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Effect of wave generation at critical time combined viscous-elastic ring

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Abstract. *In this paper we consider the problem of loss of stability for a ring as a result of external pressure. The pressure intensity is set. The ring is three-layered, composed of various materials, the ring structure is symmetrical about the middle surface, the material of the ring is viscoelastic. The article studies the critical time of stability with the aid of various geometric theories. In addition, we consider how the number of waves affects this critical time. The possibility of exact solutions of such problems is difficult, since as a result we have to solve a nonlinear boundary value problem with discontinuous coefficients. Therefore, it is necessary to resort to variational methods of solution.*

Keywords. stability · viscoelastic ring · critical time · creep · Rayleigh-Ritz method

Mathematics Subject Classification (2010): 35Q74

1 Derivation of the variational equation

We introduce the ring of radius R and thickness $2h$ in the polar coordinate system $(z,\ \varphi)$. Suppose that it is made up of s alternating connected concentric layers with different values of the modulus of elasticity E_{k+1} and creep functions

$$D_{k+1}\left\{(t-\tau)\ \sigma(\tau)\right\}\left[\,k=0,\ 1,\ldots(s-1)\right].$$

Further, we shall regard them as linear with respect to the voltage σ:

$$D_{k+1}\left\{(t-\tau)\ \sigma(\tau)\right\}=F'_{k+1}(t-\tau)\ \sigma(\tau)\ ,$$

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where $F'_{k+1}(t-\tau)$ is the difference kernel of creep, and the prime denotes differentiation with respect to the parameter $(t-\tau)$ [1]. The thickness of each layer will be denoted by δ_k. Thus, $\delta_1+\delta_2+\ldots+\delta_s=2h$ there is a full thickness of the ring.

In [4], an analogous problem was solved by using a theory in which the nonlinearity of the deflection is taken into account w and the inequality $w/R<<1$ (simple nonlinearity) is assumed to be satisfied.

The following assumptions are based on the geometrically nonlinear theory used here:

a) in the process of deformation, the nonlinearity is simultaneously taken into account both in deflection w and in tangential displacement v (total nonlinearity);

b) neglecting the tangential displacement, we confine ourselves to nonlinearity only of the deflection (partial nonlinearity).

The remaining assumptions of the theory of condensed multilayer rings are listed in [4].

Under the assumed assumptions, we write out the physical relation for the package as a whole in the form of a single equality [5]

$$\varepsilon^{\Phi}=\frac{\sigma}{E_{k+1}}+\int_0^t F'_{k+1}(t-\tau)\ \sigma(\tau)\ d\tau, a_k\le z\le a_{k+1} \tag{1.1}$$

where

$$a_k=-h+\sum_{i=0}^{s}\delta_i\left(\delta_0=0\right). \tag{1.2}$$

Consider the function $F'_{k+1}(t-\tau)$ in an exponential form:

$$F'_{k+1}(t-\tau)=\frac{A_{k+1}}{E_{k+1}}\ e^{-\alpha\,(t-\tau)}. \tag{1.3}$$

Here A_{k+1} are the creep coefficients of the materials of the layers and the indicator α is assumed to be the same for the entire package.

Let us now consider the stability of a selected ring, compressed by a compressive load uniformly distributed over the outer surface $q=const$.

Due to the hypothesis of flat sections, we write

$$\varepsilon=\varepsilon_0+kz.$$

Taking into account the assumption a) the magnitude ε_0 and curvature k are determined from the formulas of the nonlinear theory of thin shells [8]:

$$\varepsilon_0=\frac{1}{R}\left(w+\frac{\partial v}{\partial\varphi}\right)+\frac{1}{2R^2}\left\{\left(\frac{\partial v}{\partial\varphi}+w\right)^2+\left(\frac{\partial w}{\partial\varphi}-v\right)^2\right\},$$

$$\kappa=-\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial\varphi^2}-\frac{\partial v}{\partial\varphi}\right). \tag{1.4}$$

For case b) the corresponding formulas are reduced to the form:

$$\varepsilon_0=\frac{w}{R}+\frac{1}{2R^2}\left\{w^2+\left(\frac{\partial w}{\partial\varphi}\right)^2\right\}, \kappa=-\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial\varphi^2}\right). \tag{1.5}$$

We solve the problem by means of the variational method [2], in which the expressions for the functionals for cases a) and b) have the following form, respectively:

$$\mathrm{K} = R\int\limits_{-h}^{h}\int\limits_{0}^{2\pi}\left\{\dot\sigma\dot\varepsilon + \frac{\sigma}{2R^2}\left[\left(\frac{\partial\dot w}{\partial\varphi} - \dot v\right)^2 + \left(\frac{\partial\dot v}{\partial\varphi} + \dot w\right)^2\right]\right\} d\varphi dz -$$

$$-R\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\int\limits_{a_k}^{a_{k+1}} \dot\varepsilon^{\Phi}\dot\sigma\, dz\, d\varphi + \frac{R}{2}\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\int\limits_{a_k}^{a_{k+1}} \frac{\dot\sigma^2}{E_{k+1}} dz d\varphi, \tag{1.6}$$

$$\mathrm{K} = R\int\limits_{-h}^{h}\int\limits_{0}^{2\pi}\left\{\dot\sigma\dot\varepsilon + \frac{\sigma}{2R^2}\left[\left(\frac{\partial\dot w}{\partial\varphi}\right)^2 + \dot w^2\right]\right\} d\varphi dz -$$

$$-R\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\int\limits_{a_k}^{a_{k+1}} \dot\varepsilon^{\Phi}\dot\sigma\, dz\, d\varphi + \frac{R}{2}\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\int\limits_{a_k}^{a_{k+1}} \frac{\dot\sigma^2}{E_{k+1}} dz d\varphi. \tag{1.7}$$

We differentiate by the parameter t (physical time). From (1.3) we have:

$$F''(t-\tau) = -\alpha\frac{A_{k+1}}{E_{k+1}}e^{-\alpha(t-\tau)}$$

and for $\dot\varepsilon^{\Phi}$ writing

$$\dot\varepsilon^{\Phi} = \frac{1}{E_{k+1}}\left\{\dot\sigma + A_{k+1}\left[\sigma - \alpha\int\limits_{0}^{t} e^{-\alpha(t-\tau)}\sigma(\tau)\, d\tau\right]\right\}. \tag{1.8}$$

Taking into account (1.8) in (1.6) and (1.7), we obtain the following expressions for the functionals:

$$\mathrm{K} = R\int\limits_{-h}^{h}\int\limits_{0}^{2\pi}\left\{\dot\sigma\dot\varepsilon + \frac{\sigma}{2R^2}\left[\left(\frac{\partial\dot w}{\partial\varphi} - \dot v\right)^2 + \left(\frac{\partial\dot v}{\partial\varphi} + \dot w\right)^2\right]\right\} d\varphi\, dz -$$

$$-\frac{R}{2}\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\frac{1}{E_{k+1}}\int\limits_{a_k}^{a_{k+1}} \dot\sigma^2\, dz\, d\varphi - R\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\frac{A_{k+1}}{E_{k+1}}\int\limits_{a_k}^{a_{k+1}} \sigma\dot\sigma\, dz\, d\varphi +$$

$$+\alpha R\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\frac{A_{k+1}}{E_{k+1}}\int\limits_{a_k}^{a_{k+1}} \dot\sigma\left\{\int\limits_{0}^{t} e^{-\alpha(t-\tau)}\sigma(\tau)\, d\tau\right\} dz\, d\varphi \tag{1.9}$$

for case a),

$$\mathrm{K} = R\int\limits_{-h}^{h}\int\limits_{0}^{2\pi}\left\{\dot\sigma\dot\varepsilon + \frac{\sigma}{2R^2}\left[\left(\frac{\partial\dot w}{\partial\varphi}\right)^2 + \dot w^2\right]\right\} d\varphi dz -$$

$$-\frac{R}{2}\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\frac{1}{E_{k+1}}\int\limits_{a_k}^{a_{k+1}} \dot\sigma^2\, dz\, d\varphi - R\int\limits_{0}^{2\pi}\sum_{k=0}^{s-1}\frac{A_{k+1}}{E_{k+1}}\int\limits_{a_k}^{a_{k+1}} \sigma\dot\sigma\, dz\, d\varphi +$$

$$+\alpha R\int_0^{2\pi}\sum_{k=0}^{s-1}\frac{A_{k+1}}{E_{k+1}}\int_{a_k}^{a_{k+1}}\dot{\sigma}\left\{\int_0^t e^{-\alpha(t-\tau)}\sigma(\tau)\,d\tau\right\}dz\,d\varphi \tag{1.10}$$

for case b).

To obtain the final form of the functional, we use the Rayleigh-Ritz method. To this end, as approximating functions, as in [4], we set

$$w = w_0(t) + w_1(t)\cos l\varphi, v = v_0(t)\sin l\varphi, M = m(t)\cos l\varphi, \tag{1.11}$$

or after differentiation with t respect to the velocity, we have:

$$\dot{w} = \dot{w}_0 + \dot{w}_1\cos l\varphi, \dot{v} = \dot{v}_0\sin l\varphi, \dot{M} = \dot{m}\cos l\varphi, \tag{1.12}$$

where the value l takes even values (2,4,6) and characterizes the number of waves in the circumferential direction, in particular, for $l = 2$, the buckling of the ring occurs as a "figure-eight". But $\dot{w}_0$, $\dot{w}_1$, $\dot{v}_0$ and $\dot{m}$ are independent variable parameters. Because of its thinness, the circumferential voltage σ varies linearly according to thickness:

$$\sigma = -\frac{qR}{2h} + \frac{3z}{2h^3}M \text{ or } \dot{\sigma} = \frac{3z}{2h^3}\dot{M}. \tag{1.13}$$

2 The case of total nonlinearity

The subsequent course of the calculations is that the relations (1.11) - (1.13) are substituted into the expression of the functional (1.9), after integration with respect to z and φ, we find it as a function of w_0, w_1, v_0, m and their derivatives with respect to t. Carrying out the appropriate calculations, we get:

$$\begin{gathered}\mathrm{K} = \tfrac{\pi l^2}{R}\dot{w}_1\dot{m} + \tfrac{\pi l}{R}\dot{v}_0\dot{m} - \tfrac{\pi l^2}{2}q\dot{w}_1^2 - \tfrac{\pi q}{2}\dot{v}_0^2 - \tfrac{\pi l^2}{2}q\dot{v}_0^2 - \pi q\dot{w}_0^2 - \tfrac{\pi}{2}q\dot{w}_1^2 - \\ -2\pi lq\dot{w}_1\dot{v}_0 - \tfrac{9\pi R}{8h^6}\eta_2\dot{m}^2 - \tfrac{9\pi R}{4h^6}\gamma_2 m\dot{m} + \alpha\tfrac{9\pi R}{4h^6}\gamma_2\dot{m}\int_0^t e^{-\alpha(t-\tau)}m(\tau)\,d\tau.\end{gathered} \tag{2.1}$$

In (2.1), for brevity of the notation, the following notation is introduced

$$\eta_2 = \sum_{k=0}^{s-1}\frac{1}{E_{k+1}}\int_{a_k}^{a_{k+1}}z^2dz, \gamma_2 = \sum_{k=0}^{s-1}\frac{A_{k+1}}{E_{k+1}}\int_{a_k}^{a_{k+1}}z^2dz. \tag{2.2}$$

The stationarity condition - $\delta\,\mathrm{K} = 0$ is the constructed functional (2.1) corresponding to the equality of the expressions

$$\frac{\partial \mathrm{K}}{\partial \dot{w}_0} = 0, \frac{\partial \mathrm{K}}{\partial \dot{w}_1} = 0, \frac{\partial \mathrm{K}}{\partial \dot{v}_0} = 0 \text{ and } \frac{\partial \mathrm{K}}{\partial \dot{m}} = 0,$$

leads to the following system of three ordinary differential equations

$$\dot{m} = \frac{l^2-1}{l^2}Rq\dot{w}_1 \text{ or } rm = \frac{l^2-1}{l^2}Rqw_1 \text{ (i.e. } m = 0 \text{ when } q = 0),$$

$$\dot{v}_0 = -\frac{1}{l}\dot{w}_1, \frac{l^2}{R}\dot{w}_1 + \frac{\pi}{R}\dot{v}_0 l - \frac{9R}{4h^6}\eta_2\dot{m} -$$

$$-\frac{9R}{4h^6}\gamma_2 m + \alpha\frac{9R}{4h^6}\gamma_2\int_0^t e^{-\alpha(t-\tau)}m(\tau)\,d\tau = 0. \tag{2.3}$$

Combining equations (2.3), we can write

$$\dot{w}_1\left(-\frac{l^2-1}{R}+\frac{9\left(l^2-1\right)R^2}{4l^2h^6}\eta_2 q\right)+\frac{9\left(l^2-1\right)R^2}{4l^2h^6}\gamma_2 q w_1- \\ -\alpha\frac{9\left(l^2-1\right)R^2}{4l^2h^6}\gamma_2 q\int\limits_0^t e^{-\alpha(t-\tau)}w_1(\tau)\,d\tau=0. \tag{2.4}$$

Equation (2.4) must be supplemented by the initial condition

$$w_1(0)=w_1^0.$$

Here, the value w_1^0 represents the deflection value immediately after applying the load q. The consideration of the problem of stability in viscoelasticity is meaningful only if the effective load is less than the critical load. Since the instantaneous deformation is linearly elastic, it is natural to apply the variational principle [3] to determine w_1^0 and q_{cr} it is natural to set the same probable distribution of stress, displacements and angular momentum as in the analysis of viscoelasticity, that is, representing w, v, M and σ by formulas (1.11) and (1.13). Leaving basically the previous notation, in this case we write the corresponding functional in the following form:

$$\mathrm{K}=R\int\limits_{-h}^{h}\int\limits_0^{2\pi}\left\{\dot{\sigma}\dot{\varepsilon}+\frac{\sigma}{2R^2}\left[\left(\frac{\partial\dot{w}}{\partial\varphi}-\dot{v}\right)^2+\left(\frac{\partial\dot{v}}{\partial\varphi}+\dot{w}\right)^2\right]\right\}d\varphi dz-$$

$$-\frac{R}{2}\int\limits_0^{2\pi}\sum_{k=0}^{s-1}\frac{1}{E_{k+1}}\int\limits_{a_k}^{a_{k+1}}\dot{\sigma}^2\,dz\,d\varphi+R\int\limits_0^{2\pi}\dot{w}\,d\varphi.$$

However, here by a point we mean differentiation with respect to q. Calculating this functional and varying it with respect to $\dot{w}_0$, $\dot{w}_1$, $\dot{v}_0$ and $\dot{m}$, finally we arrive at the following differential equation for $\dot{w}_1$

$$\dot{w}_1=w_1\left\{\frac{3R^2\left(l^2-1\right)}{2l^2h^4}\eta_2\right\}\left\{\frac{2h^2\left(l^2-1\right)}{3R}-\frac{3R^2\left(l^2-1\right)}{2l^2h^4}q\eta_2\right\}^{-1}. \tag{2.5}$$

To determine the critical load, the denominator of equation (2.5) must be equated to zero. From here

$$q_{cr}=\frac{4l^2h^6}{9R^3}\eta_2^{-1}, \tag{2.6}$$

and the magnitude of the instantaneous deflection is found by the formula, which can be easily established from the integration of equation (2.5) by the method of separation of variables:

$$w_1^0=w_1^{\vee}\frac{1}{1-\frac{9R^3q}{4l^2h^6}\eta_2}. \tag{2.7}$$

Here, the value $w_1^{\vee}$ is the given amplitude of the initial imperfection of the ring.

We introduce the following dimensionless relations, which make it possible to significantly reduce subsequent entries

$$y=\frac{w_1}{h},\ \omega=\frac{q}{q_{cr}}=\frac{\omega_{t/n}}{l^2},$$

then

$$\omega_{t/n} = q\left\{\frac{4h^6}{9R^3}\eta_2^{-1}\right\}^{-1},$$

but q_{cr} is determined by formula (2.6).

Such dimensionization ω aims to write down the problem further in a form that depends explicitly on l. As for ω, then, as already noted above, for it we have an inequality $\omega < 1$, from which it follows that $\omega_{t/n} < l^2$. Now, equation (2.4) and the initial condition (2.7) will look like this:

$$\dot{y} - \frac{\omega_{t/n.}}{l^2 - \omega_{t/n}}\frac{\gamma_2}{\eta_2}\left\{y - \alpha\int_0^t e^{-\alpha(t-\tau)}y(\tau)\, d\tau\right\} = 0, \tag{2.8}$$

$$y_0 = y^{\vee}\frac{1}{1 - \frac{\omega_{t/n.}}{l^2}}, \tag{2.9}$$

where

$$y^{\vee} = \frac{w_1^{\vee}}{h}.$$

Taking into account the differentiation formula under the integral sign, we have

$$\left\{\int_0^t e^{-\alpha(t-\tau)}y(\tau)\, d\tau\right\}^{\bullet} = y - \alpha\int_0^t e^{-\alpha(t-\tau)}y(\tau)\, d\tau,$$

which reduces equation (2.8) to the form

$$\left\{y(t) - \frac{\omega_{t/n}}{l^2 - \omega_{t/n}}\frac{\gamma_2}{\eta_2}\int_0^t e^{-\alpha(t-\tau)}y(\tau)\, d\tau\right\}^{\bullet} = 0.$$

Integrating the expression in curly brackets, we get:

$$y(t) - \frac{\omega_{t/n}}{l^2 - \omega_{t/n}}\frac{\gamma_2}{\eta_2}\int_0^t e^{-\alpha(t-\tau)}y(\tau)\, d\tau = C. \tag{2.10}$$

We define a constant C using condition (2.9). When $t = 0$

$$C = y(0) = y_0,$$

exactly

$$C = y^{\vee}\frac{1}{1 - \frac{\omega_{t/n}}{l^2}}.$$

Then, if we denote by β a combination

$$\beta = \frac{\omega_{t/n}}{l^2 - \omega_{t/n}}\frac{\gamma_2}{\eta_2},$$

from (2.10) we have:

$$y(t) - \beta\int_0^t e^{-\alpha(t-\tau)}y(\tau)\, d\tau = y_0. \tag{2.11}$$

From this, using the substitution $-\lambda = \beta - \alpha$, we reduce equation (2.11) to the form

$$y\,(t) - \beta \int\limits_0^t e^{-(\lambda+\beta)(t-\tau)} y\,(\tau)\ d\tau = y_0. \tag{2.12}$$

Now we can immediately write the solution of the integral equation (2.12) [7]

$$y = y_0 \left\{ 1 + \beta \int\limits_0^t e^{-\lambda(t-\tau)} d\tau \right\}. \tag{2.13}$$

Calculating the integral appearing in (2.13), we obtain

$$y\,(t) = y_0 \left\{ \left(1 - \frac{\alpha}{\lambda}\right) e^{-\lambda t} + \frac{\alpha}{\lambda} \right\}. \tag{2.14}$$

According to the formula (2.14) we have that, depending on the sign λ, various solutions are possible. If $\lambda \leq 0$, then there is an unlimited growth of the deflection in time. When $\lambda < 0$ it is exponential, and for $\lambda = 0$ - linear, and

$$y = y_0\,(\,1 + \alpha\, t)\,,$$

as can easily be verified by applying the Lopital rule to (2.8). If $\lambda > 0$, then, there is a limited increase in deflection. Its limit value is determined by the value $y_* = y_0 \frac{\alpha}{\lambda}$, where, by definition $\alpha / \lambda > 1$.

3 The case of nonlinearity only of deflection

Neglecting the tangential displacement, we confine ourselves to a nonlinearity only of the deflection. The corresponding relations (1.11) - (1.13) are substituted into the expression of the functional (1.10) and after a certain procedure we have:

$$\begin{gathered} \mathrm{K} = \tfrac{\pi l^2}{R} \dot{w}_1 \dot{m} - \tfrac{\pi l^2}{2} q \dot{w}_1^2 - \pi q \dot{w}_0^2 - \tfrac{\pi}{2} q \dot{w}_1^2 - \tfrac{9\pi R}{8h^6} \eta_2 \dot{m}^2 - \\ - \tfrac{9\pi R}{4h^6} \gamma_2 m \dot{m} + \alpha \tfrac{9\pi R}{4h^6} \gamma_2 \dot{m} \int\limits_0^t e^{-\alpha(t-\tau)} m\,(\tau)\, d\tau. \end{gathered} \tag{3.1}$$

Here η_2 and γ_2 are determined by formula (2.2).

As a result of calculations performed similarly to the previous ones, we get:

$$\begin{gathered} \dot{w}_1 \left(-\tfrac{l^2}{R} + \tfrac{9\left(l^2+1\right)R^2}{4l^2h^6} \eta_2\, q \right) + \tfrac{9\left(l^2+1\right)R^2}{4l^2h^6} \gamma_2 q w_1 - \\ - \alpha \tfrac{9\left(l^2+1\right)R^2}{4l^2h^6} \gamma_2\, q \int\limits_0^t e^{-\alpha(t-\tau)} w_1\,(\tau)\ d\tau = 0. \end{gathered} \tag{3.2}$$

Here, the value q_{cr} of the critical force for the case of non-linearity of the deflection, which is determined by the formula:

$$q_{cr} = \frac{16 l^2 h^6}{45 R^3} \eta_2^{-1}, \tag{3.3}$$

and the magnitude of the instantaneous deflection is found from equation

$$w_1^0 = w_1^{\vee} \frac{1}{1 - \frac{45 R^3 q}{16 l^2 h^6} \eta_2}\ . \tag{3.4}$$

We note that the relations (3.3) and (3.4) were obtained in the same way as in paragraph 2 for the nonlinear theory adopted here.

We leave the former dimensionless relations

$$y = \frac{w_1}{h},\ \omega = \frac{q}{q_{cr}} = \frac{\omega_{p/n}}{l^2},$$

then

$$\omega_{p/n} = q\left\{\frac{16h^6}{45R^3}\eta_2^{-1}\right\}^{-1},$$

but q_{cr} is determined by formula (3.3), equation (3.2) and the initial condition (3.4) can be rewritten as follows:

$$\dot{y} - \frac{\omega_{p/n.}}{\frac{5l^4}{4(l^2+1)} - \omega_{p/n}}\frac{\gamma_2}{\eta_2}\left\{y - \alpha\int\limits_0^t e^{-\alpha(t-\tau)}y(\tau)\,d\tau\right\} = 0, \tag{3.5}$$

$$y_0 = y^{\vee}\frac{1}{1-\frac{\omega_{p/n}}{l^2}}, \tag{3.6}$$

where $y^{\vee} = \frac{w_1^{\vee}}{h}$.

Now it is not difficult to write down the solution (3.5), which has the form:

$$y(t) = y_0\left\{\left(1-\frac{\alpha}{\lambda}\right)e^{-\lambda t} + \frac{\alpha}{\lambda}\right\}. \tag{3.7}$$

Here $\beta - \alpha = -\lambda$, but β is determined from equality

$$\beta = \frac{\omega_{p/n}}{\frac{5l^4}{4(l^2+1)} - \omega_{p/n}}\frac{\gamma_2}{\eta_2}.$$

4 Numerical calculation and conclusions

The purely visual identity of the solutions (2.14) and (3.7) obtained is quite understandable by the conducted dimensionlessness, since in both cases the critical force is chosen from the solution of the corresponding linearly elastic problem. Therefore, here the question should be formulated as follows: specify values that correspond to the same value q. Obviously, then the numerical values of instantaneous deflections will be different.

Taking $\omega_{s/n} = 3$, as was borrowed from [4], where the case of geometric nonlinearity is investigated, from the preceding arguments we have the following chain of equalities:

$$\omega_{s/n}q_{cr}^{(1)} = \omega_{p/n}q_{cr}^{(2)} = \omega_{t/n}q_{cr}^{(3)},$$

where in

$$q_{cr}^{(1)} = \frac{l^2h^6}{9R^3}\eta_2^{-1},\ q_{cr}^{(2)} = \frac{16l^2h^6}{45R^3}\eta_2^{-1},\ q_{cr}^{(3)} = \frac{4l^2h^6}{9R^3}\eta_2^{-1}.$$

Here the upper indices correspond to different theories of nonlinearity, namely: (1.1) - simple nonlinearity, (1.2) - partial nonlinearity, (1.3) - total nonlinearity. Hence we have:

$$\omega_{p/n} \approx 0,94\,, \qquad \omega_{t/n} \approx 0,75.$$

The solutions (2.14) and (3.7) obtained above are, in principle, applicable to arbitrary values t. However, very large deflections in rings, which are elements of structures, are inadmissible in themselves. Therefore, it is very reasonable to limit the time of operation of the ring by the condition that a deflection of a certain value fixed from certain physically grounded considerations is reached and thereby determine the critical stability time t_{cr}. Let us $\tilde{y} = 1$, assume that it corresponds to a dimensionless deflection equal to half the thickness. According to the noted, we will write down

$$1 = y_0 \left\{ \left(1 - \frac{\alpha}{\lambda}\right) e^{-\lambda\, t_{cr}} + \frac{\alpha}{\lambda} \right\},$$

where we find

$$t_{cr} = -\frac{1}{\lambda} \ln \left| \frac{\lambda - \alpha\, y_0}{\lambda\, y_0 \left(1 - \frac{\alpha}{\lambda}\right)} \right|. \tag{4.1}$$

We confine ourselves to the case $\lambda < 0$ and give some results of calculations corresponding to different values of the physical and geometric parameters characterizing the ring.

We take $y^{\vee} = 10^{-1}$ and give the numerical values of the instantaneous deflection (y_0) corresponding to the same compressive force q, depending on the number of waveforms l (Table 1). Such a choice of values ω always ensures the condition for the fulfillment of the inequality $\omega \big/ l^2 < 1$ and is suitable for any number of waveforms l.

The ring is represented by a three-layer ($s = 3$) and having the following periodic structure

$$E_1 = E_3, \qquad A_1 = A_3, \qquad \delta_1 = \delta_3.$$

We introduce additional dimensionless notation

$$E = \frac{E_1}{E_2}, \qquad A = \frac{A_2}{A_1} \text{ and } \xi = \frac{\delta_2}{\delta_1}.$$

Proceeding from this, according to formula (1.2), for a_k we have:

$$a_0 = -h, a_1 = -\frac{\delta_2}{2}, a_2 = \frac{\delta_2}{2}, a_3 = h.$$

The above allows us to determine the ratio γ_2/η_2 that appears in the formulas for β and is written in the form

$$\frac{\gamma_2}{\eta_2} = A_1 \frac{1 + 1,5\xi + 0,75\xi^2 + 0,125EA\xi^3}{1 + 1,5\xi + 0,75\xi^2 + 0,125E\xi^3}.$$

Now give the value for the parameters:

$$A_1 = 0,8 \ \sec^{-1}; \ \ E = 0,25; \ \ \xi = 4; \ \ A = 0,25; \ \ \alpha = 0,005.$$

We represent the results of calculations performed on the basis of the dependence (4.1) obtained above (Table 1). Here it is important to indicate that in calculating the parameters of the ring were chosen in such a way that the condition $\lambda < 0$ was always preserved. As for the values α, E and A, then they are borrowed from [6].

Table 1. Numerical values of instantaneous deflection and critical time

ω	$\omega_{s/n} = 3$	$\omega_{p/n} = 0,94$	$\omega_{t/n} = 0,75$
	$l = 2$		
y_0	0,4	0,13	0,12
t_{cr}	5,3995	9,0546	12,5835
	$l = 4$		
y_0	0,123	0,106	0,105
t_{cr}	62,3997	62,3485	67,3284
	$l = 6$		
y_0	0,109	0,103	0,102
t_{cr}	174,4774	172,6963	180,3343

The values of the critical time for the homogeneous case ($E = A = \xi = 1$) for $\alpha = 0,005$ follow from (4.1):

at $l = 2$

$$t_{cr}^{(1)} = 5,01 \text{ sec}, t_{cr}^{(2)} = 8,38 \text{ sec}, t_{cr}^{(3)} = 11,53 \text{ sec},$$

at $l = 4$

$$t_{cr}^{(1)} = 57,56 \text{ sec}, t_{cr}^{(2)} = 45,32 \text{ sec}, t_{cr}^{(3)} = 48,85 \text{ sec},$$

at $l = 6$

$$t_{cr}^{(1)} = 159,06 \text{ sec}, t_{cr}^{(2)} = 120,68 \text{ sec}, t_{cr}^{(3)} = 125,69 \text{ sec} .$$

5 Conclusion

Thus, the numerical calculation allows us to conclude:

- taking into account the total nonlinearity leads to a significant increase in the critical time, from which it follows that, other things being equal, this leads to the possibility of more rational use of the bearing capacity of the ring;
- the critical time in various nonlinearity theories is calculated and comparisons of the wave formation numbers within each theory are made;
- the increase in the number of waveforms has a strong influence on the value of the critical time.

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