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Free vibrations of retaining walls consisting of orthotropic cylindrical shells

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Abstract. *In the present paper, we study free vibrations of retailing walls consisting of two orthotropic cylindrical shells dynamically contacting with soil are studied, resonance frequencies are found, and characteristic curves are built.*

Keywords. structural elements, resonance frequencies, free vibrations, retailing walls, cylindrical shells, curves are built, isotropic materials.

Mathematics Subject Classification (2010): 74H45, 74A05

1 Introduction.

The problems related to wide spread of thin-shelled cylindrical form constructions or structural elements in mechanical engineering, transmission systems, in construction spheres, investigation of their dynamic rigidity characteristics and choice of optimal variant of these constructions keep their urgency even today. The thin-shelled spatial constructions formed of junction of cylindrical shells compose the base of retaining walls and are accepted their effective form. Substituting the massive reinforced concrete retaining walls by thin-walled shells, one gets an empty-space system, for increasing its stability this space is filled with soil, and concrete is greatly saved.

Taking into account seismic activity in Azerbaijan, it is of great practical importance to study free vibrations and find resonance frequencies of such constructions with taking into account the influence of dynamic forces.

In the present paper, we study free vibrations of retailing walls consisting of two orthotropic cylindrical shells dynamically contacting with soil are studied, resonance frequencies are found, and characteristic curves are built. Let's note, that the analysis of retaining walls consisting of thin-walled spatial constructions formed of junction of cylindrical shells made of isotropic materials was performed for static case [3-5].

In [3], static deformations of retaining walls consisting of thin-walled partial constructions formed of junction of cylindrical shells made of isotropic materials, are studied. The method suggested in [6] was used in solving the problem. The analysis of retaining walls

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consisting of cylindrical shell made of three isotropic materials in the plane deformation state, was given in [4]. Solution of the problem was reduced to the solution of ordinary differential equations and the solution was structured. The paper [5] was devoted to development of the method for calculating the conditions of junction of well and cylindrical shells made of isotropic material with regard to the work of soil at compression and sliding. Analysis and investigations were carried out on the base of momentum theory of cylindrical shells. In [7-10] the problems of junction of concave shells and contour constructions were solved.

Having constructed the solutions of differential equations of momentum theory of concave shells within arbitrary boundary conditions, different junction problems were solved.

2 Problem Statement.

Let us write potential and kinetic energies of cylindrical shells [2]:

$$G_i = \frac{h_i R_i}{2} \iint\limits_{s_i} \left\{ b_{11i} \left(\frac{\partial u_i}{\partial x} \right)^2 - 2\left(b_{11i} + b_{12i} \right) \frac{w_i}{R} \frac{\partial u_i}{\partial x} \right.$$

$$+ \frac{w_i^2}{R^2} \left(b_{11i} + 2b_{12i} + b_{22i} \right) + \frac{b_{22i}}{R^2} \left(\frac{\partial \vartheta_i}{\partial \theta} \right)^2 - 2\left(b_{12i} + b_{22i} \right) \frac{w_i}{R^2} \frac{\partial \vartheta_i}{\partial \theta} + 2b_{12i} \frac{1}{R^2} \frac{\partial u_i}{\partial x} \frac{\partial \vartheta_i}{\partial \theta}$$

$$\left. + b_{66i} \frac{1}{R^2} \left(\frac{\partial u_i}{\partial \theta} \right)^2 + b_{66i} \left(\frac{\partial \vartheta_i}{\partial x} \right)^2 + b_{66i} \frac{1}{R} \frac{\partial \vartheta_i}{\partial x} \frac{\partial u_i}{\partial \theta} \right\} dx d\theta \tag{2.1}$$

$$K_i = \frac{E_i h_i}{2R_i^2 \left(1 - \nu_i^2 \right)} \iint\limits_{s_i} \left[\left(\frac{\partial u_i}{\partial t} \right)^2 + \left(\frac{\partial \vartheta_i}{\partial t} \right)^2 + \left(\frac{\partial w_i}{\partial t} \right)^2 \right] dx dy.$$

Here $i = 1$ corresponds to the first cylindrical shell forming the retaining walls, $i = 2$ to the second cylindrical shell forming the retaining walls (Fig. 1); u_i, ϑ_i, w_i are arrangement of shells, R_i, h_i are radii and thickness of cylindrical shells, respectively, $b_{11}, b_{22}, b_{12}, b_{66}$ are the main elasticity module of orthotropic material, the elasticity module E_{1i}, E_{2i} in the direction of coordinate axes are expressed by the Poisson ratio ν_{1i}, ν_{2i} as follows: $b_{11i} = \frac{E_{1i}}{1-\nu_{1i}\nu_{2i}}$; $b_{22i} = \frac{E_{2i}}{1-\nu_{1i}\nu_{2i}}$; $b_{12i} = \frac{\nu_{2i}E_{1i}}{1-\nu_{1i}\nu_{2i}} = \frac{\nu_{1i}E_{2i}}{1-\nu_{1i}\nu_{2i}}$; s_i is the surface of cylindrical shells forming the retaining walls.

The influence of soil on cylindrical shells are substituted by external forces q_{xi}, q_{yi}, q_{zi}. The work done by these forces in substitution of the points of the shell is

$$A_i = -\int\limits_0^{x_1} \int\limits_0^{2\pi} \left(q_{xi} u_i + q_{yi} \vartheta_i + q_{zi} w_i \right) dx dy. \tag{2.2}$$

As a result, the total energy is as follows:

$$\mathit{\Pi} = \sum_{i=1}^{2} \left(G_i + K_i + A_i \right). \tag{2.3}$$

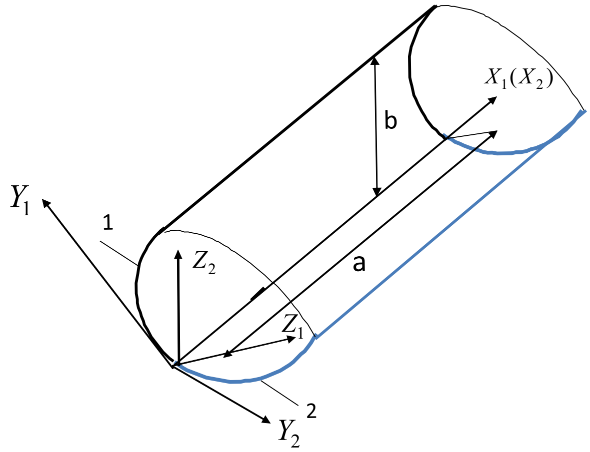

Fig. 1. The scheme of the retaining wall formed by the junction of orthotropic cylindrical shells

To expressions (2.1) and (2.2) we add the contact and boundary conditions. Assume that cylindrical wells are elastically joined, i.e. in the contact, the conditions

$$w_1(x)|_{y1=0} = \vartheta_2(x)|_{y2=0}; \ \vartheta_1(x)|_{y1=0} = w_2(x)|_{y2=0};$$
$$u_1(x)|_{y1=0} = u_2(x)|_{y2=0}; \ \left.\frac{\partial w_1(x)}{\partial x}\right|_{y1=0} = \left.\frac{\partial \vartheta_2(x)}{\partial x}\right|_{y2=0} \tag{2.4}$$

are satisfied.

It is accepted that the cylindrical shells are hingely supported on ideal diaphragms along the lines $x = 0$ and $x = a$, in this case the boundary conditions are expressed as follows:

$$u = 0, w = 0, \ T_1 = 0, \ M_1 = 0. \tag{2.5}$$

Here T_1, M_1 is a force and moment acting on the cross section of the cylindrical shell.

Using the stationary condition of Ostrogradskiy-Hamlilton action, we can determine frequency of natural vibrations of retaining walls formed of junction of cylindrical shells and get a frequency equation

$$\delta W = 0. \tag{2.6}$$

Here $W = \int_{t_0}^{t_1} \varPi dt$ is the Hamilton action. If we perform variation operation in the equality $\delta W = 0$ and take into account that the variations $\delta u, \delta v, \delta w$ are arbitrary, independent, for finding natural frequencies of retaining walls formed of the junction of cylindrical shells dynamically contacting with soil, we get a frequency equation. Thus, the solution of the problem of vibrations of retaining walls formed of the junction of cylindrical shells dynamically contacting with soil, is reduced to joint integration of total energy (2.3) of the construction within the contact (2.4) and boundary conditions (2.5).

We take the influence of soil contained in expression (2.2) on cylindrical shells, the external forces q_{xi}, q_{yi}, q_{zi} in the form:

$$q_{xi} = q_{yi} = 0; \ q_{z1} = k_1 w_1; \ q_{z2} = k_2 w_2 - k_s \left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right). \tag{2.7}$$

Here k_1, k_2, k_s are rigidity coefficients of soils at compression and sliding.

We look for the displacements of the cylindrical shell in the following way:

$$\begin{aligned} u_i &= u_{0i}\cos\chi\xi\,(\cos n\theta_i + \sin n\theta_i)\sin\omega_1 t_1 \\ \vartheta_i &= \vartheta_{0i}\sin\chi\xi\,(\cos n\theta_i + \sin n\theta_i)\sin\omega_1 t_1 \\ w_i &= w_{0i}\sin\chi\xi\,(\cos n\theta_i + \sin n\theta_i)\sin\omega_1 t_1. \end{aligned} \tag{2.8}$$

Here $u_{0i}\vartheta_{0i}$, w_{0i} are unknown constants, $\xi = \frac{x}{a}$, $t_1 = \omega_{0i}t$, $\chi, n = 2k+1$ are wave numbers of cylindrical shell in the direction of generator and circumferential directions, $\theta_i = \frac{y_i}{R}$, $0 \le \theta_i \le \frac{3\pi}{4}$. When these conditions are satisfied, the hinge-support conditions in the boundary $\theta_i = \frac{3\pi}{4}$ will be satisfied.

Substituting the solution (2.8) in (2.3), taking into account contact conditions (2.4) and expressions (2.7), expressing the constrants $u_{02}, \vartheta_{02}, w_{02}$ by the constants $u_{01}, \vartheta_{01}, w_{01}$, for the total energy (2.3) with respect to the constants $u_{01}, \vartheta_{01}, w_{01}$ we get the second order polynomial:

$$\Pi = \breve{\varphi}_{11}u_{01}^2 + \breve{\varphi}_{22}\vartheta_{01}^2 + \breve{\varphi}_{33}w_{01}^2 + \breve{\varphi}_{44}u_{01}\vartheta_{01} + \breve{\varphi}_{55}u_{01}w_{01} + \breve{\varphi}_{66}\vartheta_{01}w_{01}.$$

As the expressions for the coefficients $\breve{\varphi}_{11}, \breve{\varphi}_{22}, \breve{\varphi}_{33}, \breve{\varphi}_{44}, \breve{\varphi}_{55}, \breve{\varphi}_{66}$ are bulky, we do not give them here.

If we variate the expression Π with respect to the independent coefficients u_0, ϑ_0, w_0 and equate to zero the coefficients of independent variations, we get the following system of homogeneous algebraic equations

$$\begin{cases} 2\breve{\varphi}_{11}u_{01} + \breve{\varphi}_{44}\vartheta_{01} + \breve{\varphi}_{55}w_{01} = 0 \\ \breve{\varphi}_{44}u_{01} + 2\breve{\varphi}_{22}\vartheta_{01} + \breve{\varphi}_{66}w_{01} = 0 \\ \breve{\varphi}_{55}u_{01} + \breve{\varphi}_{66}\vartheta_{01} + 2\breve{\varphi}_{33}w_{01} = 0 \end{cases} \tag{2.9}$$

As the system (2.9) is a system of homogeneous algebraic equations, the necessary and sufficient condition for the existence of its nonzero solution is equality of its principal determinant to zero. As a result, we get the following frequency equation:

$$\begin{vmatrix} 2\breve{\varphi}_{11} & \breve{\varphi}_{44} & \breve{\varphi}_{55} \\ \breve{\varphi}_{44} & 2\breve{\varphi}_{22} & \breve{\varphi}_{66} \\ \breve{\varphi}_{55} & \breve{\varphi}_{66} & 2\breve{\varphi}_{33} \end{vmatrix} = 0. \tag{2.10}$$

We write the equation (2.10) in the following form:

$$4\breve{\varphi}_{11}\breve{\varphi}_{22}\breve{\varphi}_{33} + \breve{\varphi}_{44}\breve{\varphi}_{55}\breve{\varphi}_{66} - \breve{\varphi}_{55}^2\breve{\varphi}_{22} - \breve{\varphi}_{66}^2\breve{\varphi}_{11} - \breve{\varphi}_{44}^2\breve{\varphi}_{33} = 0. \tag{2.11}$$

The equation (2.10) was calculated by the numerical method. The parameters contained in the solution of the problem were taken as

$$k_1 = k_2 = 7 \cdot 10^8 N/m^2, k_s = 11 \cdot 10^6 N/m^2, \frac{a}{R} = 3, \frac{h}{R} = \frac{1}{6}, \nu_1 = \nu_2 = 0,35.$$

The result of the calculations is given in Fig. 2 in the form of dependence of frequency parameter on θ_1, in figure 3 in the form of dependence on the ratio a/R. As seen from figure 2, as the angle θ_1 increases, the value of the frequency parameter increases. As the length of cylindrical shells increases, as seen from Fig. 3, the value of the frequency parameter decreases. The values of the frequency parameter increases according to strengthening of orthotropic properties of the cylindrical shell.

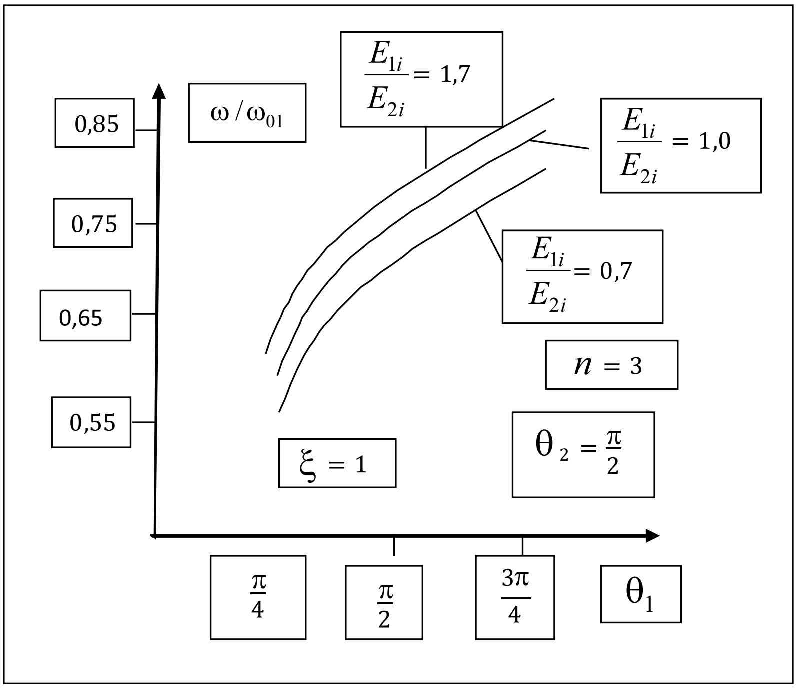

Fig. 2. Dependence of frequency parameter on θ_1.

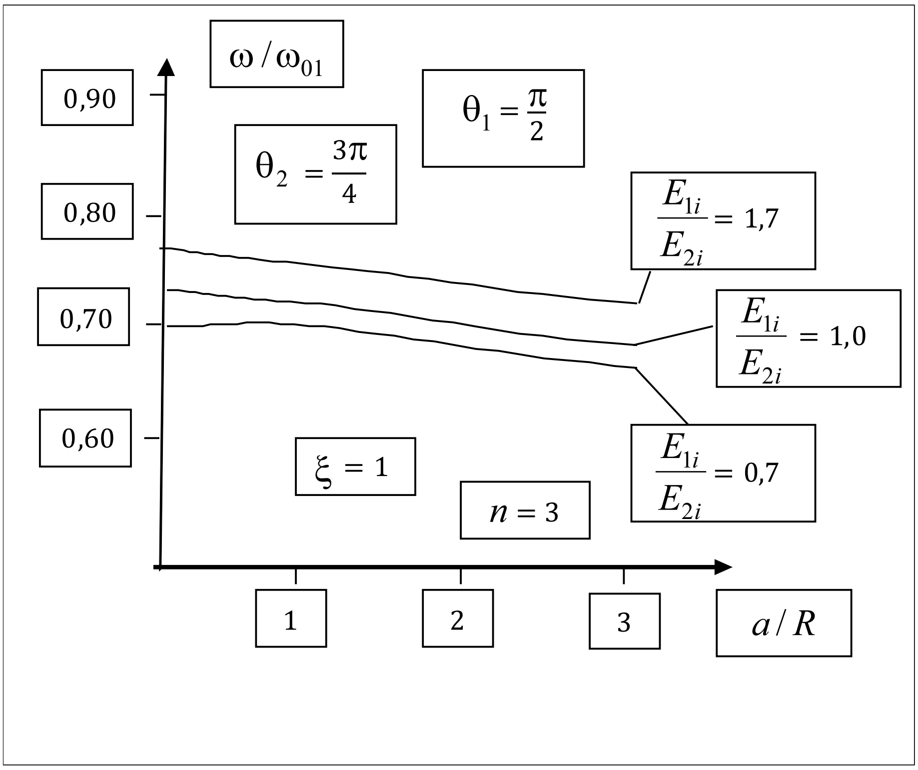

Fig. 3. Dependence of frequency parameter on the ratio a/R.

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