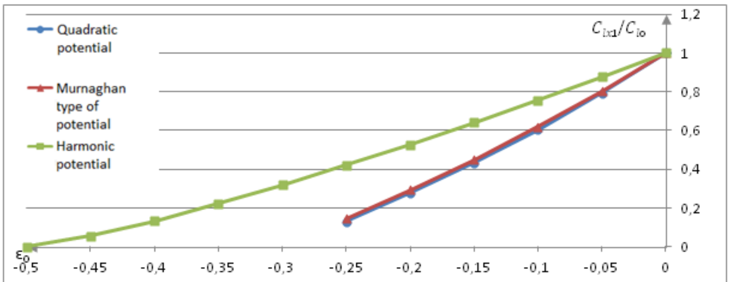

Fig. 10. Comparison of the velocity parameters of elastic pressure waves C_{lx1}/C_{lo} on ε_0, for plagiogranite in cases of quadratic, harmonic, and Murnaghan type potentials

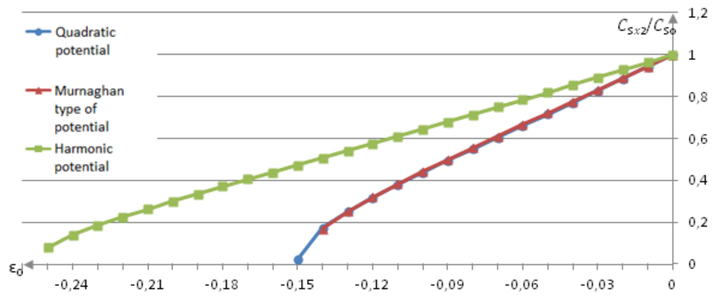

Fig. 11. Comparison of the velocity parameters of elastic shear waves C_{Sx2}/C_{So} on ε_0, for plagiogranite in cases of quadratic, harmonic and Murnaghan type potentials.

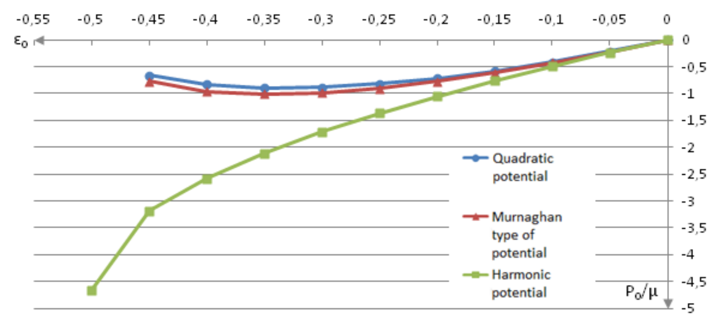

Fig. 12. Comparison of parameters P_0/μ on ε_0, for plagiogranite in cases of quadratic, harmonic, and Murnaghan type potentials.

Table 4. Critical values of parameters of Plagiogranite

Potentials	$\frac{C_{lx1}}{Cl_0}$	ε	$\frac{C_{Sx2}}{Cs_0}$	ε	$\frac{P_0}{\mu}$	ε
Quadratic potential	0,130546	-0,25	0,019666162	-0,15	-0,66318	-0,45
Murnaghan type of potential	0,144544054	-0,25	0,167209	-0,14	-0,7768614	-0,45
Harmonic potential	0	-0,5	0,078438	-0,25	-4,66038	-0,5

Table 5. Differences of values between the corresponding parameters in cases of Murnaghan and quadratic types of potentials, for plagiogranite.

Values for Murnaghan type potential						Values for quadratic potential					
$\frac{C_{lx1}}{Cl_0}$	ε	$\frac{C_{Sx2}}{Cs_0}$	ε	$\frac{P_0}{\mu}$	ε	$\frac{C_{lx1}}{Cl_0}$	ε	$\frac{C_{Sx2}}{Cs_0}$	ε	$\frac{P_0}{\mu}$	ε
1	0	1	0	0	0	1	0	1	0	0	0
0,803131	-0,05	0,943453	-0,01	-0,22527	-0,05	0,79345	-0,05	0,942032	-0,01	-0,22106	-0,05
0,61972	-0,1	0,88733	-0,02	-0,43271	-0,1	0,604594	-0,1	0,884701	-0,02	-0,41684	-0,1
0,450275	-0,15	0,831578	-0,03	-0,61829	-0,15	0,433242	-0,15	0,827954	-0,03	-0,58487	-0,15
0,294162	-0,2	0,77613	-0,04	-0,77699	-0,2	0,277988	-0,2	0,77172	-0,04	-0,72198	-0,2
0,144544	-0,25	0,720895	-0,05	-0,9023	-0,25	0,130546	-0,25	0,715912	-0,05	-0,82385	-0,25
---	---	0,665752	-0,06	-0,98529	-0,3	---	---	0,660413	-0,06	-0,88424	-0,3
---	---	0,610538	-0,07	-1,01252	-0,35	---	---	0,605064	-0,07	-0,89341	-0,35
---	---	0,555023	-0,08	-0,9607	-0,4	---	---	0,54965	-0,08	-0,83367	-0,4
---	---	0,498875	-0,09	-0,77686	-0,45	---	---	0,493861	-0,09	-0,66318	-0,45
---	---	0,441594	-0,1	---	---	---	---	0,437231	-0,1	---	---
---	---	0,38237	-0,11	---	---	---	---	0,379023	-0,11	---	---
---	---	0,319768	-0,12	---	---	---	---	0,317943	-0,12	---	---
---	---	0,250821	-0,13	---	---	---	---	0,251383	-0,13	---	---
---	---	0,167209	-0,14	---	---	---	---	0,172373	-0,14	---	---
---	---	---	---	---	---	---	---	0,019666	-0,15	---	---

4 Discussion of the numerical results.

We discuss the numerical results sequentially for plexiglas, steel and plagiogranite involving harmonic, Murnaghan and quadratic types of potentials, respectively.

Plexiglas:

1) Harmonic potential

The value of parameter $\frac{C_{lx1}}{Cl_0}$ on ε_0 for plexiglass, also for steel and plagiogranite, are shown in Figure 1.

The parameter $\frac{C_{Sx2}}{Cs_0}$ for plexiglassdecreases linearly to the value $\frac{C_{Sx2}}{Cs_0} = 0,237$ at $\varepsilon_0 = -0,15$. Then the parameter $\frac{C_{Sx2}}{Cs_0}$ quickly reaches its limit value $\frac{C_{Sx2}}{Cs_0} = 0,1070$ at $\varepsilon_0 = -0,17$. Further this critical value of the compressive parameter of deformation ε_0 waves cannot propagate in material with real values of wave velocities(Fig. 2,5).

Plexiglas shows non-linear increase of pressure parameter $\frac{P_0}{\mu}$ of the compressive parameter of deformation ε_0. Material undergoes sharp growth of $\frac{P_0}{\mu}$ with the value of $\varepsilon_0 = -0,45$. When $\varepsilon_0 = -0,5$ the value of the parameter of pressure is $\frac{P_0}{\mu} = -2,875$(Fig. 3,6).

2) Murnaghan type of potential

Plexiglas, in the case of a potential of the Murnaghan type, shows a slight increase in the velocities of propagation of elastic pressure waves at the initial stage of deformation. Its peak value of $\frac{C_{lx1}}{Cl_0}$ when $\varepsilon_0 = -0,05$ equals: $1,059877619$. After reaching the value of the deformation parameter of $\varepsilon_0 = -0,05$ the parameter of velocity of wave propagation decreases to the critical value: $\frac{C_{lx1}}{Cl_0} = 0,3561$ at $\varepsilon_0 = -0,25$ (Fig. 4).

The value of the velocity of propagation of elastic shear waves $\frac{C_{Sx2}}{Cs_0}$ first increases to $1,0103$ when $\varepsilon_0 = -0,012$. Then, the graphs for Murnaghan and harmonic types of potentials intersect at the point $\varepsilon_0 = -0,11231$ and $\frac{C_{Sx2}}{Cs_0} = 0,47799$. After that, the graph decreases to a critical value $\frac{C_{Sx2}}{Cs_0} = 0,1741$ at $\varepsilon_0 = -0,13$(Fig. 5).

There is a clear extreme value of $\frac{P_0}{\mu}$ which is -5,9457 at $\varepsilon_0 = -0,4$. The value of $\frac{P_0}{\mu}$ at $\varepsilon_0 = -0,45$ equals: $-6,5079$ (Fig.6).

3) Quadratic potential

The parameter of $\frac{C_{lx1}}{Cl_0}$, in the case of a quadratic potential, only smoothly decreases down to the critical value. This value is: $\frac{C_{lx1}}{Cl_0} = 0,2324$ at $\varepsilon_0 = -0,2$ (Fig. 4).

The value of the velocity parameters of the shear waves decreases linearly down to the value of the deformation parameter: $\varepsilon_0 = -0,07$, where$\frac{C_{Sx2}}{Cs_0} = 0,8296$. After which there is a rapid decline in the value of $\frac{C_{Sx2}}{Cs_0}$. The critical value at the same time is: $\frac{C_{Sx2}}{Cs_0} = 0,200043$ at $\varepsilon_0 = -0,09$ (Fig. 5).

In the case of a quadratic potential, there is a small extreme value of $\frac{P_0}{\mu}$, which is -1,60664 at $\varepsilon_0 = -0,35$. We note that the harmonic potential does not have such extreme values (Fig. 6).

All the critical values of the corresponding parameters for the plexiglas are given in the Table 2.

Steel

1) Harmonic potential

The value of the velocity of propagation of parameter of elastic shear waves for steel decreases linearly to the value $\frac{C_{Sx2}}{Cs_0} = 0,2286$ at $\varepsilon_0 = -0,17$. Then there is a rapid decline to the limit value $\frac{C_{Sx2}}{Cs_0} = 0,1070$ at $\varepsilon_0 = -0,22$(Fig. 2,8).

For steel, the increase of the parameter of pressure $\frac{P_0}{\mu}$ at the compressive parameter of deformation ε_0 happens nonlinearly. At the value $\varepsilon_0 = -0,45$,the material undergoes a sharp increase in the parameter of $\frac{P_0}{\mu}$. Then this parameter for the steel reaches $\frac{P_0}{\mu} = -5,584$ at $\varepsilon_0 = -0,5$ (Fig. 3,9).

2) Murnaghan type of potential.

Steel, in the case of a potential of the Murnaghan type, also shows an increase of the parameter of velocity of propagation of elastic pressure waves at the initial stage of deformation. Its peak values of $\frac{C_{lx1}}{Cl_0}$ at $\varepsilon_0 = -0,05$ is: 1,2125. After reaching the value of the parameter of deformation $\varepsilon_0 = -0,05$a decrease in the parameter of velocity of propagation of pressure waves occurs down to the critical value: $\frac{C_{lx1}}{Cl_0} = 0,0717$ at $\varepsilon_0 = -0,3$ (Fig. 7).

The value of $\frac{C_{Sx2}}{Cs_0}$ increasesup to $1,054$ at $\varepsilon_0 = -0,025$ at the initial stage of deformation. Then the graphs for Murnaghan and harmonic types of potentials also intersect. Intersection point is $\varepsilon_0 = -0,118$ at $\frac{C_{Sx2}}{Cs_0} = 0,5529$. Critical value is $\frac{C_{Sx2}}{Cs_0} = 0,1671$ at $\varepsilon_0 = -0,14$ (Fig.8).

In the case of a Murnaghan type potential, the steel has an extremum of the parameter of pressure at $\varepsilon_0 = -0,4$, where $\frac{P_0}{\mu}$ respectively equals: -8,5701. Further, this parameter reaches the value $\frac{P_0}{\mu} = -7,5703$ at $\varepsilon_0 = -0,45$ (Fig.9).

3) Quadratic potential

Steel, in the case of a quadratic potential, monotonously decreases without remarkable features. Limit value of $\frac{C_{lx1}}{Cl_0}$for the steel is reachedat $\varepsilon_0 = -0,25$ and accordingly equal to: $\frac{C_{lx1}}{Cl_0} = 0,1055$ (Fig. 7).

The values of the parameters of velocities of the share waves decreases linearly to the value of the deformation parameter: $\varepsilon_0 = -0,1$. After which the parameter $\frac{C_{Sx2}}{Cs_0}$ decreases down to the critical value. This value is: $\frac{C_{Sx2}}{Cs_0} = 0,08116$ at $\varepsilon_0 = -0,13$(Fig. 8).

For the parameter $\frac{P_0}{\mu}$ there is a small extreme value of $\frac{P_0}{\mu} = -1,0706$ with $\varepsilon_0 = -0,35$, after which the parameter $\frac{P_0}{\mu}$ smoothly reaches to the value: $\varepsilon_0 = -0,45$; $\frac{P_0}{\mu} = -0,7947$ (Fig. 9).

All the critical values of the corresponding parameters for the steel are given in the Table3.

Plagiogranite

1) Harmonic potential

In the case of plagiogranite, the decrease in the parameters of the velocity of propagation of elastic shear waves occurs down to $\frac{C_{Sx2}}{Cs_0} = 0,2996$ at $\varepsilon_0 = -0,2$. Then the rate of decrease of the parameter of velocity of the shear waves increases slightly and reaches the value $\frac{C_{Sx2}}{Cs_0} = 0,0784$ at $\varepsilon_0 = -0,25$, which is critical (Fig. 2,11).

Parameter $\frac{P_0}{\mu}$ non-linearly monotonously decreasing down to $\frac{P_0}{\mu}$=$-4,6603$ at $\varepsilon_0 = -0,5$ (Fig. 3,12).

2) Murnaghan type of potential.

Plagiogranite,in the case of a Murnaghan type potential, shows a smooth decrease in the values of the parameters of the velocity of pressure waves to $\frac{C_{lx1}}{Cl_0} = 0,1445$ at $\varepsilon_0 = -0,25$ (Fig. 10).

The values of parameter of shearwaves for plagiogranite decreases to $\frac{C_{Sx2}}{Cs_0} = 0,3197$ at $\varepsilon_0 = -0,12$. Andthen quickly reaches the critical value $\frac{C_{Sx2}}{Cs_0} = 0,1672$ at $\varepsilon_0 = -0,14$, almost coinciding with the critical value of steel ($\frac{C_{Sx2}}{Cs_0} = 0,1671$ at $\varepsilon_0 = -0,14$)(Fig. 11).

In the case of a potential of Murnaghan type for plagiogranite, there is an extreme value $\frac{P_0}{\mu}$, at $\varepsilon_0 = -0,35$ which is: -1,0125. At $\varepsilon_0 = -0,45$ the parameter of pressure is:$\frac{P_0}{\mu} = -0,7768$ (Fig. 12).

3) Quadratic potential

Just as in the case of the Murnaghan type potential, plagiogranite shows a smooth decrease in the values of the parameters of the velocity of pressure waves to $\frac{C_{lx1}}{Cl_0} = 0,1305$ at $\varepsilon_0 = -0,25$ (Fig. 10).

In the case of a quadratic potential, the values of the parameter of velocity of the shear waves decreases linearly to:$\frac{C_{Sx2}}{Cs_0} = 0,2513$ at $\varepsilon_0 = -0,13$. After that $\frac{C_{Sx2}}{Cs_0}$ decreases rapidly down to critical value, that is: $\frac{C_{Sx2}}{Cs_0} = 0,0196$ at $\varepsilon_0 = -0,15$ (Fig. 11).

In the case of plagiogranite,there is also an extreme value for $\frac{P_0}{\mu} = -0,8934$ at $\varepsilon_0 = -0,35$. At $\varepsilon_0 = -0,45$, the pressure parameter is $\frac{P_0}{\mu} = -0,6631$ (Fig. 12).

All the critical values of the corresponding parameters for the plagioranite are given in the Table 4.

The results for plagiogranite in the cases of application of quadratic and Murnaghan types of potentials practically coincide (Fig. 10,11,12). The differences in their values are shown in the Table 5.

The nature of the dependences of the velocities of propagation of elastic waves on the change in strain for the materials considered within the framework of various types of potentials differs qualitatively and quantitatively among themselves. In the case of a harmonic potential, nonlinear growth of the parameter occurs $\frac{P_0}{\mu}$ (in a negative direction of the axis).

5 Conclusion

The quantitative data of the relationship between the parameters of deformation ε_0 and elastic parameters varies significantly depending on the specific type of the applied elastic potential.

In the case of a harmonic potential, the velocity of propagation of elastic waves also have limiting values for specific values of deformations. Elastic waves cannot propagate in a medium with real values of velocity, when these values of deformations are reached. This is consistent with previous conclusions of other authors, where the data are performed using the quadratic and Murnaghan type potentials. Specific critical values of velocities differ only quantitatively in the case of applying the appropriate type of potential.

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